

Mechanics of Materials: Axial Loads & Torsion

Axial Loads:

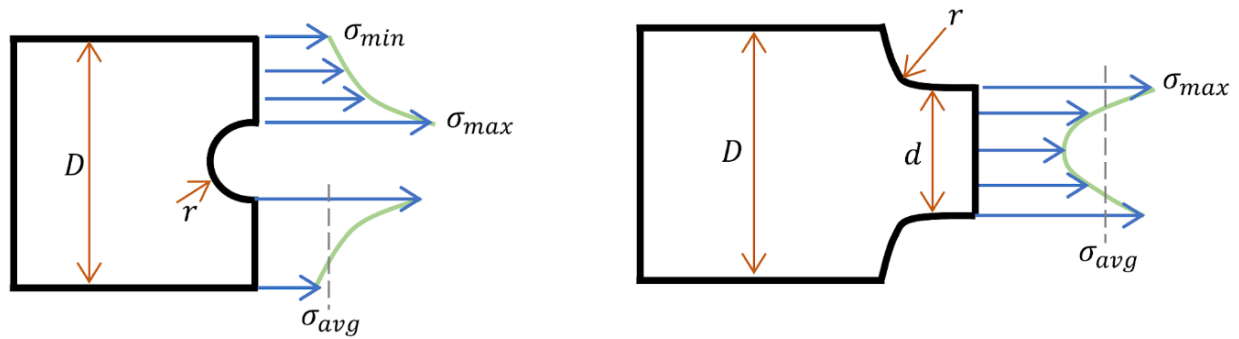
$$\varepsilon = \frac{\delta}{L} = \frac{\sigma}{\tau} = \frac{1}{E} \cdot \frac{P}{A} \quad \text{such that} \quad \delta = \frac{PL}{AE} \quad \text{for homogenous and uniform cross - sections}$$

$$\delta = \int_0^{L_0} \frac{P(x)}{A(x) \cdot E(x)} dx = \sum_{i=0}^n \frac{P_i L_i}{A_i E_i}$$

- Superposition allows the problem to be worked by evaluating how much a body *would* move if it could. Then, the reaction force must be equal and opposite.
 1. Remove one wall support
 2. Break the body into sections where P , L , A , or E changes
 3. Find δ for each section and the resulting $\delta_{total} = \sum \delta_i$ (careful to mind signs)
 4. Find $\delta_{reaction}$ in terms of the reaction force R_{wall}
 5. Equate the deformations due to the true deflection being
 $0 (\delta = 0 = \delta_{no\ wall} + \delta_{reaction})$
 6. Solve for reaction force at the wall, R_{wall} , and consequently other reaction forces.
 7. Solve for the stress in each section (from step 2), where $P = R_{wall}$ for all sections and A varies.
- Thermal Stress is a result of heating or cooling and is dependent on the material's coefficient of thermal expansion, (found in tables, measured in $^{\circ}\text{F}^{-1}$ or $^{\circ}\text{C}^{-1}$):

$$\delta_T = \alpha L \Delta T \quad \varepsilon_T = \alpha \Delta T$$

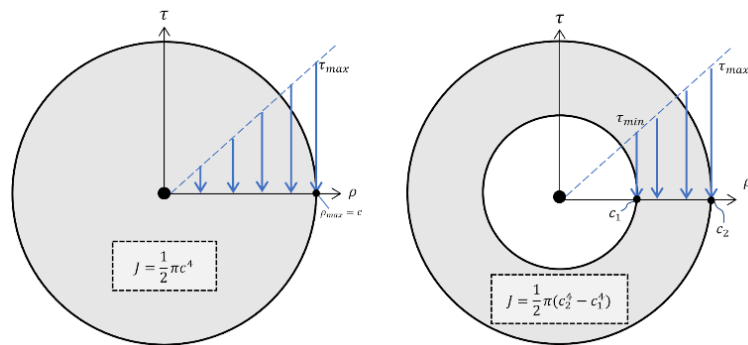
- Stress concentrations arise when stress flow is abruptly "pinched" around a corner. Two common instances are holes and fillets in a flat plate. This is the reason sidewalk corners crack first.



Given D , d , and r/d , K can be found using provided charts. K tells the ratio of the maximum stress observed to the average.

Torsion

- Twisting engenders a shear stress:

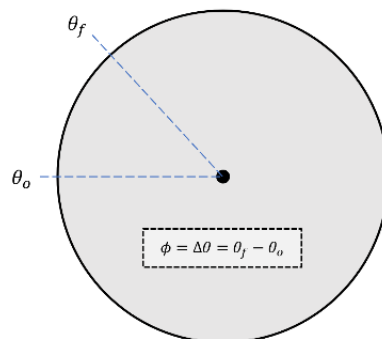


*for any given point on circle *for max stress occurring at the outermost point

$$T = \int r dF \text{ where } \tau = \frac{dF}{dA}, \text{ thus } T = \int r \cdot \tau dA$$

- Angle of twist measures deformation and is dependent on the length and diameter. Shear strain is independent of both diameter and length:

$$\gamma_{max} = \frac{c\Phi}{L} = \frac{\tau_{max}}{G} = \frac{TC}{JG} \text{ and } \Phi = \frac{TL}{JG}$$



- Power transmissions can be analyzed to understand the minimum shaft diameter for a given power requirement (or max power for given shaft diameter):

$$P = t \cdot \omega = 2\pi \text{ such that } T = \frac{P}{2\pi f} = \frac{\tau_{max} J}{c}$$

$$\frac{J}{C} = \frac{P}{2\pi f} \cdot \frac{1}{\tau_{max}}$$

$$J = \frac{TL}{G\Phi} = \frac{P}{2\pi f} \cdot \frac{L}{G\Phi}$$

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