

Calculus I Review

Derivative Definition

$$\frac{d}{dx}(f(x)) = f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Basic Properties of Derivatives

- $(cf(x))' = c(f'(x))$
- $(f(x) \pm g(x))' = f'(x) \pm g'(x)$
- $\frac{d}{dx}(c) = 0$

Product Rule

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Power Rule

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Chain Rule

$$\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$$

L'Hopital's Rule

$$\text{If } \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0} \text{ or } \frac{\pm\infty}{\pm\infty} \text{ then } \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

Limit Properties

- $\lim_{x \rightarrow a}[cf(x)] = c \lim_{x \rightarrow a} f(x)$
- $\lim_{x \rightarrow a}[f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$

- $\lim_{x \rightarrow a} [f(x)g(x)] = \lim_{x \rightarrow a} f(x) \lim_{x \rightarrow a} g(x)$
- $\lim_{x \rightarrow a} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$ if $\lim_{x \rightarrow a} g(x) \neq 0$
- $\lim_{x \rightarrow a} [f(x)]^n = \left[\lim_{x \rightarrow a} f(x) \right]^n$

Common Derivative Formulas

- $\frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx} (\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2}$
- $\frac{d}{dx} (a^x) = a^x \ln(a)$
- $\frac{d}{dx} (e^x) = e^x$
- $\frac{d}{dx} (\ln(x)) = \frac{1}{x}, x > 0$
- $\frac{d}{dx} (\ln|x|) = \frac{1}{x}$
- $\frac{d}{dx} (\log_a(x)) = \frac{1}{x \ln(a)}$
- $\frac{d}{dx} (x) = 1$
- $\frac{d}{dx} (\sin x) = \cos x$
- $\frac{d}{dx} (\cos x) = -\sin x$
- $\frac{d}{dx} (\tan x) = \sec^2 x$
- $\frac{d}{dx} (\sec x) = \sec x \tan x$
- $\frac{d}{dx} (\csc x) = -\csc x \cot x$
- $\frac{d}{dx} (\cot x) = -\csc^2 x$

Integral Definition

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k) \Delta x \text{ where } \Delta x = \frac{b-a}{n} \text{ and } x_k = a + k\Delta x$$

Fundamental Theorem

$$\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a) \text{ where } f \text{ is continuous on } [a, b] \text{ and } F' = f$$

Integral Properties

- $\int_a^b c f(x) dx = c \int_a^b f(x) dx$
- $\int_a^b f(x) \pm g(x) dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$

- $\int_a^a f(x)dx = 0$ and $\int_a^b f(x)dx = -\int_b^a f(x)dx$
- $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$

Integration by Parts

$$\int u dv = uv - \int v du \text{ where } v = \int dv \text{ or } \int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$

Integration by Substitution

- $\int_a^b f(g(x)) g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$ where $u = g(x)$ and $du = g'(x)dx$
- $\int k dx = kx + C$
- $\int x^n dx = \frac{1}{n+1} x^{n+1} + C, n \neq -1$
- $\int x^{-1} dx = \int \frac{1}{x} dx = \ln|x| + C$
- $\int \frac{1}{ax+b} dx = \frac{1}{a} \ln|ax+b| + C$
- $\int \ln(x) dx = x \ln(x) - x + C$
- $\int e^x dx = e^x + C$
- $\int \cos x dx = \sin x + C$
- $\int \sin x dx = -\cos x + C$

Common Integrals

- $\int \sec^2 x dx = \tan x + C$
- $\int \sec x \tan x dx = \sec x + C$
- $\int \csc x \cot x dx = -\csc x + C$
- $\int \csc^2 x dx = -\cot x + C$
- $\int \tan x dx = \ln|\sec x| + C$
- $\int \sec x dx = \ln|\sec x + \tan x| + C$
- $\int \frac{1}{a^2+u^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + C$
- $\int \frac{1}{\sqrt{a^2-u^2}} dx = \sin^{-1}\left(\frac{u}{a}\right) + C$

For more information on Calculus I, make an appointment with a [content tutor](#). Appointments are available in person at the [Student Success Center](#) or online.