



Calculus I: Extreme Values

Relative Extreme Values

First Derivative Test

- Local maximum at $x=c$ if $f'(c)=0$ and the function changes from increasing to decreasing at $x=c$ and if $f(c)$ is defined
- Local minimum at $x=c$ if $f'(c)=0$ and the function changes from decreasing to increasing at $x=c$ and if $f(c)$ is defined

Second Derivative Test

- Local maximum at $x=c$ if $f'(c)=0$ and $f''(c)<0$ and $f(c)$ is defined
- Local minimum at $x=c$ if $f'(c)=0$ and $f''(c)>0$ and $f(c)$ is defined

Example

Find relative max/min of $f(x)=3x^2-12x+4$

First Derivative Test

- $f'(x)=6x-12=0$
 - Critical point at $x=2$
- On $(-\infty, 2)$, let the test point be $x=0$
- $f'(0)=6(0)-12=-12<0$
 - $f(x)$ is decreasing on interval $(-\infty, 2)$
- On $(2, \infty)$, let the test point be $x=3$
- $f'(3)=6(3)-12=18-12=6>0$
 - $f(x)$ is increasing on the interval $(2, \infty)$
- Since $f(x)$ changes from decreasing to increasing at the critical point $x=2$, $x=2$ is a relative minimum
- Define $f(c)$

Second Derivative Test

- $f'(x)=6x-12=0$
 - Critical point at $x=2$
- $f''(x)=6$
- $f''(2)=6>0$
 - $f(x)$ is concave up at $x=2$
- Since $f(x)$ is concave up at the critical point, $x=2$, $x=2$ is a relative minimum
 - Define $f(c)$

Absolute Extreme Values

1. Find critical points
2. Evaluate $f(x)$ for critical points and endpoints
3. Compare $f(x)$ values to determine maximum and minimum

Example

Find absolute max/min of $f(x)=x^3-3x+5$ on the interval $[0,2]$

- $f'(x)=3x^2-3=0$
 - Critical points are $x=-1$ and $x=1$
- $f(0)=(0)^3-3(0)+5=5$
 - $f(1)=(1)^3-3(1)+5=3$
 - $f(2)=(2)^3-3(2)+5=7$
 - Critical point $x=-1$ is not included because -1 is not on the interval $[0,2]$
- Absolute maximum at $x=2$ and absolute minimum at $x=1$

For more information on extreme values, make an appointment with a [content tutor](#).
Appointments are available in person at the [Student Success Center](#) or online.

Adapted from Hass, J., Weir, M.D., & Thomas, G.B. (2012). *University Calculus: Early Transcendentals* (2nd ed.). Boston: Pearson Education.