

Calculus II: Integration Methods

Integration by Parts

$$\int u dv = uv - \int v du \text{ where } v = \int dv \text{ OR } \int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$

- Example:

$$\int x \cos(x) dx \rightarrow u = x, du = 1 dx, dv = \cos(x) dx, v = \sin(x) \rightarrow x \sin(x) - \int \sin(x) dx = x \sin(x) + \cos(x) + c$$

Trig Integrals

Let's assume an integral of the form $\int \sin^m x \cos^n x dx$ such that m and n are nonnegative integers (positive or zero). We will divide the appropriate substitution into three cases according to m and n being odd or even.

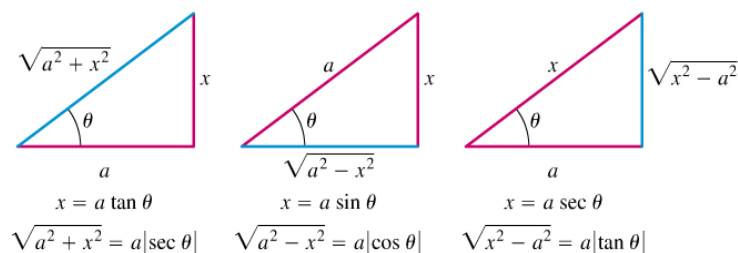
Case 1: If m is odd, let $m = 2k + 1$ and use the identity $\sin^2 x = 1 - \cos^2 x$ to get $\sin^m x = \sin^{2k+1} x = (\sin^2 x)^k \sin x = (1 - \cos^2 x)^k \sin x$ and do u -sub where $u = \cos(x)$

Case 2: If m is even and n is odd, let $n = 2k + 1$ and use the identity $\cos^2 x = 1 - \sin^2 x$ to get $\cos^n x = \cos^{2k+1} x = (\cos^2 x)^k \cos x = (1 - \sin^2 x)^k \cos x$ and do u -sub where $u = \sin(x)$

Case 3: If m is even and n is even, substitute $\sin^2(x) = \frac{1 - \cos(2x)}{2}$, $\cos^2(x) = \frac{1 + \cos(2x)}{2}$

For Integrals of Powers of $\tan(x)$ and $\sec(x)$: To integrate higher powers, use the identities $\tan^2(x) = \sec^2(x) - 1$ and $\sec^2(x) = \tan^2(x) + 1$ and integration by parts when necessary.

Trig Substitution



Case 1: If the integral contains $\sqrt{a^2 + x^2}$, then $x = a \tan \theta$, $dx = a \sec^2(\theta) d\theta$

Case 2: If the integral contains $\sqrt{a^2 - x^2}$, then $x = a \sin \theta$, $dx = a \cos(\theta) d\theta$

Case 3: If the integral contains $\sqrt{x^2 - a^2}$, then $x = a \sec \theta$, $dx = a \sec(\theta) \tan(\theta) d\theta$

Partial Fraction Decomposition

We assume a rational function $\frac{f(x)}{g(x)}$ such that the degree of $f(x)$ is less than the degree of $g(x)$ and we know the factors of $g(x)$.

1. Let $x - r$ be a linear factor of $g(x)$ and let $(x - r)^m$ be the highest power of $x - r$ that divides $g(x)$. Then we rewrite as $\frac{A_1}{(x-r)} + \frac{A_2}{(x-r)^2} + \dots + \frac{A_m}{(x-r)^m}$ and do this for each distinct factor of $g(x)$.
2. Let $x^2 + px + q$ be a non-reducible quadratic factor of $g(x)$ where it has no real roots and let $(x^2 + px + q)^n$ be the highest power of $x^2 + px + q$ that divides $g(x)$. Then we rewrite as $\frac{B_1x+C_1}{(x^2+px+q)} + \frac{B_2x+C_2}{(x^2+px+q)^2} + \dots + \frac{B_nx+C_n}{(x^2+px+q)^n}$ and do this for each distinct quadratic factor of $g(x)$.
3. Set the function $\frac{f(x)}{g(x)}$ equal to the sum of all of these partial fractions and clear the equation of fractions, arranging the terms in decreasing powers of x .
4. Equate the coefficients of corresponding powers of x and solve.

Improper Integrals

Type I (Infinite Bounds):

If $f(x)$ is continuous on $[a, \infty)$, then $\int_a^\infty f(x)dx = \lim_{b \rightarrow \infty} \int_a^b f(x)dx$

If $f(x)$ is continuous on $(-\infty, b]$, then $\int_{-\infty}^b f(x)dx = \lim_{a \rightarrow -\infty} \int_a^b f(x)dx$

If $f(x)$ is continuous on $(-\infty, \infty)$, then $\int_{-\infty}^\infty f(x)dx = \lim_{a \rightarrow -\infty} \int_a^c f(x)dx + \lim_{b \rightarrow \infty} \int_c^b f(x)dx$ for some real number c .

Type II (Discontinuity):

If $f(x)$ is cont. on $(a, b]$ and discontinuous at a , then $\int_a^b f(x)dx = \lim_{c \rightarrow a^+} \int_c^b f(x)dx$

If $f(x)$ is cont. on $[a, b)$ and discontinuous at b , then $\int_a^b f(x)dx = \lim_{c \rightarrow b^-} \int_a^c f(x)dx$

If $f(x)$ is discontinuous at c , where $a < c < b$, and cont. on $[a, c) \cup (c, b]$, then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

In each case, if the limit is finite, the improper integral **converges and that the limit is the **value** of the improper integral. If the limit does not exist, the integral **diverges**.**

For more information on integration methods, make an appointment with a [content tutor](#).

Appointments are available in person at the [Student Success Center](#) or online.