



Useful Terminology for Understanding Differential Equations

Notation for Derivatives:

The most common notation methods are Lagrange notation (aka prime notation), Newton notation (aka dot notation), and Leibniz's notation (aka dy/dx notation).

Ex 1: Lagrange Notation: $y''(x) = 0$

Newton Notation: $\ddot{y} = 0$

Leibniz Notation: $\frac{d^2-y}{dx^2} = 0$

The example above shows three different ways to write the second derivative of y is equal to zero. Note that Leibniz notation is the notation used for the rest of the reference sheet.

Independent Variable:

The variable in an equation that can be freely chosen and does not depend on another variable.

Dependent Variable

The variable that depends on the value of at least one independent variable.

Ex 2: $\frac{dy}{dx} = x + 2$

The variable y is the dependent variable. Variable x is the independent variable. y is a function of x and can be denoted $y = y(x)$. Note how y is in the numerator and x is in the denominator of the derivative.

Differential Equation:

An equation that contains an unknown function and its derivatives.

Ex 3: $\frac{dy}{dx} + y = 0$

The example contains the dependent variable y and its derivative. Again, remember that y is a function of x and can be denoted $y = y(x)$.

Ordinary Differential Equation (ODE):

A differential equation that is written in terms of one independent variable.

$$\text{Ex 4: } \frac{d^2y}{dx^2} + \frac{dy}{dx} + y = x$$

The example above is written in terms of independent variable x , where y is a function of x . All examples given so far have been ODEs.

Partial Differential Equation (PDE):

In contrast to an ODE, a partial differential equation is a differential equation written in terms of more than one independent variable.

$$\text{Ex 5: } \frac{dy}{dx} + \frac{dy}{dv} = x$$
$$y = f(x, v)$$

The example above is written in terms of independent variables x and v . The dependent variable is y , where y is a function of both x and v .

Order:

The value of the highest derivative of an ODE. If given a system of equations, the order of the system is the sum of the order of each equation.

$$\text{Ex 6: } \frac{d^2y}{dx^2} = x$$

The highest derivative in the example is two. Therefore, it is a second-order equation. Examples 1 and 4 also show second order equations. Examples 2,3 and 5 are first order equations. The term 'Higher order' refers to an order of three or more.

Separable:

When the variables of an ODE can be rearranged on to opposite sides of the equal sign.

$$\text{Ex 7: } \frac{dy}{dx} = x + xy$$

$$\frac{dy}{dx} = x(1 + y)$$

$$\frac{dy}{(1+y)} = x dx$$

The example ODE equation was first factored into x and (1+y). The term (1+y) was divided to the left-hand side. The x variable was multiplied to the right-hand side. Separating equations in this way allows for easy integration.

Linear:

An equation that forms a line when plotted. The dependent variable should always be to a power of 1 and should not be multiplied by another dependent variable term.

Ex 8: $a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = 0$

Ex 9: $a \frac{d^2y}{dx^2} \frac{dy}{dx} + b \frac{dy}{dx} + cy^2 = 0$

Example 8 is the form of a second-order linear equation with coefficients a, b, and c. Example 9 is a non-linear second-order equation with the same coefficients. Note why the equations are different.

Homogeneous:

A linear equation that is equal to zero when only the dependent variable terms are on the left-hand side of the equal sign.

Ex 10: $\frac{dy}{dx} + y = 0$

The example above is homogenous. Examples 1,3, and 8 are also homogeneous. Examples 2,4-7, and 9 are not homogenous.

For more information on useful terminology for understanding differential equations, make an appointment with a [content tutor](#). Appointments are available in person at the [Student Success Center](#) or online.