STUDENT SUCCESS CENTER
THE UNIVERSITY OF ALABAMA IN HUNTSVILLE

Calculus I: Limits and Integrals

Limits

Limit Properties

- $\lim_{x\to a}[c\cdot f(x)] = c\cdot \lim_{x\to a} f(x)$
- $\lim_{x\to a}[f(x)\pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$
- $\lim_{x\to a}[f(x)\cdot g(x)] = \lim_{x\to a} f(x)\cdot \lim_{x\to a} g(x)$
- $\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ if $\lim_{x\to a} g(x) \neq 0$
- $\lim_{x\to a}[f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$

Continuity Test

Used to determine if a function is continuous at a point

1. $f(c)$ exists
2. $\lim_{x\to c} f(x)$ exists, so $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)$
3. $\lim_{x\to c} f(x) = f(c)$

Example: Determine if $f(x) = x^2 + 2x - 6$ is continuous at $x = 1$

- $f(1) = (1)^2 + 2(1) - 6 = -3$
- $x^2 + 2x - 6 = -3$
- The limit exists at $x = 1$ because $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)$
- The function is continuous at $x = 1$ because $\lim_{x\to 1} f(x) = f(1)$

Integrals

Definition: The sum of infinitely many, infinitesimally small areas

$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\,\Delta x$ where $\Delta x = \frac{b-a}{n}$ and $x_k = a + k\cdot\Delta x$

Integral Properties

- $\int_a^a f(x)dx = 0$
- $\int_b^a f(x)dx = -\int_a^b f(x)dx$
- $\int_a^b c\cdot f(x)dx = c\cdot\int_a^b f(x)dx$
- $\int_a^b [f(x)\pm g(x)]\,dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

- $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$

Fundamental Theorem of Calculus

If *f(x)* is continuous on *[a,b]* and *F'(x) = f(x)*:

- $F'(x) = \frac{d}{dx}\int_a^x f(t)dt = f(x)$
- $\int_a^b f(x)dx = F(b) - F(a)$

Integration by Substitution

$\int_a^b [f(g(x)) \cdot g'(x)]\,dx = \int_{g(a)}^{g(b)} f(u)du$ where $u = g(x)$ and $du = g'(x)dx$

Example: Evaluate $\int_1^3 3(3x-1)^2 dx$

- Let $u = 3x - 1$, so $du = 3dx$
- $\int_{3(1)-1}^{3(3)-1} (u)^2 du = \int_2^8 u^2 du = \left[\frac{1}{3}u^3\right]_2^8$
- $= \frac{1}{3}(8)^3 - \frac{1}{3}(2)^3 = 168$
- So, $\int_1^3 3(3x-1)^2 dx = 168$

Common Integrals

- $\int k\,dx = k \cdot x + C$
- $\int e^x dx = e^x + C$
- $\int \sin\sin(x)\,dx = -\cos(x) + C$
- $\int sec^2(x)dx = \tan\tan(x) + C$
- $\int \sec\sec(x) \cdot \tan\tan(x)\,dx = \sec\sec(x) + C$
- $\int x^n dx = \frac{1}{n+1}x^{n+1} + C$
- $\int \frac{1}{x}dx = ln|x| + C$
- $\int \cos\cos(x)\,dx = \sin\sin(x) + C$
- $\int csc^2(x)dx = -\cot\cot(x) + C$
- $\int \csc(x) \cdot \cot(x)\,dx = -\csc(x) + C$

For more information on limits and integrals, make an appointment with a content tutor. Appointments are available in person at the Student Success Center or online.

Adapted from Hass, J., Weir, M.D., & Thomas, G.B. (2012). *University Calculus: Early Transcendentals* (2nd ed.). Boston: Pearson Education.

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