



Calculus III: Functions, Derivatives, and Gradient

Functions of Several Variables

Just as a typical one-dimensional function maps an input (x , horizontal axis) to an output (y , vertical axis), so a two-dimensional function maps inputs (x and y) to an output (z , height). All rules still apply to higher-order functions and many properties scale.

Directional Derivatives and Gradient

- Directional derivatives describe how rapidly a function is changing in a certain direction as defined by the unit vector of v . A typical partial derivative is a specified directional derivative, with the unit vector in the i or j direction. If you are given a vector u that is not a unit vector, then the unit vector v would be $u/[u]$.
- Gradients define the rate of change in the direction that water would flow along the surface; that is, the direction of greatest increase/decrease at a specific point. A vector orthogonal to the gradient experiences no change. This is analogous to a contour map.

The directional derivative at point P_0 in the direction of $u = \frac{v}{|v|} \cdot \left(\frac{\partial f}{\partial s} \right) |_{u, P_0} = (\nabla f) |_{P_0} \cdot u$

The gradient of f is noted by the Greek letter "del": $\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j$

Extreme Values

First, determine the location (a,b) such that $f_x = f_y = 0$. Note that this often requires factoring.

Next, apply the second derivative test to determine which feature is located at (a,b) .

If the discriminant of f is < 0 (negative), f has a saddle point at (a,b) .

If the discriminant of f is $= 0$, the test is inconclusive.

If the discriminant of f is > 0 (positive), evaluate the following rules: If $f_{xx} < 0$ (negative), then there is a local maximum at (a,b) . If $f_{xx} > 0$ (positive), then there is a local minimum at (a,b) .

The discriminant is defined as $f_{xx} \cdot f_{yy} - f_{xy}^2$, which can be remembered using $|f_{xx} f_{xy} f_{xy} f_{yy}|$.

For more information on functions, derivatives, and gradient, make an appointment with a [content tutor](#). Appointments are available in person at the [Student Success Center](#) or online.

Adapted from Hass, J., Weir, M.D., & Thomas, G.B. (2012). *University Calculus: Early Transcendentals* (2nd ed.). Boston: Pearson Education.

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