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THE UNIVERSITY OF ALABAMA IN HUNTSVILLE

RFAL
RELIABILITY AND FAILURE ANALYSIS LABORATORY

Characterizing Reliability Data Using the 3-Parameter Weibull Distribution

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Reliability and Failure Analysis Lab

Objective

- Fit Reliability Data to the 3-Parameter Weibull
  - Calculate Reliability Functions
  - Plot Reliability Functions

- Why?

Background

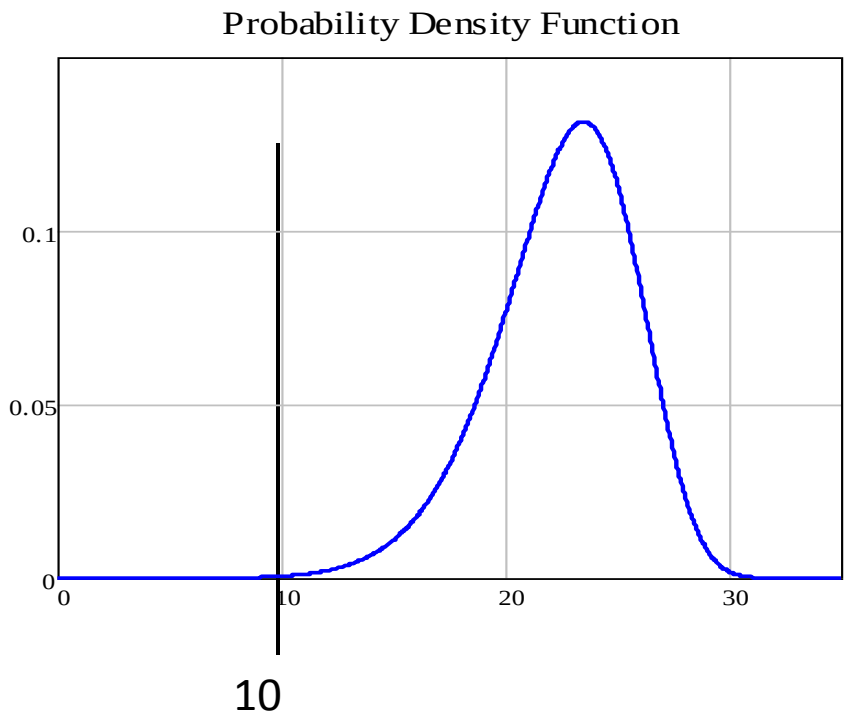
- Most failure and all repair models have a region from 0 – γ in which no sample data exists.
- Many distributions begin at 0.
  - Exponential, lognormal, & 2-Parameter Weibull
  - Normal distribution have negative values for data that can only be positive.
- The 3-Parameter Weibull addresses this issue.

3-Parameter Weibull

$$f(t) = \frac{\beta}{\eta} \cdot \left(\frac{t - \gamma}{\eta} \right)^{\beta - 1} \cdot e^{-\left(\frac{t - \gamma}{\eta} \right)^{\beta}}$$

- β (Beta) – Shape Parameter
- η (Eta) – Scale Parameter
- γ (Gamma) - Location Parameter

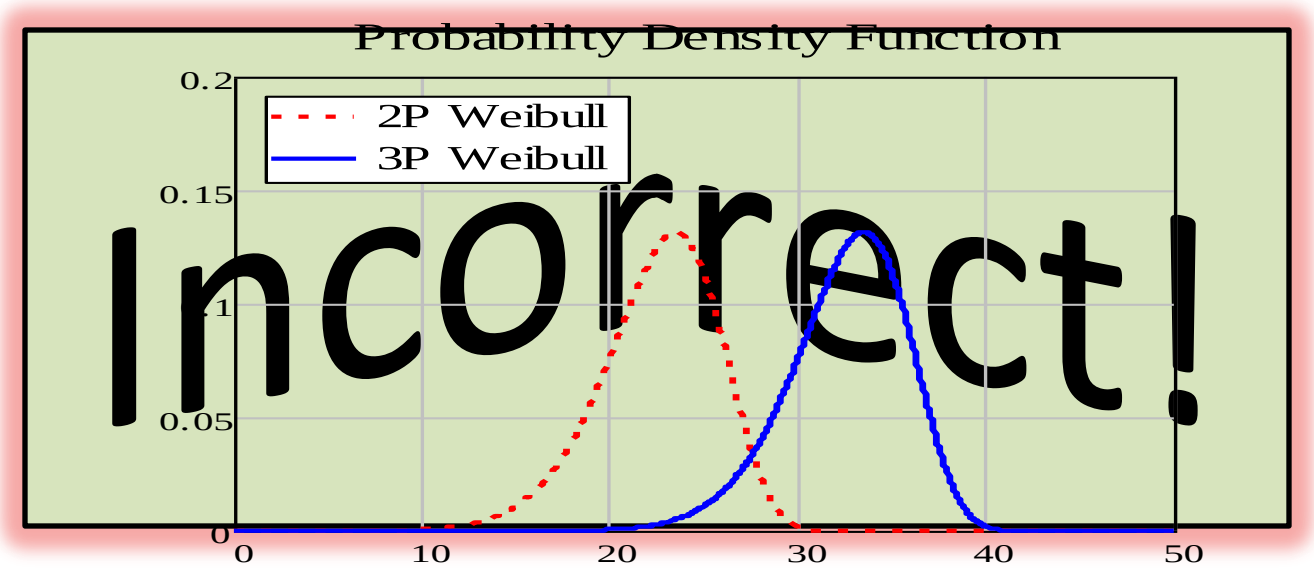
Misconceptions



Distribution	Shape (β)	Scale (η)
2-Parameter Weibull	8.46	23.86

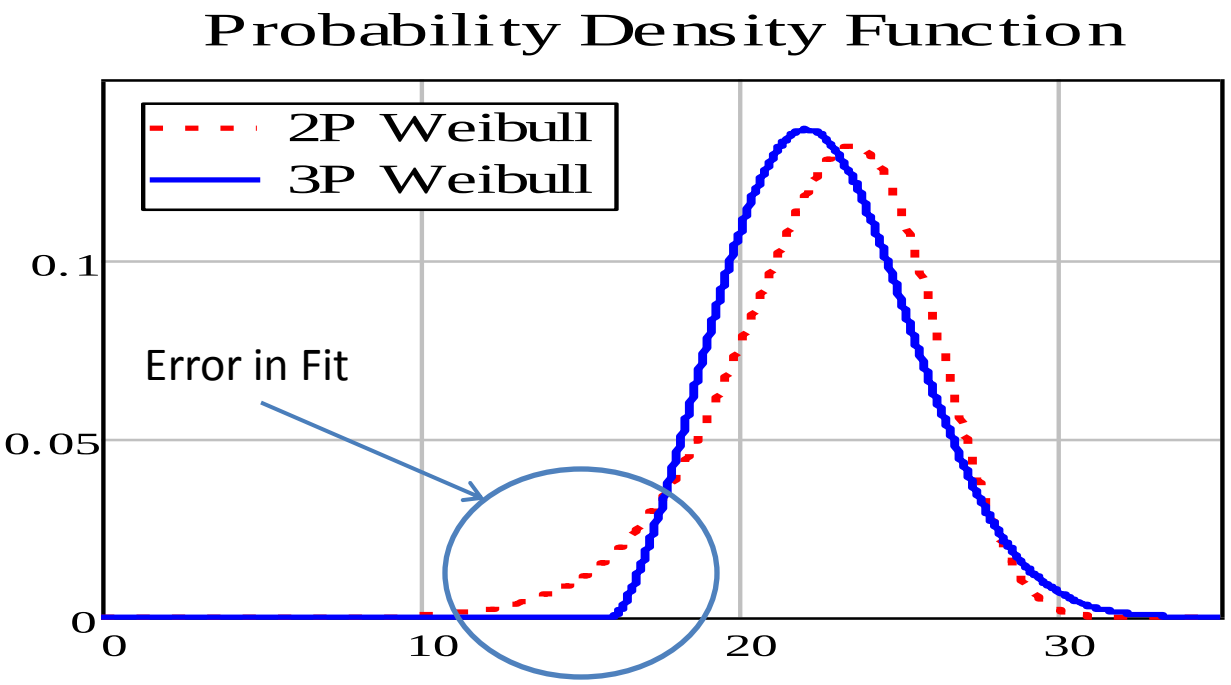
Distribution	Shape (β)	Scale (η)	Location (γ)
3-Parameter Weibull	8.46	23.86	10

Misconceptions



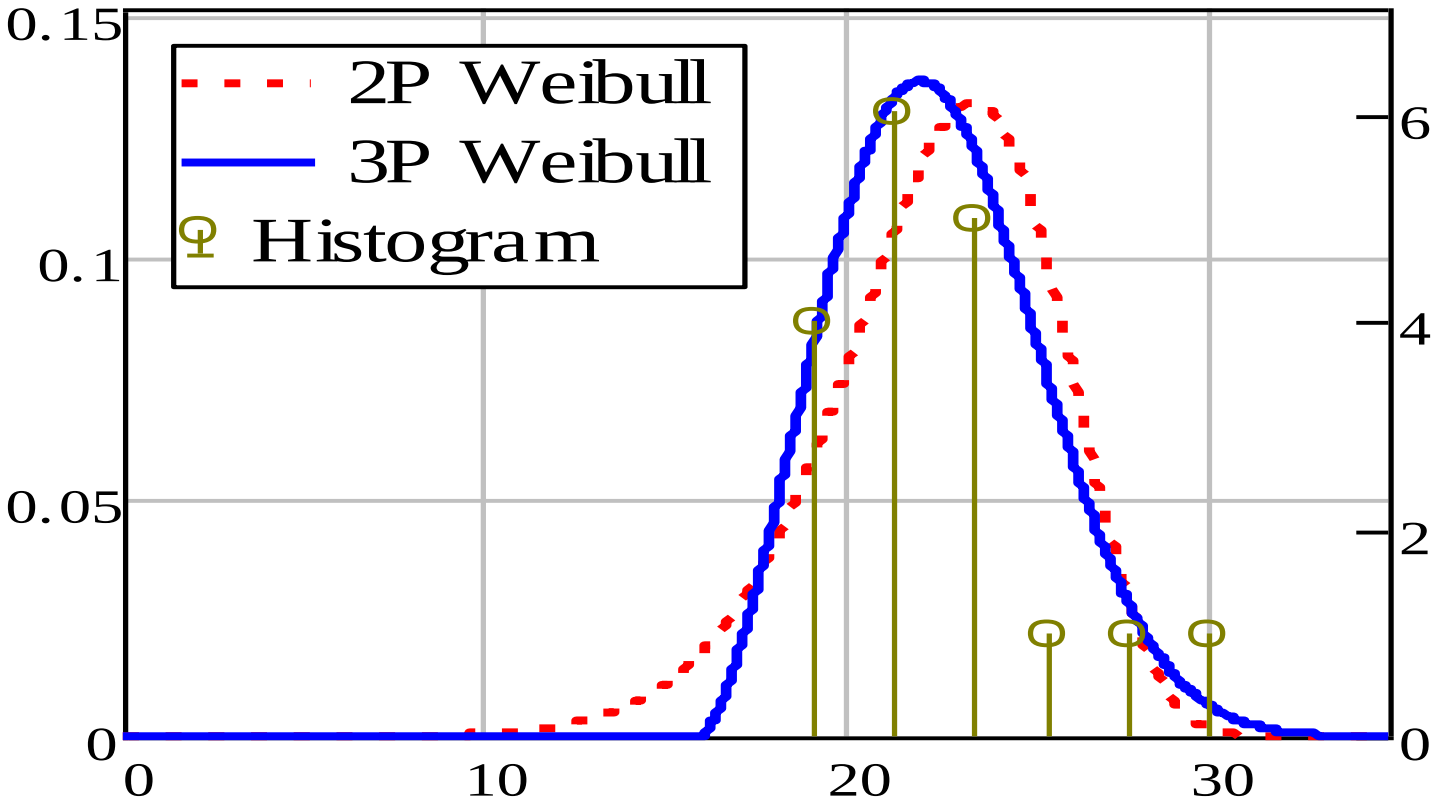
Distribution	Shape	Scale	Threshold
2-Parameter Weibull	8.46	23.86	0
3-Parameter Weibull	8.46	23.86	10

Correctly Fit

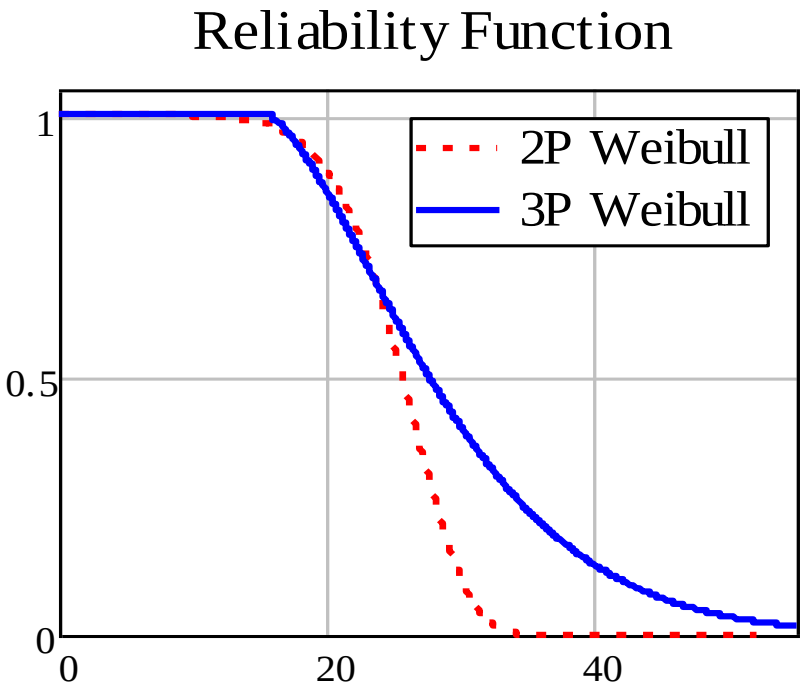


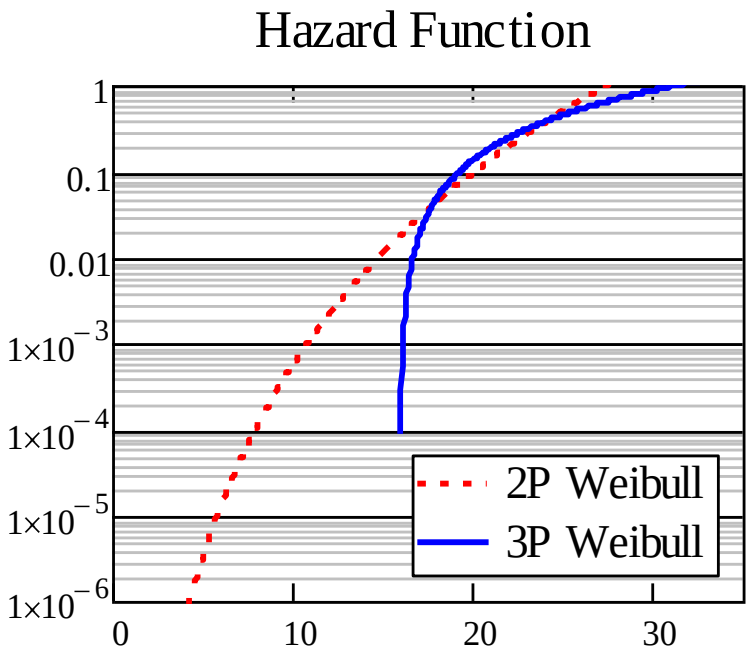
Distribution	Shape	Scale	Threshold
2-Parameter Weibull	8.46	23.86	0
3-Parameter Weibull	2.211	6.834	16.557

Why 3-Parameter Weibull?



Why 3-Parameter Weibull?





Why 3-Parameter Weibull?

	Reliability of Single Part	Parts in System	System Reliability
2-Parameter Weibull	0.9991	100	0.9139
3-Parameter Weibull	0.9995	100	0.9512

- System Requirement: Reliability of 0.95
- Parts are in series

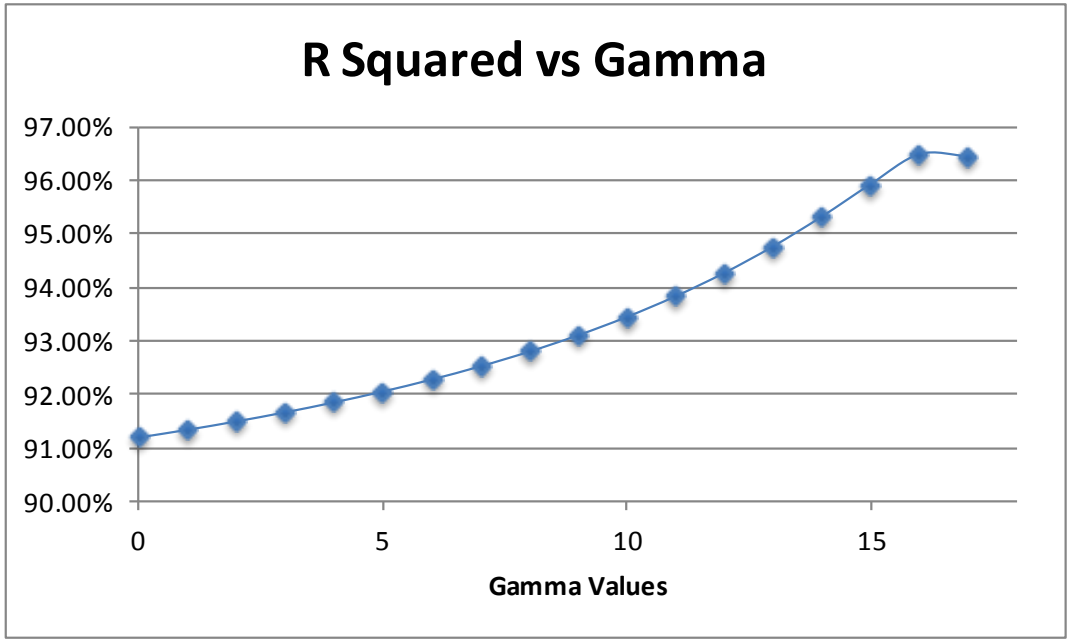
Tools

- Excel – Common Data analysis tool
- Minitab – Statistical software package
- Mathcad – Mathematical program
- Relex – Reliability Modeling program
- Reliasoft – Reliability Modeling program

Excel Example

	Gamma		Rsquared		
Guess	12		94.2586%		

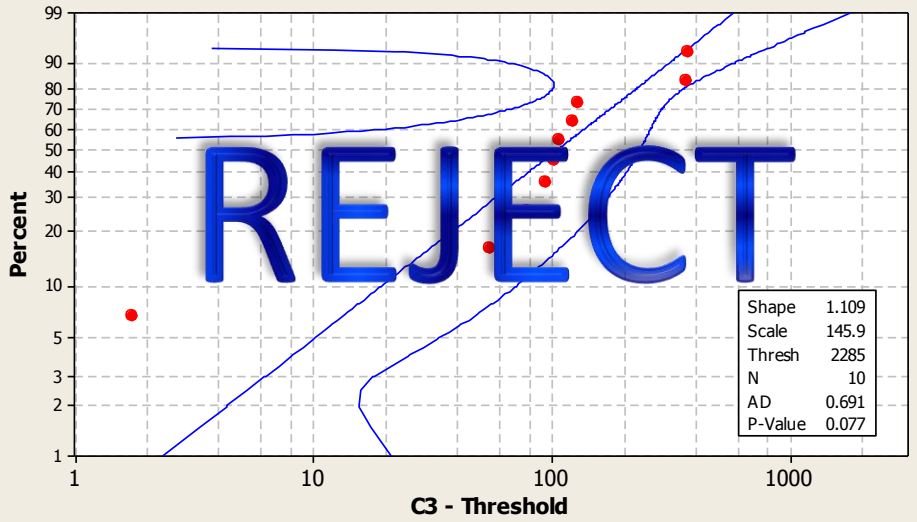
Gamma	R Squared		Data	Y	X = ln(TTF-γ)
0	91.1925%		18.0000	-3.2497	1.791759469
1	91.3363%		20.0000	-2.3336	2.079441542
2	91.4921%		20.0000	-1.8408	2.079441542
3	91.6615%		20.0000	-1.4939	2.079441542
4	91.8463%		21.0000	-1.2209	2.197224577
5	92.0487%		21.0000	-0.9922	2.197224577
6	92.2713%		21.0000	-0.7924	2.197224577
7	92.5172%		21.0000	-0.6123	2.197224577
8	92.7902%		22.0000	-0.4459	2.302585093
9	93.0949%		22.0000	-0.2890	2.302585093
10	93.4368%		23.0000	-0.1380	2.397895273
11	93.8222%		23.0000	0.0102	2.397895273
12	94.2586%		23.0000	0.1586	2.397895273
13	94.7533%		24.0000	0.3110	2.48490665
14	95.3097%		24.0000	0.4725	2.48490665
15	95.9133%		25.0000	0.6519	2.564949357
16	96.4709%		27.0000	0.8678	2.708050201
17	96.4292%		31.0000	1.1845	2.944438979



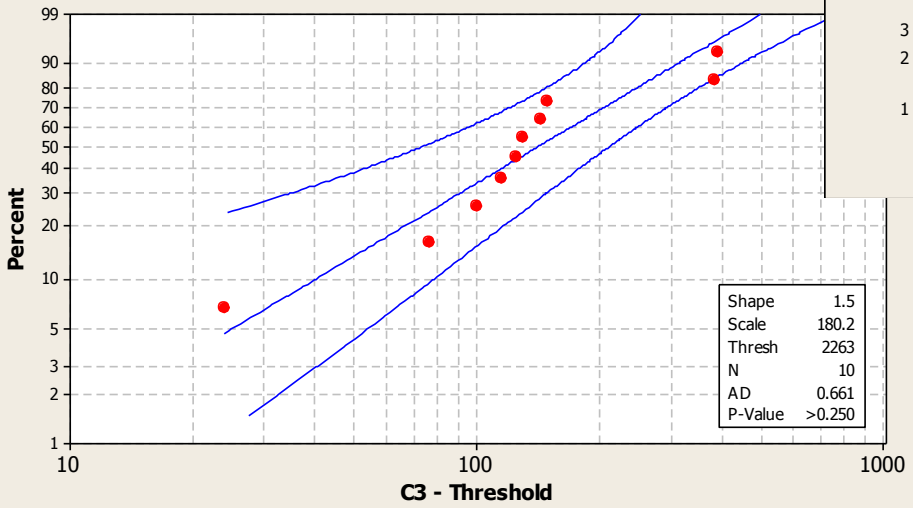
Example 1

	Shape	Scale	Threshold	r
Minitab	1.11	145.90	2285.00	
Mathcad	1.34	187.65	2262.91	94.69%
Relex	1.50	180.19	2262.82	94.65%

Minitab Solution



Relex Solution



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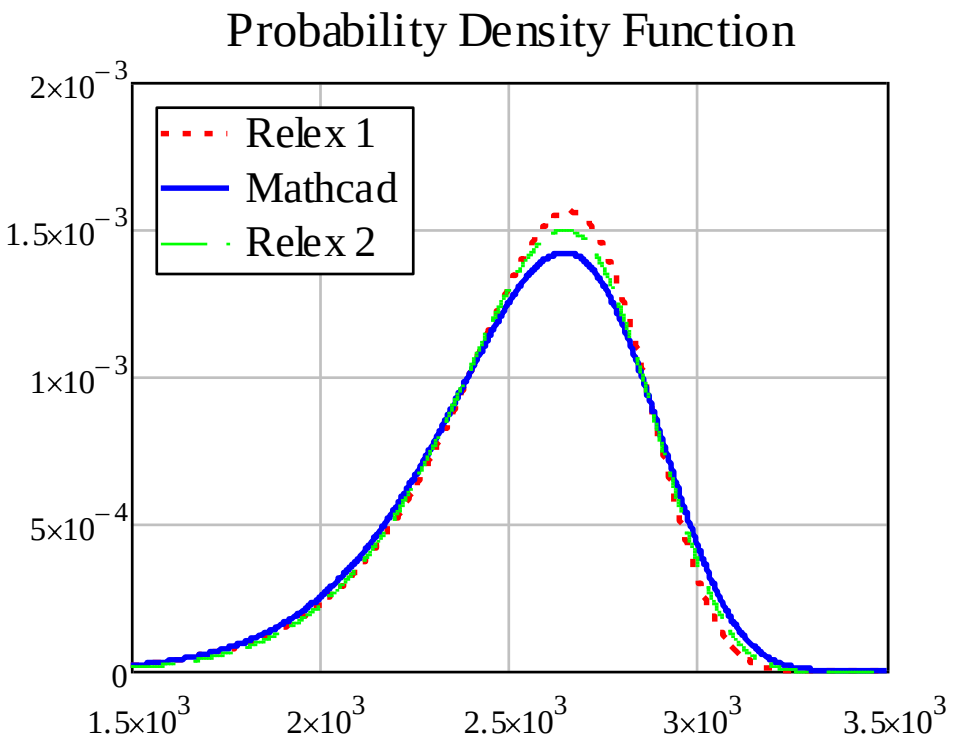
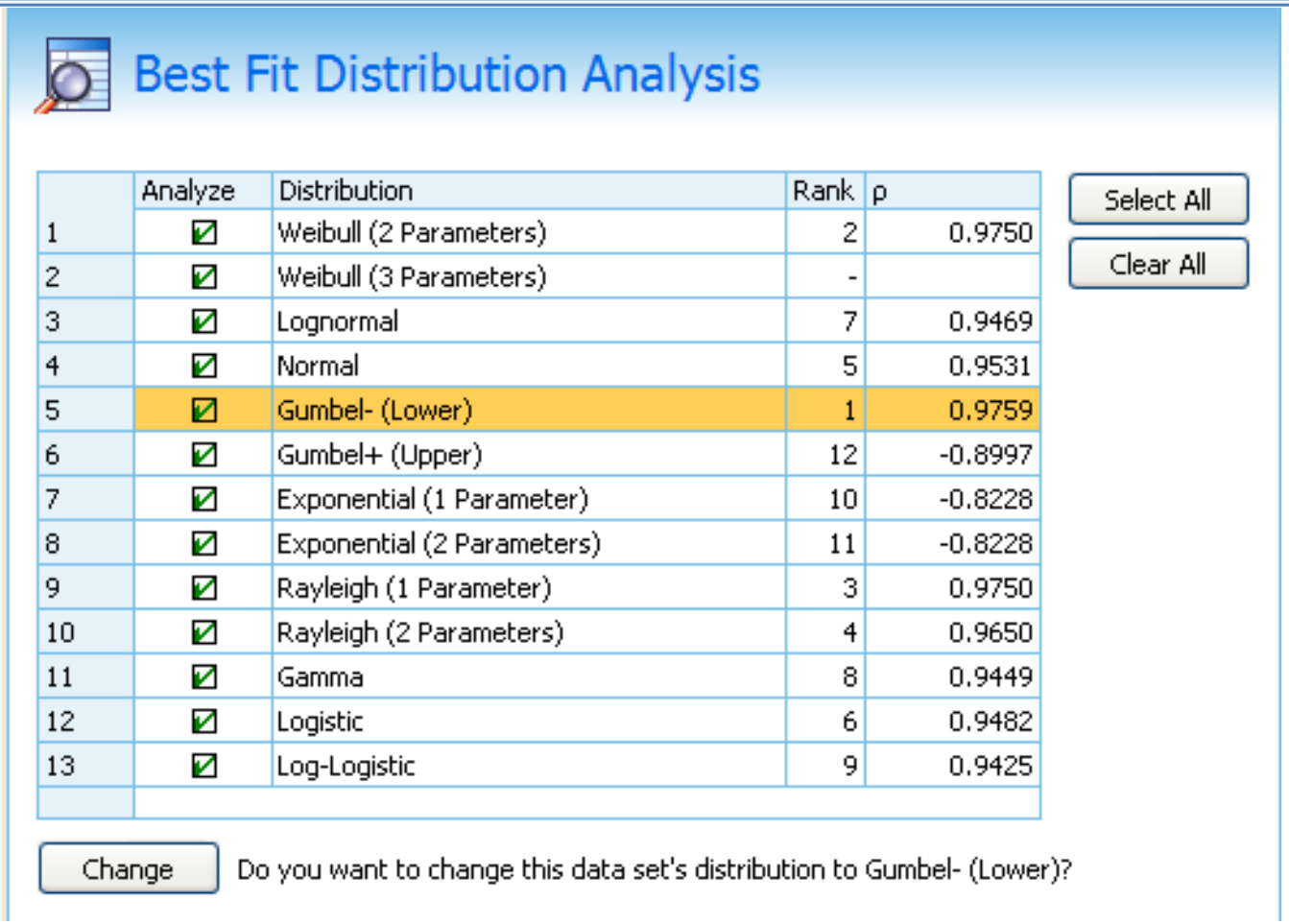
Example 2

	Shape	Scale	Threshold	r
Minitab	6398	1183489	-1180822	
Mathcad	10.27	2672.77	3.65E-10	97.48%
Relex	23.27	5489.07	-2820.07	97.56%
Relex (2)	10.84	2665.85	0.00	97.50%

Relex Weibull package allows the user to assign fixed values for the Weibull distribution.



Example 2



	Analyze	Distribution	Rank	ρ
1	☑	Weibull (2 Parameters)	2	0.9750
2	☑	Weibull (3 Parameters)	-	
3	☑	Lognormal	7	0.9469
4	☑	Normal	5	0.9531
5	☑	Gumbel- (Lower)	1	0.9759
6	☑	Gumbel+ (Upper)	12	-0.8997
7	☑	Exponential (1 Parameter)	10	-0.8228
8	☑	Exponential (2 Parameters)	11	-0.8228
9	☑	Rayleigh (1 Parameter)	3	0.9750
10	☑	Rayleigh (2 Parameters)	4	0.9650
11	☑	Gamma	8	0.9449
12	☑	Logistic	6	0.9482
13	☑	Log-Logistic	9	0.9425

Caution

- Results from Software packages
  - Not always applicable
  - Check For Sanity

Tools

- Excel – Long Iterative Process. Solution is not exact.
- Minitab – Good Statistical software package. Minitab's solution for fitting a 3-Parameter Weibull is suspect.
- Mathcad – Statistical tools are lacking. The built-in 2-Parameter Weibull function is not well defined and does not solve for the parameters. I wrote a program to solve for the 3-Parameter Weibull.
- Relex – Expensive Reliability software package which produces valid results and most of the graphs for reliability functions, can give gamma values that are suspect. Can fix by checking one box.
- Reliasoft – Expensive Reliability software package Reliability modeling tool. Will fit data to the 3-Parameter Weibull. Do not have 1st hand experience with the software.

Questions

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