

Linear Algebra Review

Matrix Properties

Matrix Addition

If $A = [a_{ij}]$ and $B = [b_{ij}]$ are matrices of size $m \times n$, then their sum is the $m \times n$ matrix given by $A + B = [a_{ij} + b_{ij}]$. The sum of two different size matrices is undefined.

Matrix Scalar Multiplication

If $A = [a_{ij}]$ is an $m \times n$ matrix and c is a scalar, then the multiple scalar of A by c is the $m \times n$ matrix given by $cA = [ca_{ij}]$.

Matrix Multiplication

If $A = [a_{ij}]$ is an $m \times n$ and $B = [b_{ij}]$ is an $n \times p$ matrix, then the product of AB is an $m \times p$ matrix $AB = [c_{ij}]$ where $c_{ij} = \sum_{k=1}^n a_{ik}b_{kj}$.

Properties of Matrix Addition and Scalar Multiplication

- $A + B = B + A$
- $A + (B + C) = (A + B) + C$
- $(cd)A = c(dA)$
- $1A = A$
- $c(A + B) = cA + cB$
- $(c + d)A = cA + dA$

Properties of Matrix Multiplication

- $A(BC) = (AB)C$
- $A(B + C) = AB + AC$
- $(A + B)C = AC + BC$
- $c(AB) = (cA)B = A(cB)$

Properties of Transposes

- $(A^T)^T = A$
- $(A + B)^T = A^T + B^T$

- $(cA)^T = c(A^T)$
- $(AB)^T = B^T A^T$

Inverse of a Matrix

An $n \times m$ matrix A is invertible (or nonsingular) when there exists an $n \times n$ matrix B such that $AB = BA = I_n$ where I_n is the identity matrix. B is called the inverse of A. A matrix that does not have an inverse is noninvertible (or singular).

Properties of Inverse Matrices

If A is an invertible matrix, k is a positive integer, and c is a nonzero scalar, then A^{-1} , A^k , cA , and A^T are invertible and the following are true:

- $(A^{-1})^{-1} = A$
- $(A^k)^{-1} = (A^{-1})^k$
- $(cA)^{-1} = \frac{1}{c}A^{-1}$
- $(A^T)^{-1} = (A^{-1})^T$

Inverse of a Product

If A and B are invertible matrices of order n, then AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$.

Systems of Equations with Unique Solutions

If A is an invertible matrix, then the system of linear equations $Ax = b$ has a unique solution given by $x = A^{-1}b$.

Properties of Determinants

Determinant of a Triangular Matrix

If A is a triangular matrix of order n, then its determinant is the product of the entries on the main diagonal. That is, $\det(A) = |A| = a_{11}a_{22} \dots a_{nn}$.

Conditions that Yield a Zero Determinant

If A is a square matrix and any one of the following conditions are true, then $\det(A) = 0$.

- An entire row or column consists of zeroes.
- Two rows or columns are equal.
- One row or column is a multiple of another row or column.

Determinant of Matrix Multiplication

If A and B are square matrices of order n, then $\det(AB) = \det(A)\det(B)$.

Determinant of a Scalar Multiple of a Matrix

If A is a square matrix of order n and c is a scalar, then the determinant of cA is

$$\det(cA) = c^n \det(A).$$

Determinant of an Invertible Matrix

A square matrix is invertible if and only if $\det(A) \neq 0$.

Determinant of an Inverse Matrix

If A is an $n \times n$ invertible matrix, then $\det(A^{-1}) = \frac{1}{\det(A)}$.

Equivalent Conditions for a Nonsingular Matrix

If A is an $n \times n$ matrix, then the following statements are equivalent:

- A is invertible.
- $Ax = b$ has a unique solution for every $n \times 1$ column matrix b .
- $Ax = 0$ has only the trivial solution.
- A is row-equivalent to I_n .
- A can be written as the product of elementary matrices.
- $\det(A) \neq 0$.

Inverse of a Matrix Given Its Adjoint

If A is an $n \times n$ matrix, then $A^{-1} = \frac{1}{\det(A)} \text{adj}(A)$.

Cramer's Rule

If a system of n linear equations in n variables has a coefficient matrix A with a nonzero

determinant, then the solution of the system is $x_1 = \frac{\det(A_1)}{\det(A)}, x_2 = \frac{\det(A_2)}{\det(A)}, \dots, x_n = \frac{\det(A_n)}{\det(A)}$ where the i th column of A is the column of constants in the system of equations.

For more information on linear algebra, make an appointment with a [content tutor](#).

Appointments are available in person at the [Student Success Center](#) or online.