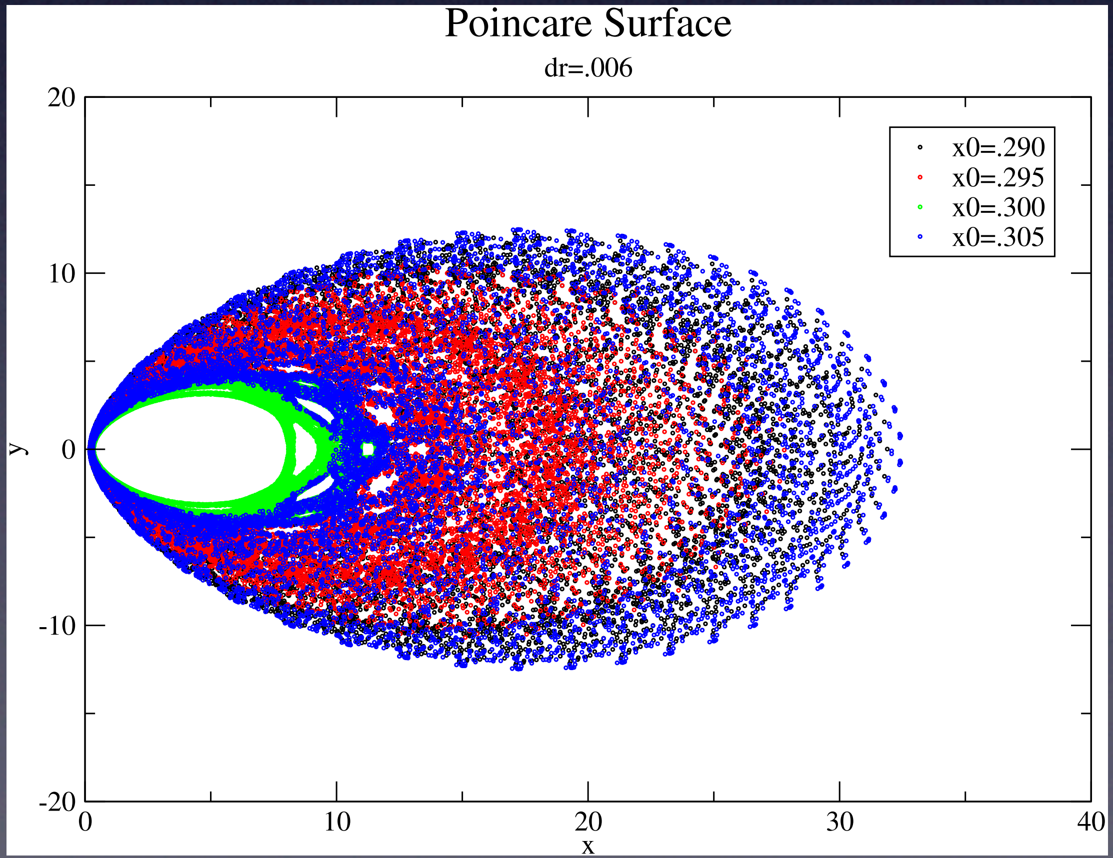
Retarded Vector Potentials and Chaotic Magnetic Fields

Scott Ripperda
PH 662 Final Project
University of Alabama in Huntsville



Overview

- Reasons to Study Chaotic Magnetic Fields
- How to Obtain Chaotic Magnetic Fields
- Time Independent Vector Potential and Magnetic Field
- Retarded Vector Potential and Magnetic Field for the Wire.
- Numerical Modeling
- Results
- Summary and Future Work

Reasons to Study Chaotic Magnetic Fields

- Magnetic Fields are involved in nearly all of physics, so understanding their structure is vital.
- Chaotic magnetic fields likely exist in the real world.
- Chaotic magnetic fields may play crucial roles in several plasma and astrophysical process.

How to Obtain Chaotic Magnetic Fields

Several papers have shown that wire and loop configurations can generate magnetic fields that can become chaotic despite the simple configuration. The example studied in this project is shown on the right.

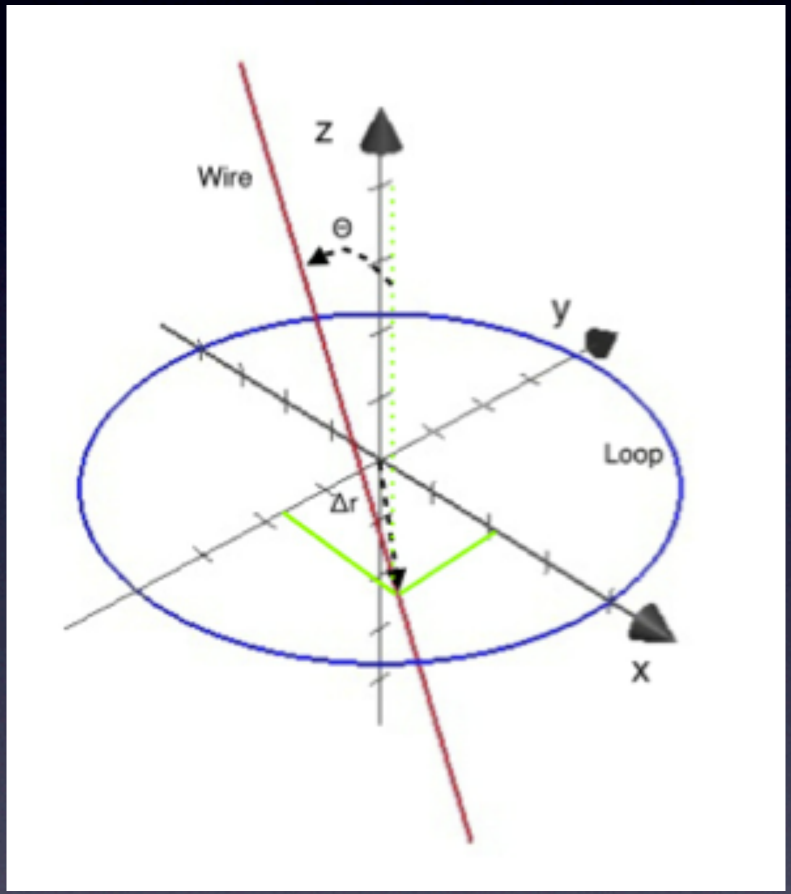


Diagram taken from Li, Dasgupta, Webb, and Ram[4]

Time Independent Vector Potential and Magnetic Field

Loop Formulas

$$A_x = -\frac{2C_1 ay}{\beta r}\left[\frac{(2-k^2)K(k)-2E(k)}{k^2}\right] \quad (1)$$

$$A_y = \frac{2C_1 ax}{\beta r}\left[\frac{(2-k^2)K(k)-2E(k)}{k^2}\right] \quad (2)$$

$$B_x = \frac{C_1 xz}{\alpha^2\beta s^2}\left[(a^2+r^2)E(k)-\alpha^2 K(k)\right] \quad (3)$$

$$B_y = \frac{C_1 yz}{\alpha^2\beta s^2}\left[(a^2+r^2)E(k)-\alpha^2 K(k)\right] \quad (4)$$

$$B_z = \frac{C_1}{\alpha^2\beta}\left[(a^2+r^2)E(k)-\alpha^2 K(k)\right] \quad (5)$$

where a is the loop radius, $s^2 = x^2+y^2$, $r^2 = s^2+z^2$, $\alpha^2 = a^2+r^2-2as$, $\beta^2 = a^2+r^2+2as$, $k^2 = 1-\alpha^2/\beta^2$, and $C_1 = \mu_0 I_{loop}/(2\pi)$. K(k) and E(k) are complete elliptical integrals of the first and second kind respectively.

Wire Formulas

$$Az = -C_2 ln(s)$$

$$B_x = -C_2\frac{y}{s^2}$$

$$B_y = C_2\frac{x}{s^2}$$

$C_2=\mu_0 I_{wire}/2\pi$

$$\vec{A} = A_{x_{loop}}\hat{x} + A_{y_{loop}}\hat{y} + A_{z_{wire}}\hat{z}$$

$$\vec{B} = (B_{x_{wire}} + B_{x_{loop}})\hat{x} + (B_{y_{wire}} + B_{y_{loop}})\hat{y} + B_{z_{loop}}\hat{z}$$

Retarded Vector Potential and Magnetic Field for the Wire

Instead of using a constant current , I_{wire} it would be more interesting to turn on the wire at t=0. Since information does not instantly, we must account for this using the retarded time.

$$t_r=t-r/c$$

Using the retarded time, we can find retarded vector potential and magnetic field as shown on the right.

$$I(t) = \begin{cases} I_{wire} & \text{for } t > 0 \\ 0 & \text{for } t \leq 0. \end{cases}$$

$$\vec{A} = \begin{cases} C_2 ln\left(\frac{ct+\sqrt{(ct)^2-s^2}}{s}\right)\hat{z} & \text{for } ct > \zeta \\ 0 & \text{for } ct < \zeta \end{cases}$$

$$\vec{B} = \begin{cases} C_2 \frac{ct}{s\sqrt{(ct)^2-s^2}}\hat{\phi} & \text{for } ct > \zeta \\ 0 & \text{for } ct < \zeta \end{cases}$$

$$\zeta=|r-r'|$$

Numerical Modeling

We can map the fields lines for both the vector potential and the magnetic field by solving the differential equation below.

$$\frac{\partial \vec{x}}{\partial l} = \frac{\vec{F}}{|\vec{F}|}.$$

This can solved several different ways. One of the most popular algorithms is the 4th order Runge-Kutta algorithm due to its speed, stability, and accuracy.

Numerical Modeling: RK4 method

$$y_{n+1}=y_n+1/6(k_1 +k_2 +k_3 + k_4)$$

where $k_1 =hf(t_n, y_n)$

$k_2=hf(t_n +h/2, y_n + k_1/2)$

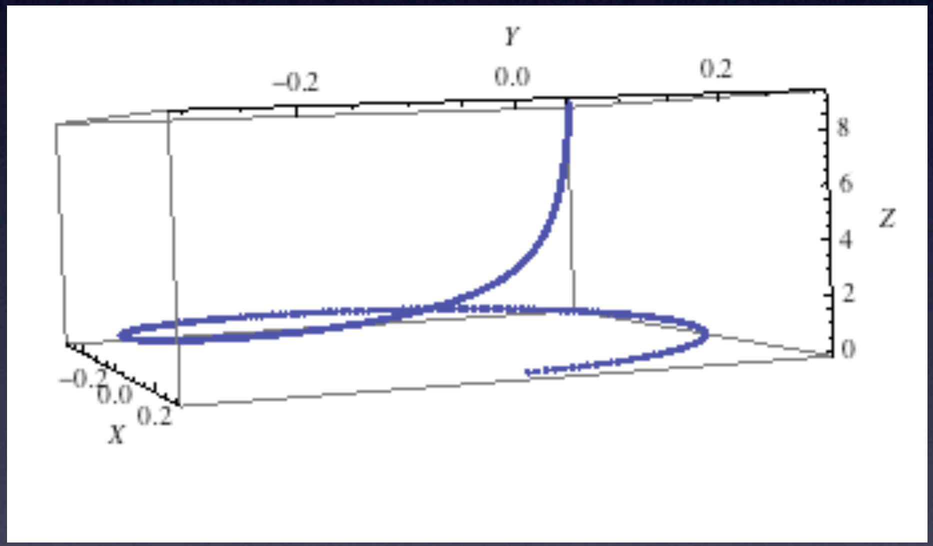
$k_3=hf(t_n +h/2, y_n + k_2/2)$

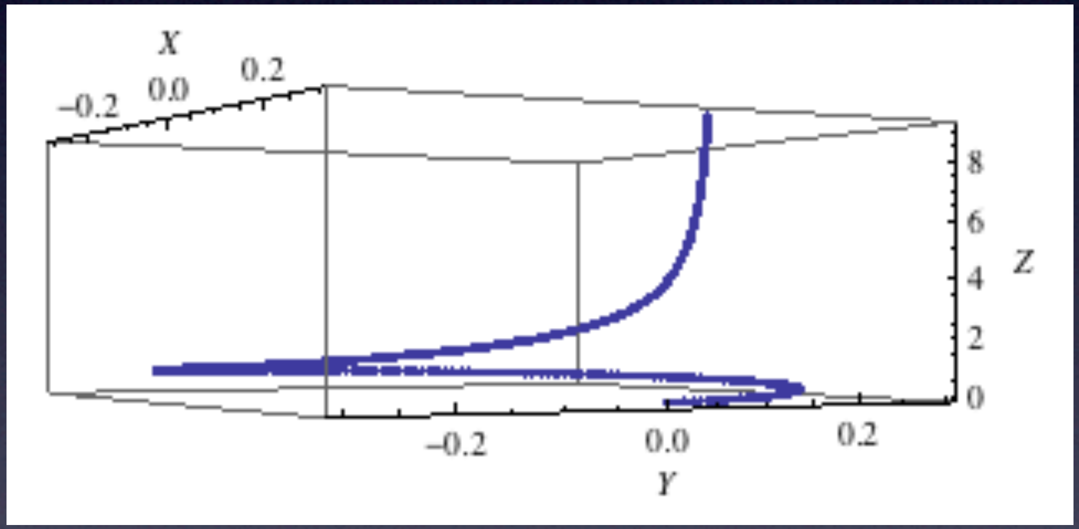
$k_4=hf(t_n +h, y_n + k_3)$

Time Independent Results: Vector Potential

Symmetric Case

Antisymmetric Case





x_0=0.3, Δr=0

x_0=0.3, Δr=0.05

Time Independent Vector Potential and Magnetic Field

Loop Formulas

$$A_x = -\frac{2C_1 ay}{\beta r}\left[\frac{(2-k^2)K(k)-2E(k)}{k^2}\right] \tag{1}$$

$$A_y = \frac{2C_1 ax}{\beta r}\left[\frac{(2-k^2)K(k)-2E(k)}{k^2}\right] \tag{2}$$

$$B_x = \frac{C_1 xz}{\alpha^2\beta s^2}\left[(a^2+r^2)E(k)-\alpha^2 K(k)\right] \tag{3}$$

$$B_y = \frac{C_1 yz}{\alpha^2\beta s^2}\left[(a^2+r^2)E(k)-\alpha^2 K(k)\right] \tag{4}$$

$$B_z = \frac{C_1}{\alpha^2\beta}\left[(a^2+r^2)E(k)-\alpha^2 K(k)\right] \tag{5}$$

where a is the loop radius, $s^2 = x^2+y^2$, $r^2 = s^2+z^2$, $\alpha^2 = a^2+r^2-2as$, $\beta^2 = a^2+r^2+2as$, $k^2 = 1-\alpha^2/\beta^2$, and $C_1 = \mu_0 I_{loop}/(2\pi)$. K(k) and E(k) are complete elliptical integrals of the first and second kind respectively.

Wire Formulas

$$Az = -C_2 ln(s)$$

$$B_x = -C_2\frac{y}{s^2}$$

$$B_y = C_2\frac{x}{s^2}$$

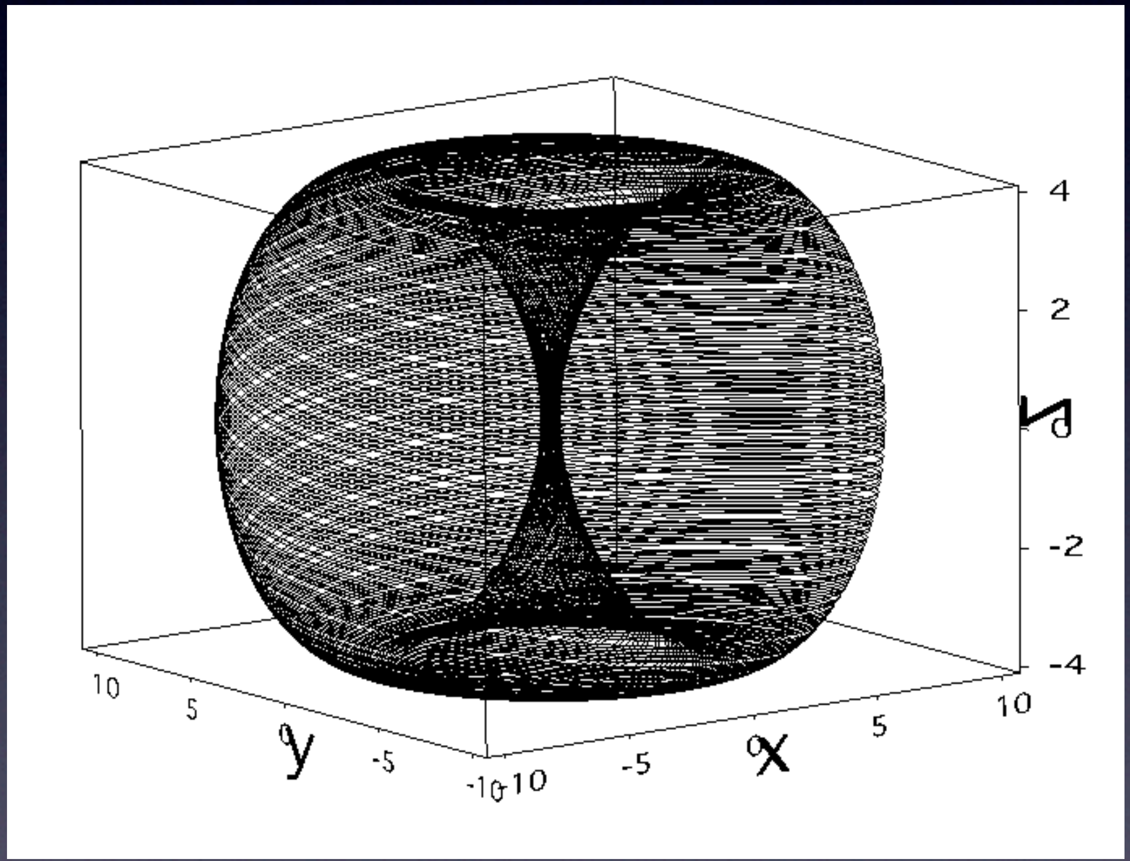
$C_2 = \mu_0 I_{wire}/2\pi$

$$\vec{A} = A_{x_{loop}}\hat{x} + A_{y_{loop}}\hat{y} + A_{z_{wire}}\hat{z}$$

$$\vec{B} = (B_{x_{wire}} + B_{x_{loop}})\hat{x} + (B_{y_{wire}} + B_{y_{loop}})\hat{y} + B_{z_{loop}}\hat{z}$$

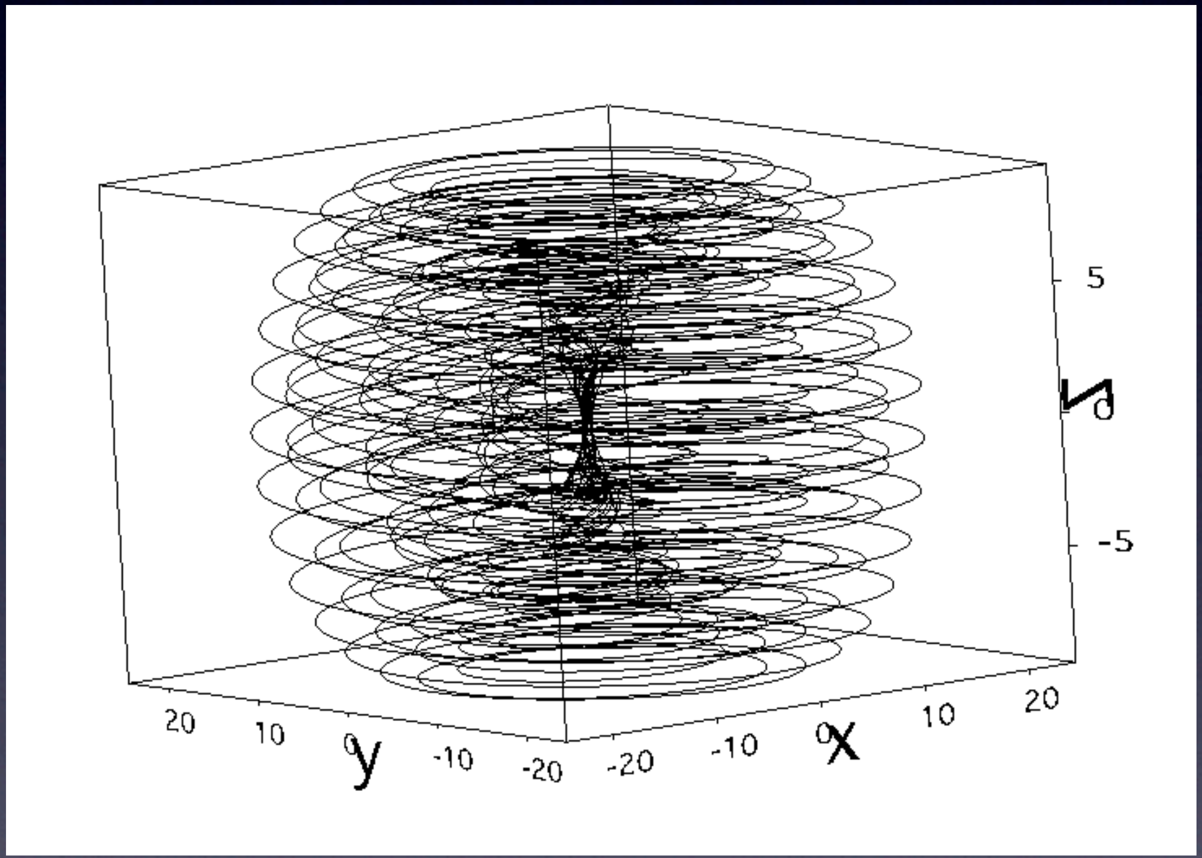
Time Independent Results: Magnetic Field

Symmetric Case



$x_0=0.3$, $\Delta r=0$

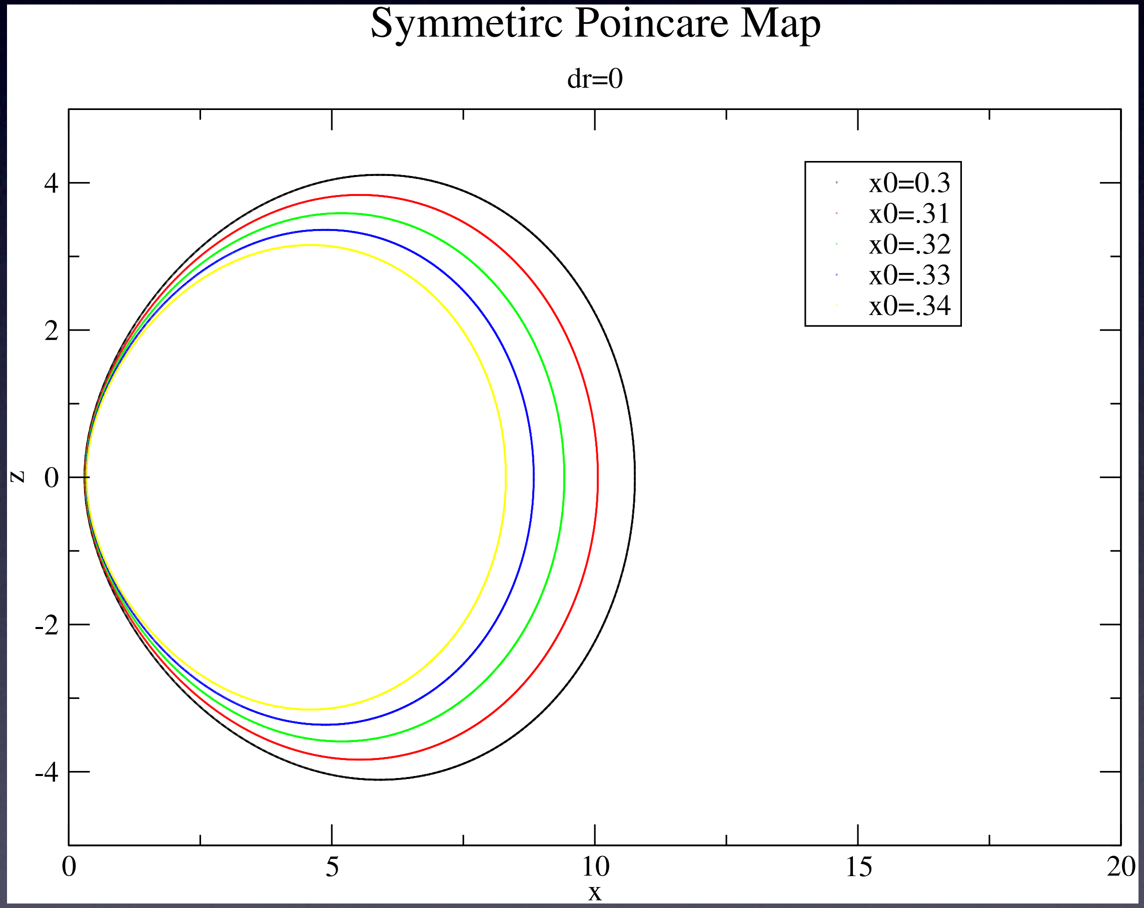
Antisymmetric Case



$x_0=0.3$, $\Delta r=0.05$

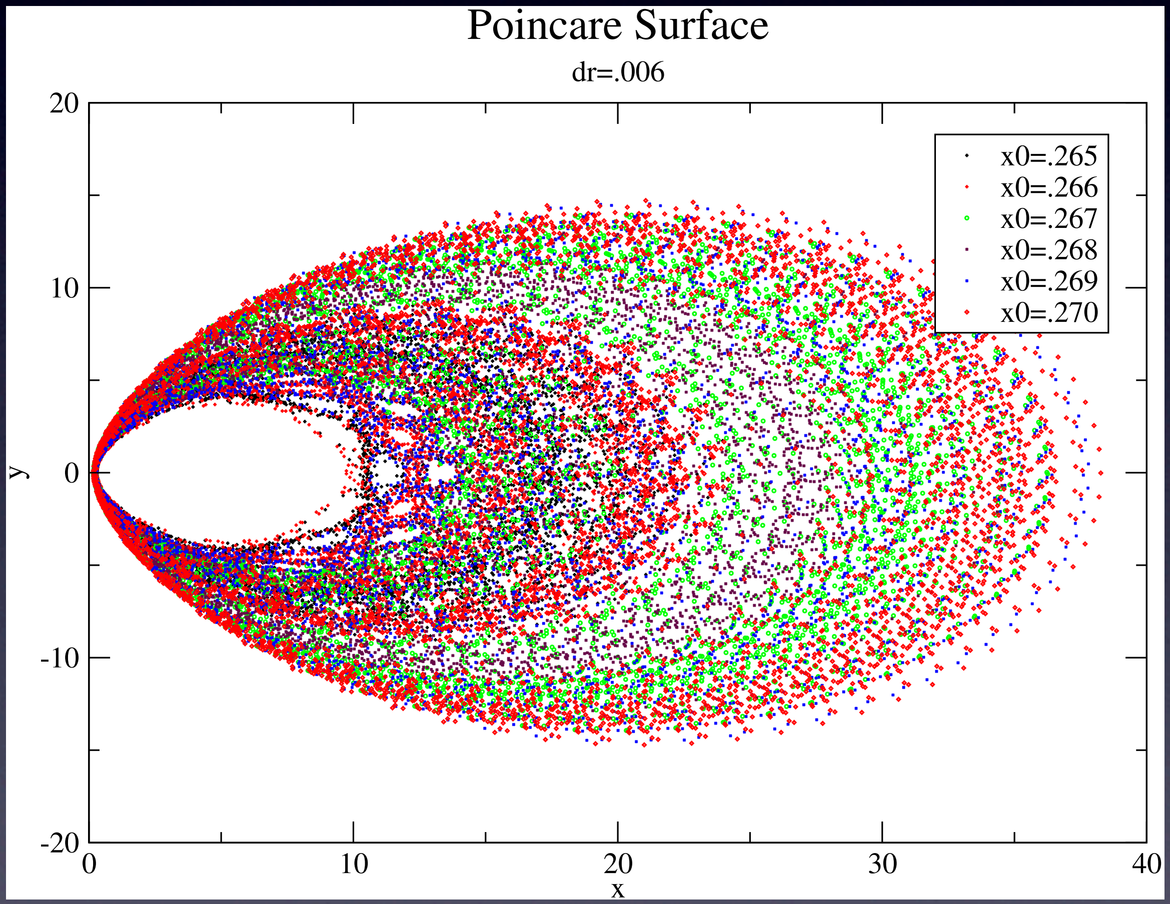
Time Independent Results: Poincare' Maps

Symmetric



$\Delta x=0$

Antisymmetric

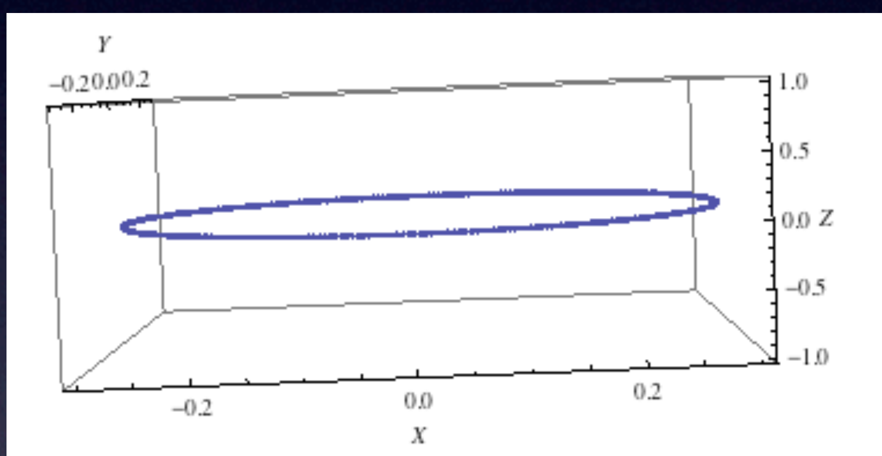


$\Delta x=0.006$

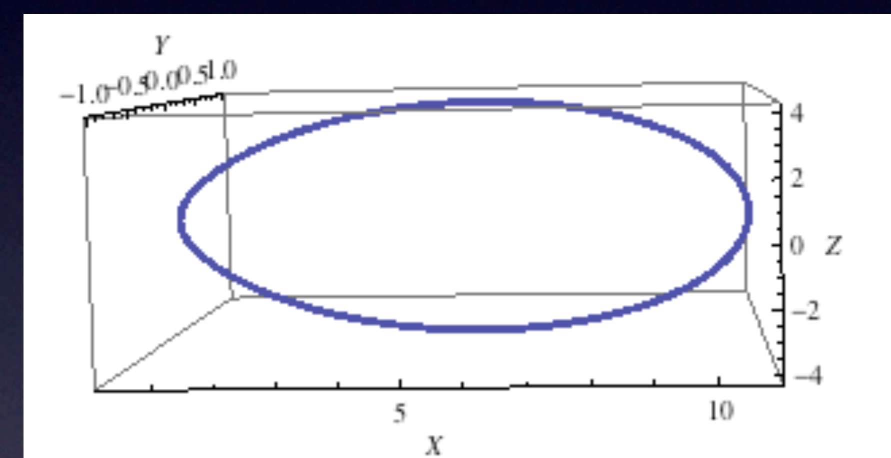
Time Dependent Results: Symmetric $\Delta x=0$, $x_0=0.3$

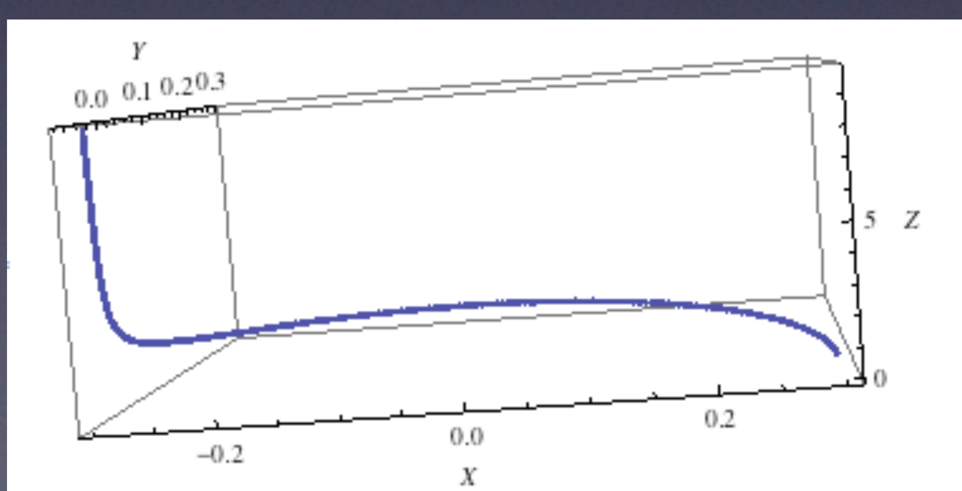
Vector Potential

Magnetic Field

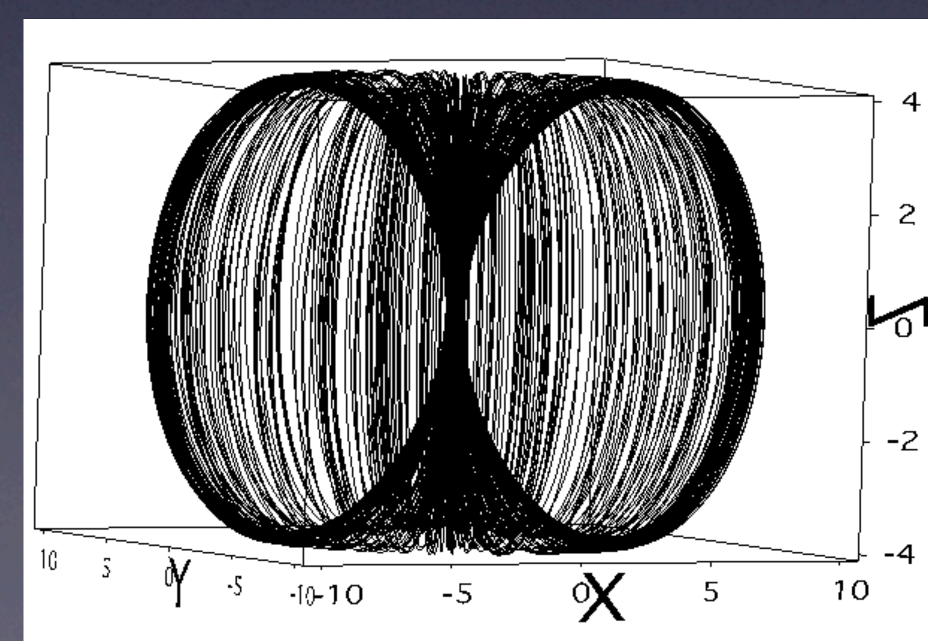


$t=.9x_0c$





$t=5x_0c$



Retarded Vector Potential and Magnetic Field

Instead of using a constant current , I_{wire} it would be more interesting to turn on the wire at t=0. Since information does not instantly, we must account for this using the retarded time.

$$t_r = t - r/c$$

The retarded potential and magnetic field are shown on the right.

$$I(t) = \begin{cases} I_{wire} & \text{for } t > 0 \\ 0 & \text{for } t \leq 0. \end{cases}$$

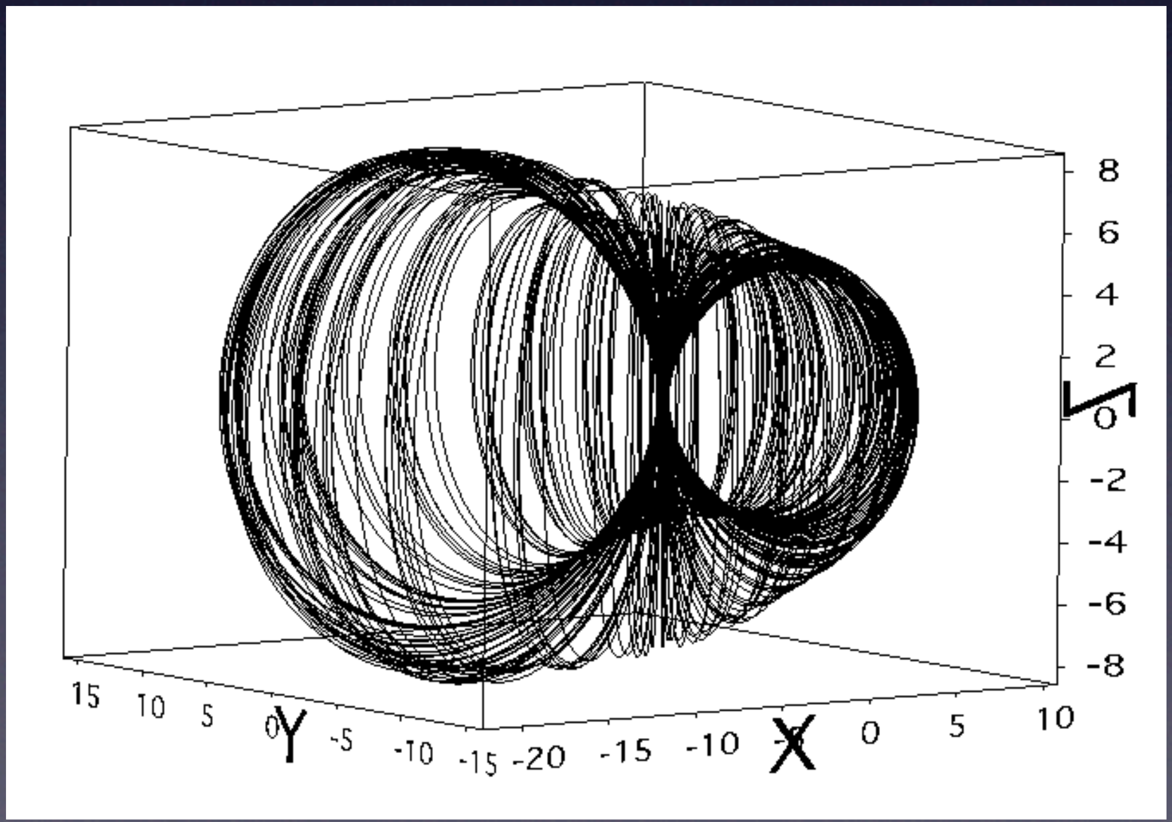
$$\vec{A} = \begin{cases} C_2 ln\left(\frac{ct+\sqrt{(ct)^2-s^2}}{s}\right)\hat{z} & \text{for } ct > \zeta \\ 0 & \text{for } ct < \zeta \end{cases}$$

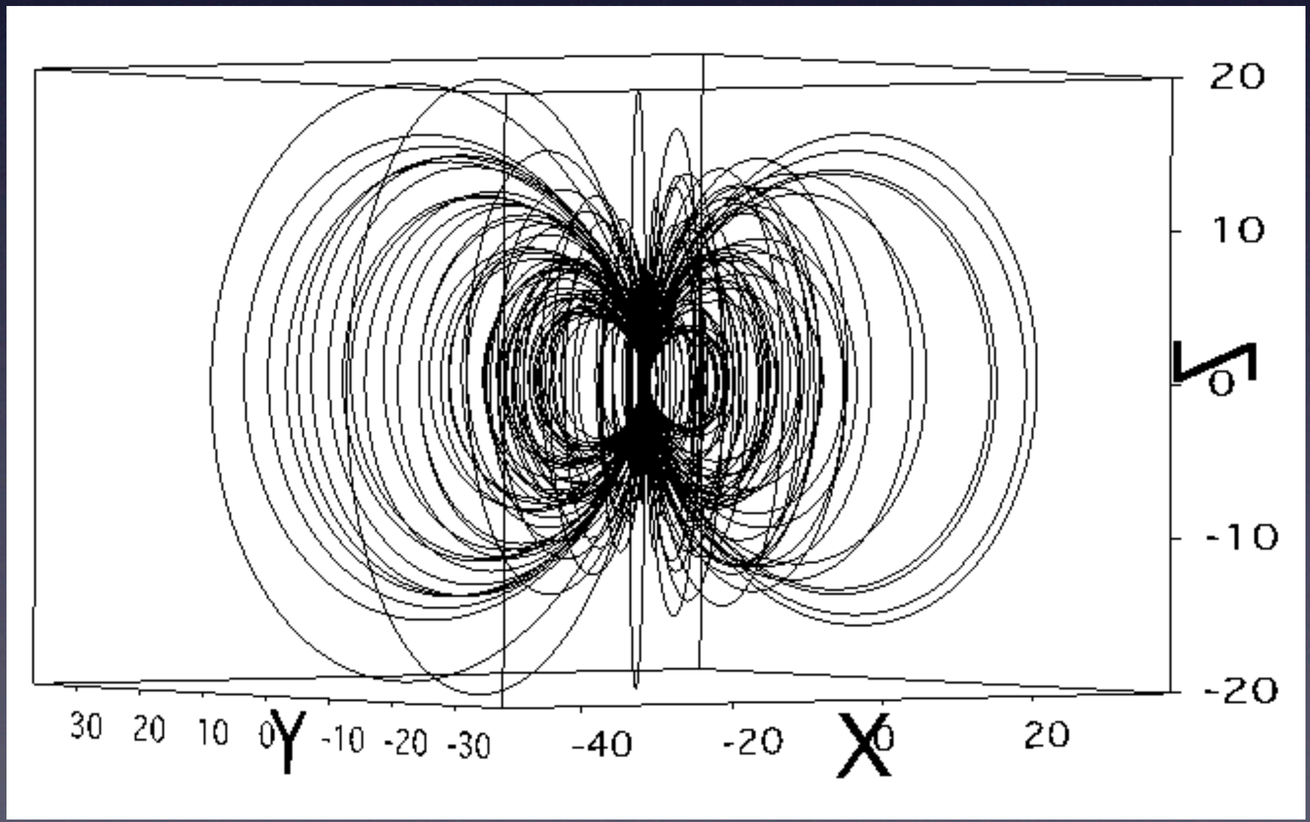
$$\vec{B} = \begin{cases} C_2 \frac{ct}{s\sqrt{(ct)^2-s^2}}\hat{\phi} & \text{for } ct > \zeta \\ 0 & \text{for } ct < \zeta \end{cases}$$

$$\zeta = |r - r'|$$

Time Dependent Results: Asymmetric Magnetic Field $\Delta x=0.05$, $x_0=0.3$

$t=1.1x_0c$ $t=5.0x_0c$





Summary and Future Work

- The loop wire system can lead to chaotic magnetic fields when an asymmetry is introduced.
- The vector potential shows no signs of chaos and behaves as we expect.
- The Poincare’ map is similar to that of Li et al.
- The time dependent case still required an asymmetry for chaos, but produced very odd maps when chaos occurred.
- The method could be expanded to more complicated time varying currents such as sinusoidal currents.

Appendix

(1) Griffiths, D. J.,Introduction to Electrodynamics, Prentice-Hall, New Jersey, p. 425-426, 1981, 1989, and 1999
(2) Jackson, J. D., Classical Electrodynamics, Wiley, New York, p. 181-182, 1999
(3) Landau, R., M. Paez, C. Bordeianu, Computational Physics, Wiley, 2007

(4) Li, G., B. Dasgupta, G. Webb, and A. K. Ram, Particle Motion and Energization in a Chaotic Magnetic Field
(5) Simpson, J., J. Lane, C. Immer, and R. Youngquist, Simple Analytic Expressions for the Magnetic Field of a Circular Current Loop, nasa techdoc 20010038494, 2001

(6) Weaver, R., Complete Elliptic Intergrals of the First and Second Kind, http://www.oooforum.org/forum/vie 2011