

Calculus II: Vectors

Vector Notation for 3-Dimensional Vector

- Component form: $\vec{v} = \langle v_1, v_2, v_3 \rangle$
- "i-j-k" notation: $\vec{v} = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$

Magnitude or Length of a Vector

$$|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Vector Operations

Let $\vec{u} = \langle u_1, u_2, u_3 \rangle$ and $\vec{v} = \langle v_1, v_2, v_3 \rangle$ with c being a scalar. Then vector addition is defined by $\vec{u} + \vec{v} = \langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle$ and scalar multiplication is defined by $c\vec{u} = \langle cu_1, cu_2, cu_3 \rangle$.

Unit Vector

Given that $\vec{v}$ is a nonzero vector, $\frac{\vec{v}}{|\vec{v}|}$ is the unit vector, also called the direction, of $\vec{v}$. The equation $\vec{v} = |\vec{v}| \frac{\vec{v}}{|\vec{v}|}$ expresses $\vec{v}$ as its length times its direction.

Dot Product of Two Vectors

$$\vec{u} \cdot \vec{v} = u_1v_1 + u_2v_2 + u_3v_3$$

Properties of Dot Product

- $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$
- $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$
- $0 \cdot \vec{v} = 0$
- $(c\vec{u}) \cdot \vec{v} = \vec{u} \cdot (c\vec{v}) = c(\vec{u} \cdot \vec{v})$
- $\vec{u} \cdot \vec{u} = |\vec{u}|^2$
- Two vectors are orthogonal if $\vec{u} \cdot \vec{v} = 0$

Angle Between Two Nonzero Vectors

$$\theta = \cos^{-1} \left(\frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|} \right)$$

Projection of $\vec{u}$ onto $\vec{v}$

$$\text{proj}_{\vec{v}} \vec{u} = \left(\frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \right) \vec{v}$$

Cross Product of Two Vectors

$\vec{u} \times \vec{v} = (|\vec{u}||\vec{v}|\sin\theta)\hat{n}$ such that $\hat{n}$ is a unit vector perpendicular to the plane determined by $\vec{u}$ and $\vec{v}$

Determinant Formula for Cross Product of Two Vectors

Suppose $\vec{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$ and $\vec{v} = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$.

Then $\vec{u} \times \vec{v} = (u_2v_3 - u_3v_2)\hat{i} - (u_1v_3 - u_3v_1)\hat{j} + (u_1v_2 - u_2v_1)\hat{k}$.

Properties of Cross Product

- $(r\vec{u}) \times (s\vec{v}) = (rs)(\vec{u} \times \vec{v})$
- $\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \vec{v} + \vec{u} \times \vec{w}$
- $\vec{u} \times \vec{v} = -(\vec{v} \times \vec{u})$
- $(\vec{v} + \vec{w}) \times \vec{u} = \vec{v} \times \vec{u} + \vec{w} \times \vec{u}$
- $\vec{0} \times \vec{v} = \vec{0}$
- $\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w}$
- Two vectors are parallel if $\vec{u} \times \vec{v} = \vec{0}$

For more information on vectors, make an appointment with a [content tutor](#). Appointments are available in person at the [Student Success Center](#) or online.