Turbulence Transport Model Applied Space Physics And Astrophysics

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Background

-Magnetohydrodynamics (MHD)Turbulence:

Magneto – magnetic fluid

hydro - Liquid

dynamics - movement

-MHD deals with dynamics of electrically conducting fluids.

-MHD Turbulence is observed when the Reynolds number of magnetofluid is large.

Background

- Sun - source of solar wind which is highly inhomogeneous magnetofluid expanding radially outward from it.
- The velocity and magnetic field in terms of fluctuating fields,

$$V =< V > +v$$

$$B =< B > +b$$

Where v is fluctuation on mean velocity $< V >$ and b is fluctuation on mean magnetic field $< B >$

- Elsässer variables (Elsässer 1950),

$$z^{\pm} = u \pm \frac{b}{\sqrt{4\pi\rho}}$$

Where z^{+} and z^{-} describe wave propagating outward and inward with respect to sun.

- The transport equation of evolution of fluctuations of u and b from their mean velocity and magnetic field in terms of elsässer variables (Zhou and Matthaeus, 1990a,b)

$$\frac{\partial z^{\pm}}{\partial t} + (U \mp V_A) \cdot \nabla z^{\pm} + \frac{1}{2} \nabla \cdot (U/2 \pm V_A) z^{\pm} + z^{\mp} \cdot \left[\nabla U \pm \frac{\nabla B}{\sqrt{4\pi\rho}} - \frac{1}{2} I \nabla \cdot (U/2 \pm V_A) \right] = NL_{\pm} + S^{\pm}$$

Where I is the identity matrix, $NL_{\pm}$ are the non-linear terms, $S^{\pm}$ is the source term.

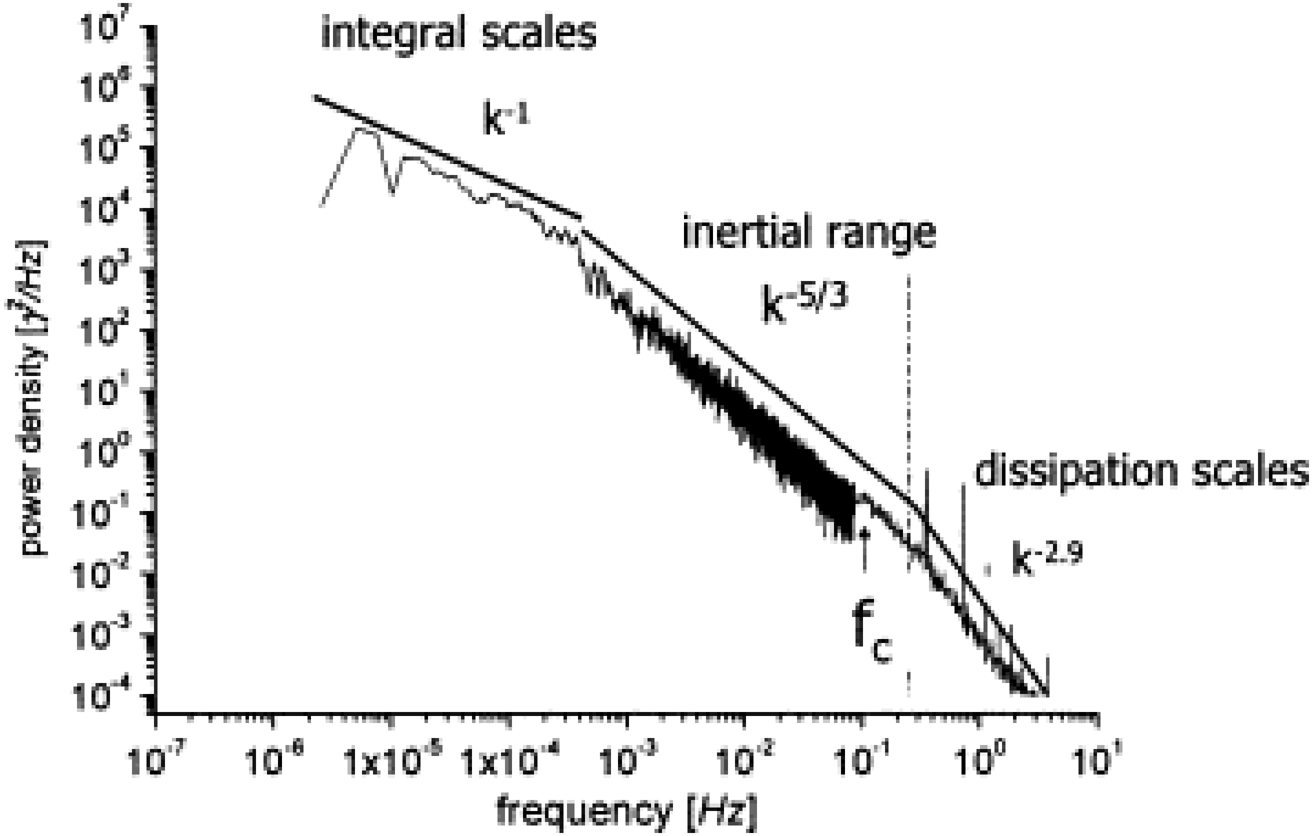
- Transport equations of Normalized energy density of magnetic fluctuation($\bar{E}_b$) and correlation length($\bar{\lambda}$)

$$\frac{\partial \bar{E}_b}{\partial \bar{t}} + \bar{U} \frac{\partial \bar{E}_b}{\partial \bar{r}} + (1 - \Gamma) \frac{\bar{U}}{\bar{r}} \bar{E}_b = -\frac{D \bar{E}_b^{\frac{3}{2}}}{\bar{\lambda}} + S \frac{r_0}{u_0 E_{bo}}$$

$$\frac{\partial \bar{\lambda}}{\partial \bar{t}} + \bar{U} \frac{\partial \bar{\lambda}}{\partial \bar{r}} + \Gamma \frac{\bar{U}}{\bar{r}} \bar{\lambda} = \frac{D \bar{E}_b^{\frac{1}{2}}}{2} - \frac{\bar{l}}{2\bar{E}_b} \frac{l_0 r_0}{E_{bo} u_0} S$$

Where $D = \frac{r_0}{l_0} \frac{\sqrt{E_{bo}}}{u_0}$ is a constant ($r_0 = 1AU$, $l_0 = 0.01AU$, $u_0 = 350km/s$, $E_{b0} = 540(km/s)^2$). I have put D=3.9, $\bar{U} = 1$ in the numerical solution.

Background
-The spectrum of magnetic/velocity fluctuation is kolmogorov.



Undriven Models

- No source to drive the turbulence

$$\frac{\partial \bar{E}_b}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{E}_b}{\partial \bar{r}} + (1-\Gamma)\,\frac{\bar{U}}{\bar{r}}\bar{E}_b = -\frac{D\bar{E}_b^{\frac{3}{2}}}{\bar{\lambda}}$$

$$\frac{\partial \bar{\lambda}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{\lambda}}{\partial \bar{r}} + \Gamma\frac{\bar{U}}{\bar{r}}\bar{\lambda} = \frac{D\bar{E}_b^{\frac{1}{2}}}{2}$$

- The discretization of $\bar{E}_b$ and $\bar{\lambda}$ based on explicit finite difference method,

$$\bar{E}_{bi}^{(n+1)} = \bar{E}_{bi}^{n} - \nu\left[\bar{E}_{bi}^{n} - \bar{E}_{(i-1)}^{n}\right] + \left[-D\frac{\bar{E}_i^{\frac{3n}{2}}}{\bar{\lambda}_i^n} - (1-\Gamma)\bar{U}\frac{\bar{E}_i^n}{\bar{r}_i^n}\right]\Delta\bar{t}$$

$$\bar{\lambda}_i^{(n+1)} = \bar{\lambda}_i^n - \nu\left[\bar{\lambda}_i^n - \bar{\lambda}_{(i-1)}^n\right] + \left[D\frac{\bar{E}_i^{\frac{n}{2}}}{2} - \Gamma\frac{\bar{U}}{\bar{r}_i^n}\bar{\lambda}_i^n\right]\Delta\bar{t}$$

  Where $\Delta\bar{r}(=0.045)$ is the step size, $\Delta\bar{t}(=0.036)$ is the time step and $\nu = \bar{U}\frac{\Delta\bar{t}}{\Delta\bar{r}} > 0$
- Steady state solution

$$\bar{E}_b = \frac{\left(\frac{1}{\bar{r}}\right)^{1-\Gamma}}{(\bar{r}_0)^{2\Gamma}\,\bar{l}_0^2 + \frac{2D}{(1+3\Gamma)}\left[\bar{r}^{\frac{1+3\Gamma}{2}} - \bar{r}_0^{\frac{1+3\Gamma}{2}}\right]}$$

$$\bar{\lambda} = \left(\frac{1}{\bar{r}}\right)^{\Gamma}\left[(\bar{r}_0)^{2\Gamma}\,\bar{l}_0^2 + \frac{2D}{(1+3\Gamma)}\left\{\bar{r}^{\frac{1+3\Gamma}{2}} - \bar{r}_0^{\frac{1+3\Gamma}{2}}\right\}\right]^{\frac{1}{2}}$$

  I have assumed $\bar{r}_0 = 1 = \frac{r_0}{r_0}$, and $\bar{l}_0 = 1 = \frac{l_0}{l_0}$ in the numerical solution.

- Initial and boundary condition $\bar{E}_b = (\frac{1}{\bar{r}})^{(\frac{3}{2})}$, $\bar{l} = (\bar{r})^{(\frac{1}{4})}$ for $\Gamma = 0$ and $\bar{E}_b = (\frac{1}{\bar{r}})^2$, $\bar{l} = 1$ for $\Gamma = 1$. And B.C $\bar{E}_b = 1$, $\bar{\lambda} = 1$

Stream Interaction Driven Models

- There is a Source to drive turbulence
  $S = c_{sh}\frac{U}{r}E_b$ where c_{sh} represents the strength of the stream shear interaction

$$\frac{\partial \bar{E}_b}{\partial \bar{t}} + \ \bar{U}\frac{\partial \bar{E}_b}{\partial \bar{r}} + (1-\Gamma)\,\frac{\bar{U}}{\bar{r}}\bar{E}_b = -\frac{D\bar{E}_b^{\frac{3}{2}}}{\bar{\lambda}} + C_{sh}\frac{\bar{U}}{\bar{r}}\bar{E}_b$$

$$\frac{\partial \bar{\lambda}}{\partial \bar{t}} + \ \bar{U}\frac{\partial \bar{\lambda}}{\partial \bar{r}} + \Gamma\frac{\bar{U}}{\bar{r}}\bar{\lambda} = \frac{D\bar{E}_b^{\frac{1}{2}}}{2} - C_{sh}\frac{\bar{U}}{2\bar{r}}\bar{\lambda}$$

  $C_{sh} \sim 10$ for typical solar wind particle and $D = \frac{r_0}{l_0}\frac{\sqrt{E_{bo}}}{u_0}$
- The discretization of $\bar{E}_b$ and $\bar{\lambda}$ based on explicit finite difference method,

$$\bar{E}_{bi}^{(n+1)} = \bar{E}_{bi}^{n} - \nu\left[\bar{E}_{bi}^{n} - \bar{E}_{b(i-1)}^{n}\right] + \left[-D\frac{\bar{E}_{bi}^{\frac{3n}{2}}}{\bar{\lambda}_i^n} - (1-\Gamma)\bar{u}_r\frac{\bar{E}_{bi}^{n}}{\bar{r}_i^n} + C_{sh}\bar{u}_r\frac{\bar{E}_{bi}^{n}}{\bar{r}_i^n}\right]\Delta\bar{t}$$

$$\bar{\lambda}_i^{(n+1)} = \bar{\lambda}_i^n - \nu\left[l_i^n - l_{(i-1)}^n\right] + \left[D\frac{\bar{E}_{bi}^{\frac{n}{2}}}{2} - \Gamma\frac{\bar{u}_r}{\bar{r}_i^n}\bar{\lambda}_i^n - C_{sh}\bar{u}_r\frac{\lambda_i^n}{2\bar{r}_i^n}\right]\Delta\bar{t}$$

- Steady state solution

$$\bar{E}_b = \frac{\left(\frac{1}{\bar{r}}\right)^{1-\Gamma-C_{sh}}}{1 + DF^{-1}(\bar{r}^F - 1)}$$

$$\bar{\lambda} = \left(\frac{1}{\bar{r}}\right)^{\Gamma+\frac{C_{sh}}{2}}\left[1 + DF^{-1}(\bar{r}^F - 1)\right]^{\frac{1}{2}}$$

  Where $F = \frac{1+2C_{sh}+3\Gamma}{2}$

Pickup Ion Driven Models

- The source to drive turbulence is Pickup Ion
- Turbulent Transport of Normalized energy density magnetic fluctuation ($\bar{E}_b$) and correlation length ($\bar{\lambda}$)

$$\frac{\partial \bar{E}_b}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{E}_b}{\partial \bar{r}} + (1-\Gamma)\frac{\bar{U}}{\bar{r}}\bar{E}_b = -\frac{D\bar{E}_b^{\frac{3}{2}}}{\bar{\lambda}} + G\bar{U}exp(-\frac{H}{\bar{r}})$$

$$\frac{\partial \bar{\lambda}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{\lambda}}{\partial \bar{r}} + \Gamma\frac{\bar{U}}{\bar{r}}\bar{\lambda} = \frac{D\bar{E}_b^{\frac{1}{2}}}{2} - G\bar{U}\frac{\bar{\lambda}}{2\bar{E}_b}exp(-\frac{H}{\bar{r}})$$

Where $G = \frac{r_0 C_{PI}}{E_{b0}} = 0.4$, $H = \frac{\lambda_I}{r_0} = 8$. Also, $C_{PI} = \frac{V_A n_H^{\infty}}{\tau_{ion} n_{SW}^0}$, $\lambda_I = \frac{\lambda\theta}{sin\theta}$

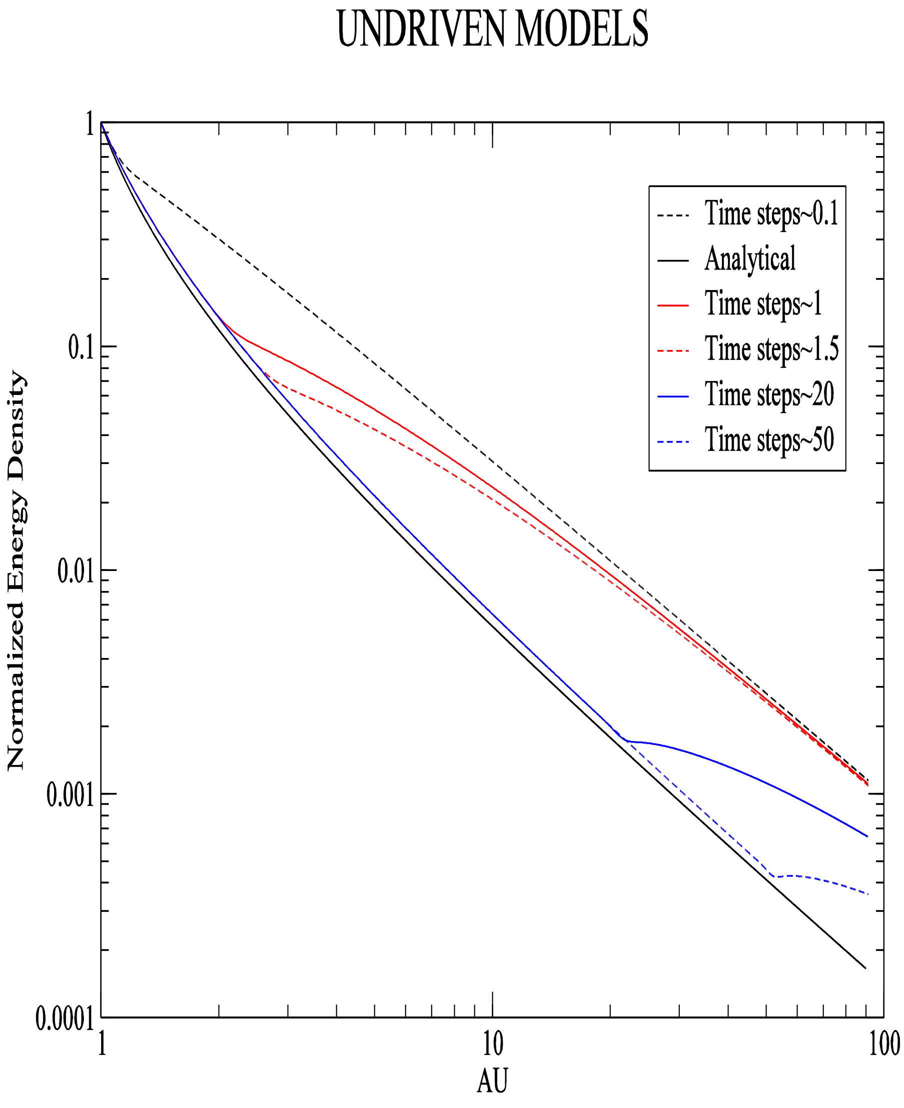
- the discretization of $\bar{E}_b$ and $\bar{\lambda}$ based on explicit finite difference method,

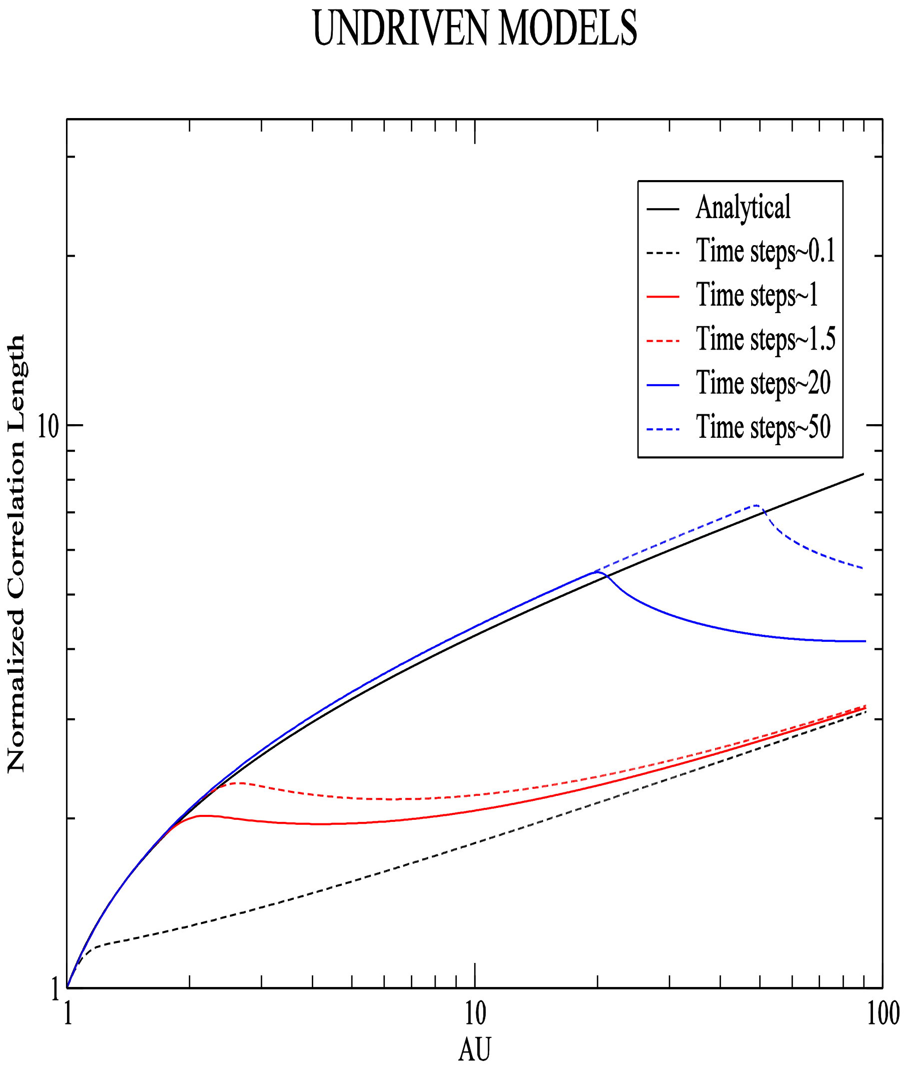
$$\bar{E}_{bi}^{(n+1)} = \bar{E}_{bi}^n - \nu\left[\bar{E}_{bi}^n - \bar{E}_{b(i-1)}^n\right] + \left[-D\frac{\bar{E}_{bi}^{\frac{3n}{2}}}{\bar{\lambda}_i^n} - (1-\Gamma)\bar{u}_r\frac{\bar{E}_{bi}^n}{\bar{r}_i^n} + G\bar{u}_r exp(-\frac{H}{\bar{r}_i^n})\right]\Delta\bar{t}$$

$$\bar{\lambda}_i^{(n+1)} = \bar{\lambda}_i^n - \nu\left[l_i^n - l_{(i-1)}^n\right] + \left[D\frac{\bar{E}_{bi}^{\frac{n}{2}}}{2} - \Gamma\frac{\bar{u}_r}{\bar{r}_i^n}\bar{\lambda}_i^n - G\bar{u}_r\frac{\bar{\lambda}_i^n}{2\bar{E}_{bi}^n}exp(-\frac{H}{\bar{r}_i^n})\right]\Delta\bar{t}$$

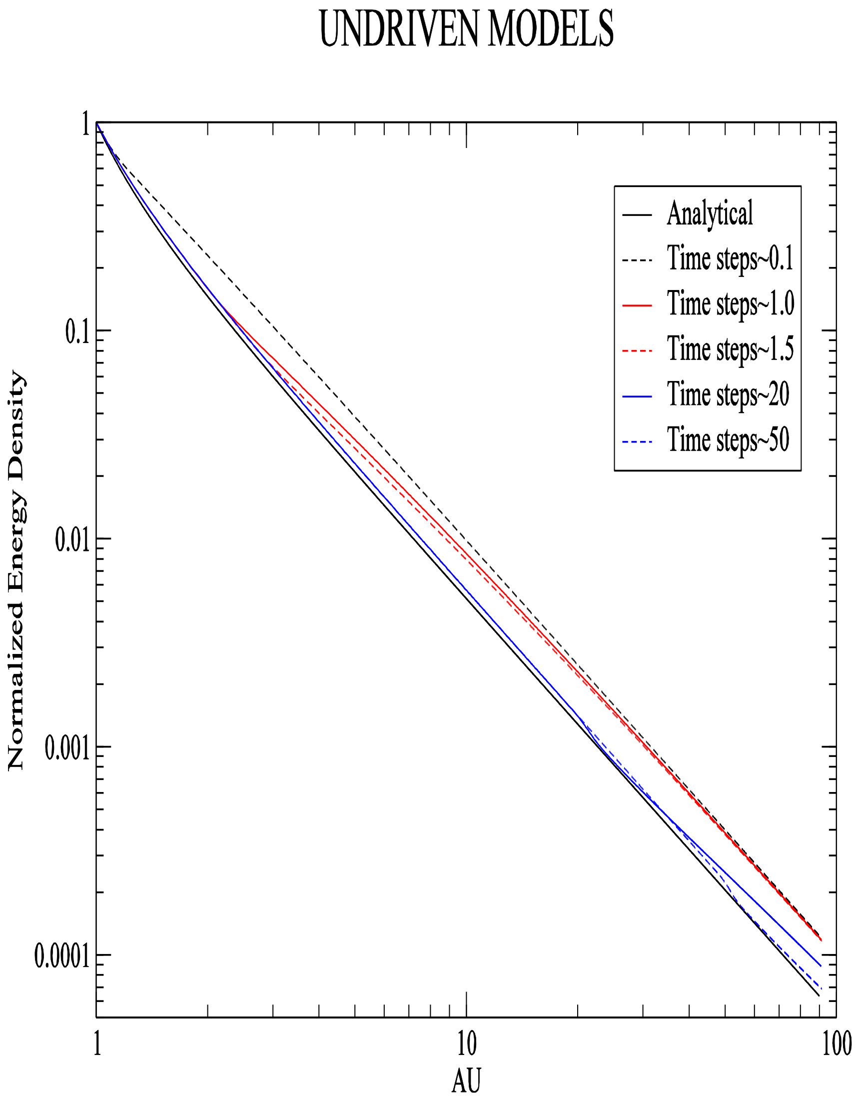
I have used $\Delta\bar{t} = 0.0001$ and $\Delta\bar{r} = 0.045$

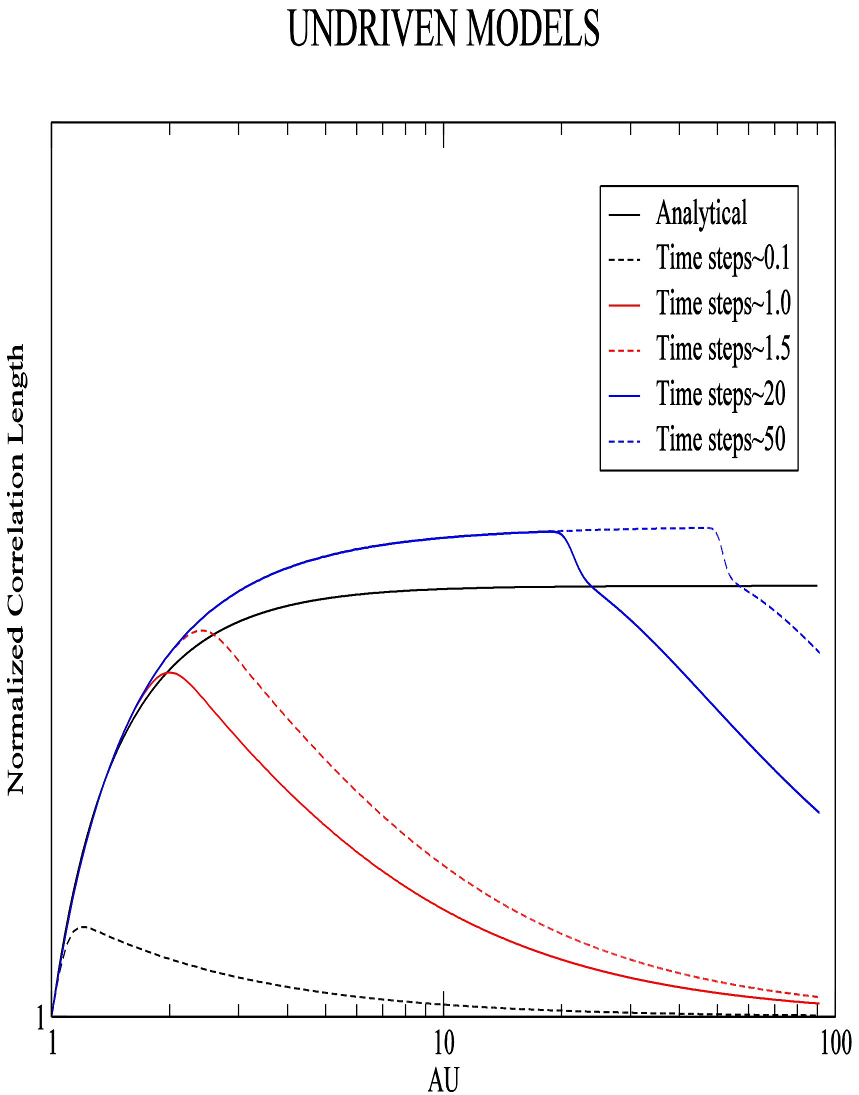
Undriven Models(No Mixing)



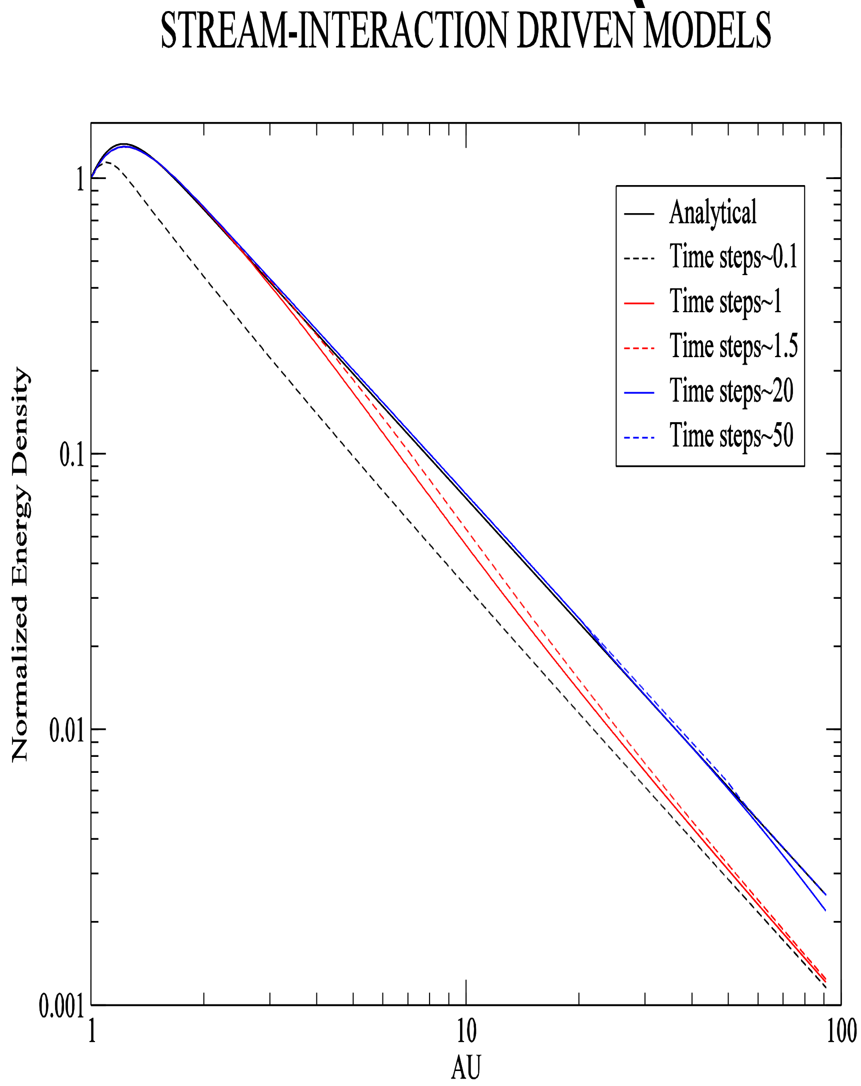


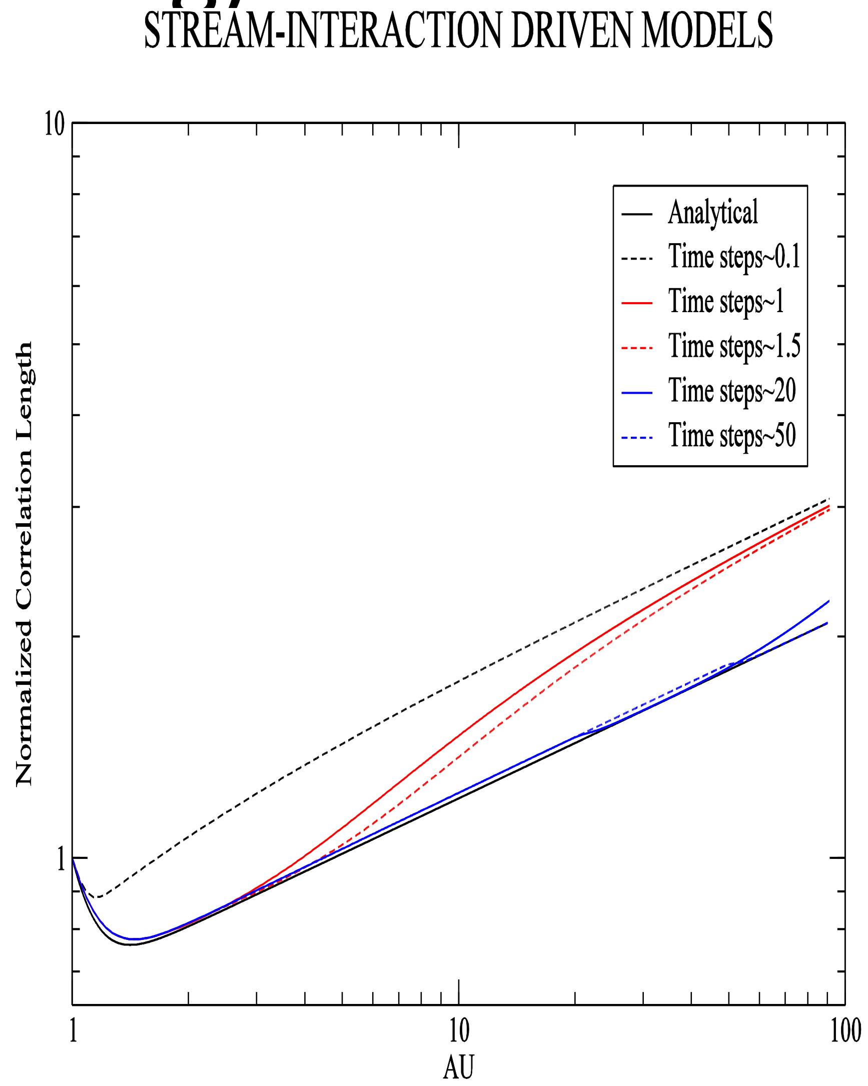
Undriven Models (Strong Mixing)





Stream Interaction Driven Models (No Mixing)





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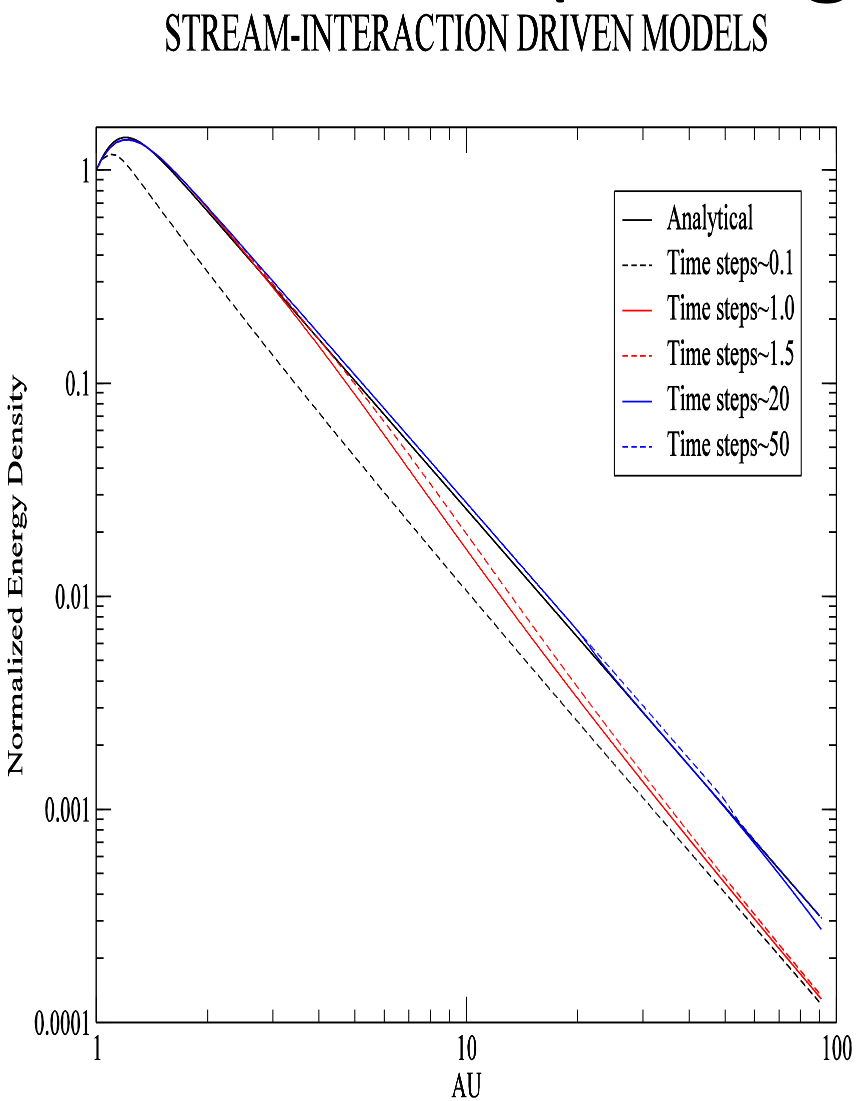
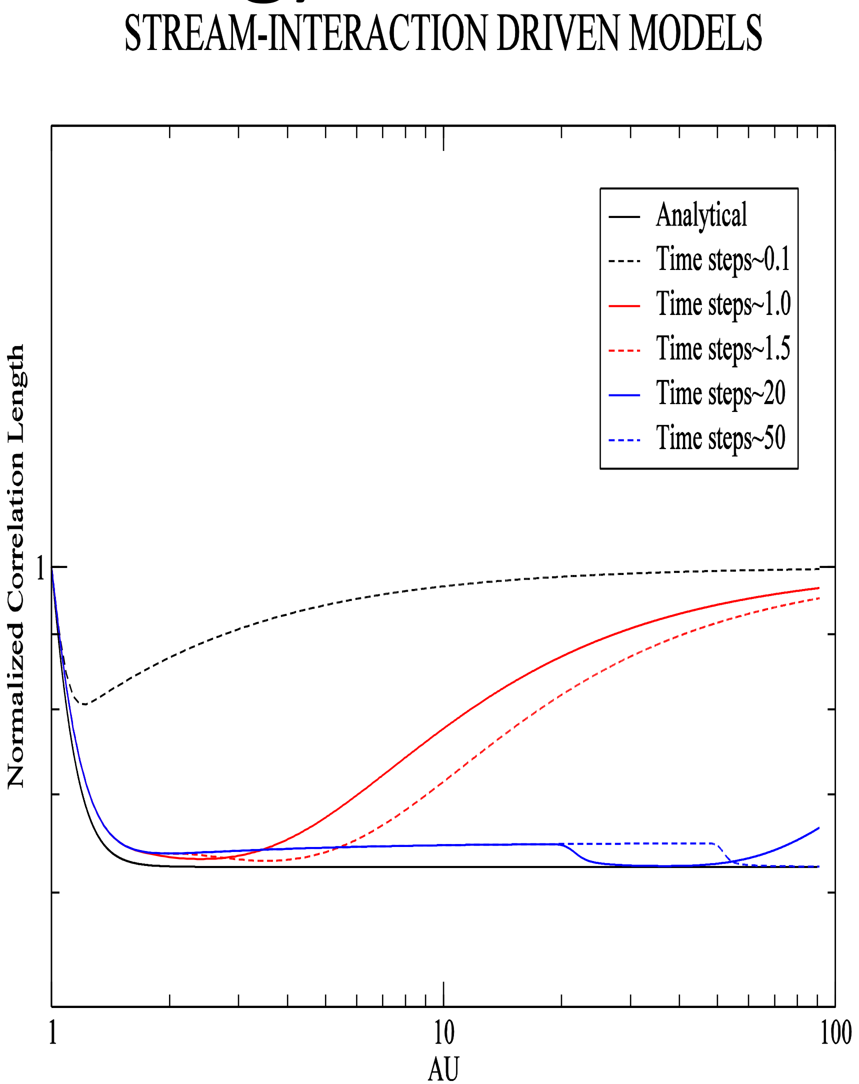
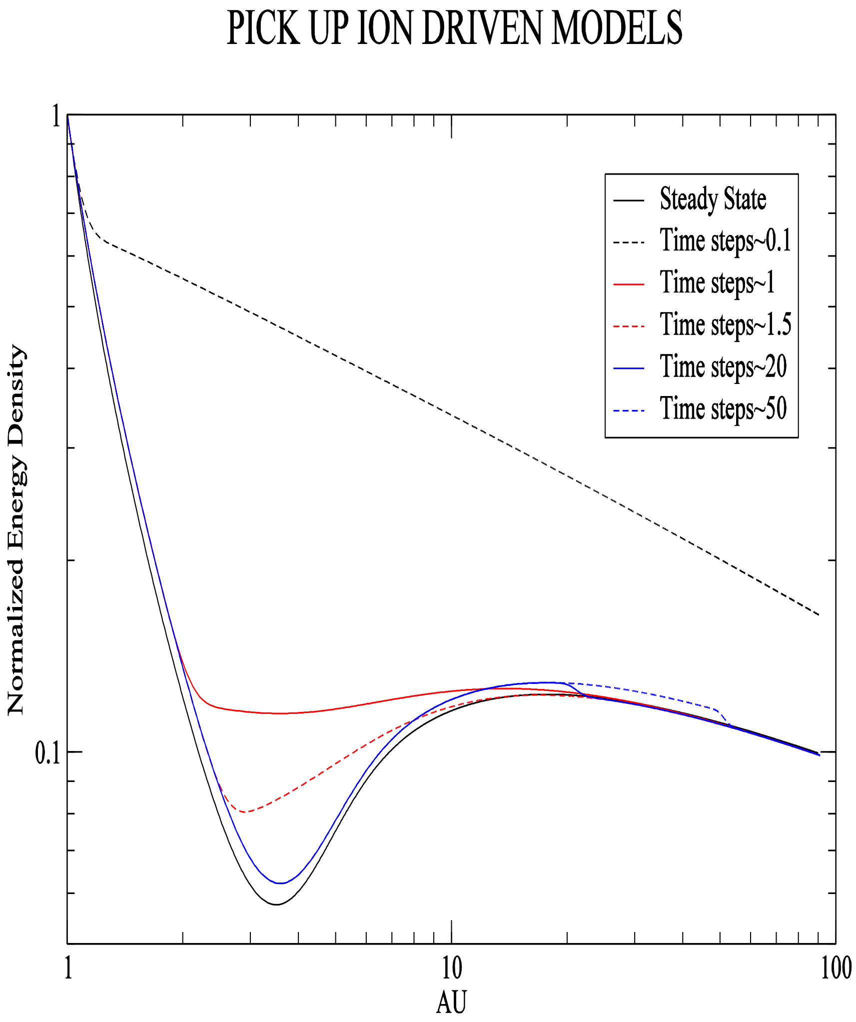
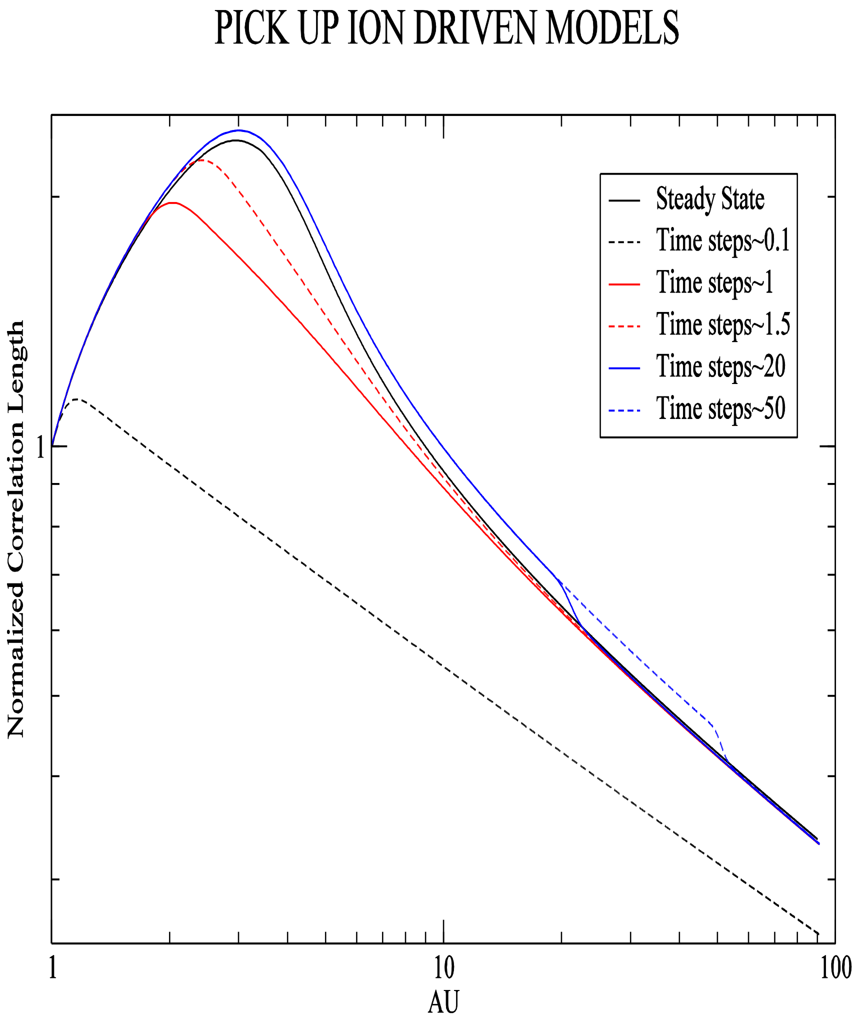


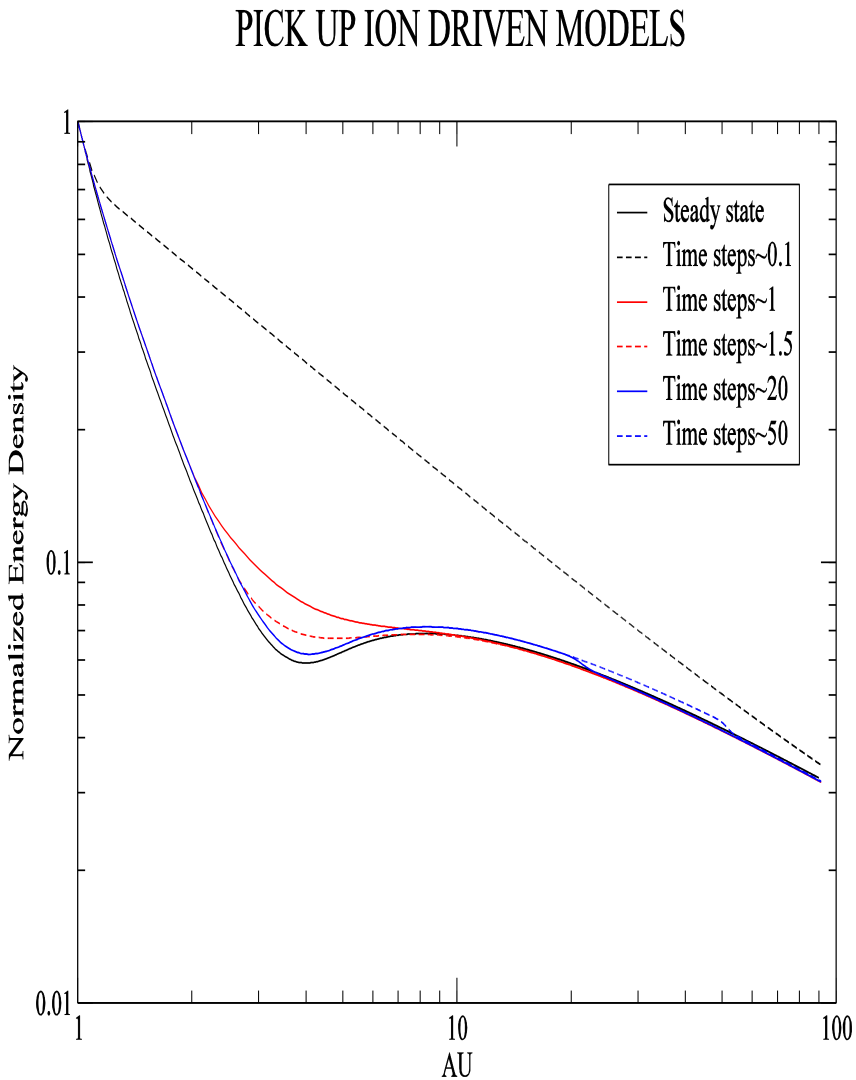


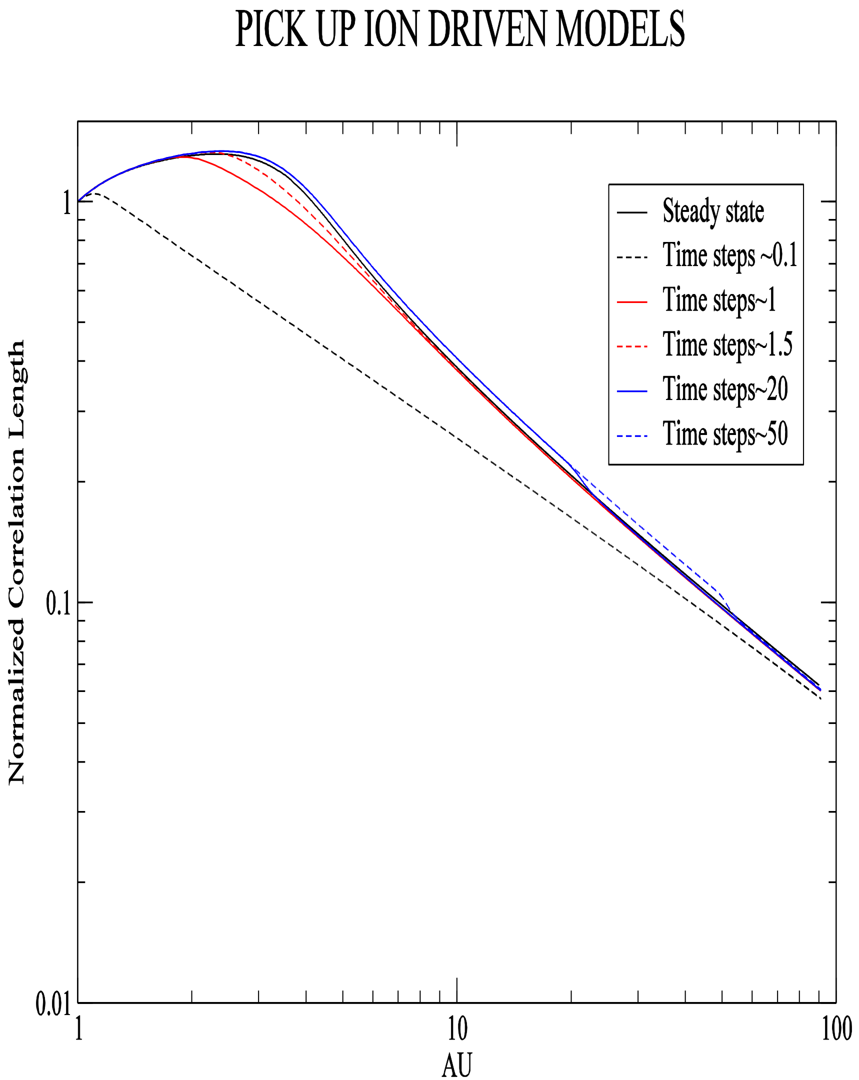
Pickup Ion Driven Models (No Mixing)





Pickup Ion Driven Model (Strong Mixing)





Summary

-Generally, the fluctuation of magnetic energy decreases with the increase of radial distance. On the other hand the correlation length increases which we can see in undriven models.

-There is a slight increase of magnetic energy density in driven model which is because of the source term.

-The correlation length in the case of pickup ion driven models decreases with increase of radial distance which is different than other two models.

- Most of the systems are in unsteady state first and as the time passes it is getting to steady state which we can see in the result.