Computational Methods for Kinetic Processes in Plasma Physics

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Context

- Three-dimensional current deposit
  by Villasenor & Buneman
- Zigzag scheme in two-dimensional systems
  by Umeda

Current deposit scheme (2-D)

$$\nabla \bullet E = 4\pi\rho,\ \nabla \bullet \frac{\partial E}{\partial t} = 4\pi\frac{\partial \rho}{\partial t},\ \nabla \bullet (c\nabla \times B - 4\pi J) = 4\pi\frac{\partial \rho}{\partial t},$$

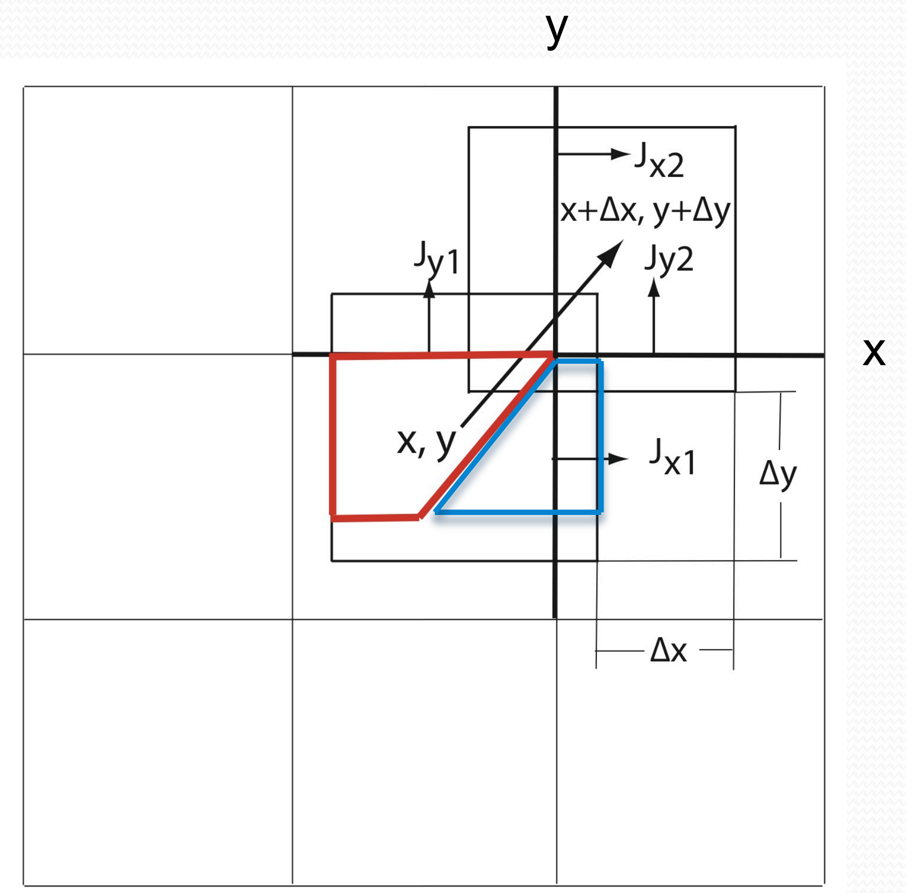
$$\frac{\partial \rho}{\partial t} = -\nabla \bullet J$$

$$J_{x1} = q\Delta x(\frac{1}{2} - y - \frac{1}{2}\Delta y)$$

$$J_{x2} = q\Delta x(\frac{1}{2} + y + \frac{1}{2}\Delta y)$$

$$\rightarrow\ J_{y1} = q\Delta y(\frac{1}{2} - x - \frac{1}{2}\Delta x)$$

$$\rightarrow\ J_{y2} = q\Delta y(\frac{1}{2} + x + \frac{1}{2}\Delta x)$$



3D current deposit

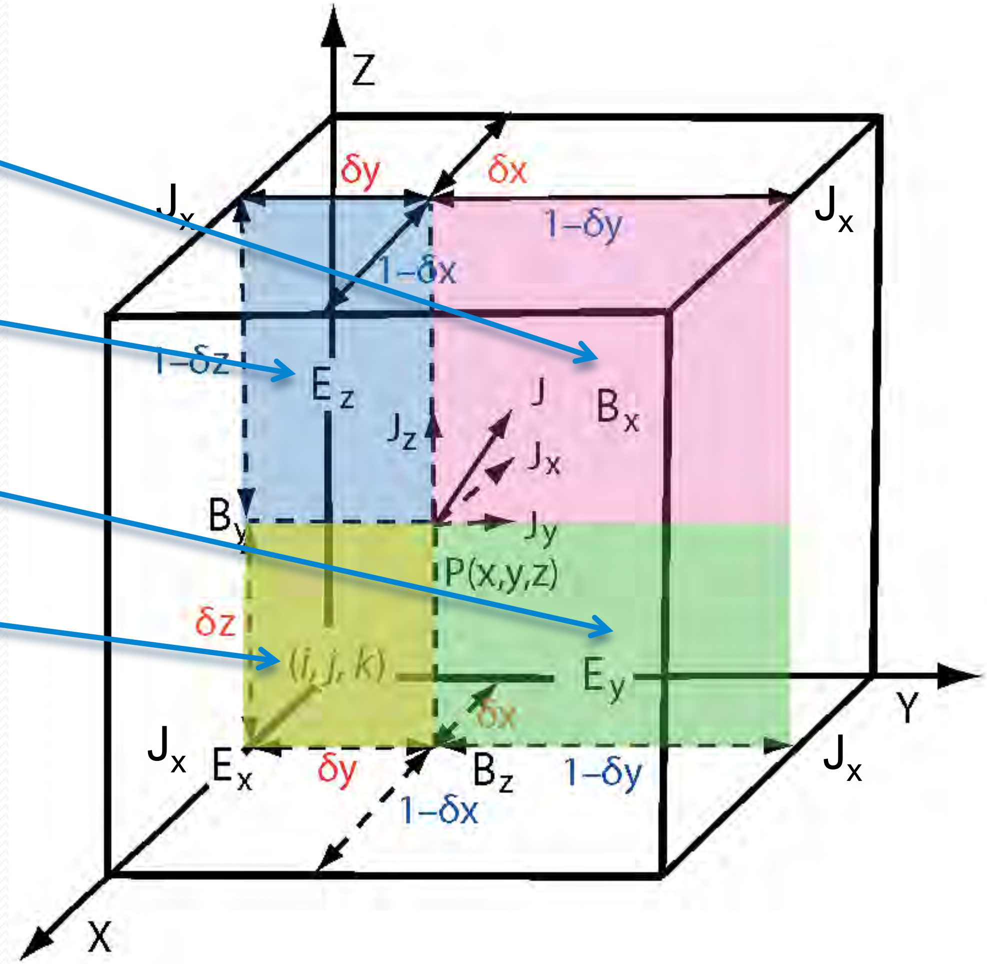
$$x = i + \delta x = i + \xi, y = j + \delta y = j + \eta, z = k + \delta z = k + \zeta$$

$$(1-\eta)(1-\zeta)$$

$$\eta(1-\zeta)$$

$$(1-\eta)\zeta$$

$$\eta\zeta$$



3D current deposit

Particle moves from $(i+\xi_1, j+\eta_1, k+\zeta_1)$ to $(i+\xi_2, j+\eta_2, k+\zeta_2)$

$$\Delta x = \xi_2 - \xi_1,\ \Delta y = \eta_2 - \eta_1,\ \Delta z = \zeta_2 - \zeta_1$$ between t= −1/2 and t = +1/2

at t = 0 $\bar{\zeta} = (\xi_2 + \xi_1)/2, \bar{\eta} = (\eta_2 + \eta_1)/2, \bar{\zeta} = (\zeta_2 + \zeta_1)/2$

- $J_x = \int_{\xi_1}^{\xi_2} \eta(t)\zeta(t)\,d\xi$

$$= \int_{-1/2}^{1/2} (\bar{\eta} + t\Delta y)(\bar{\zeta} + t\Delta z)\Delta x\,dt$$

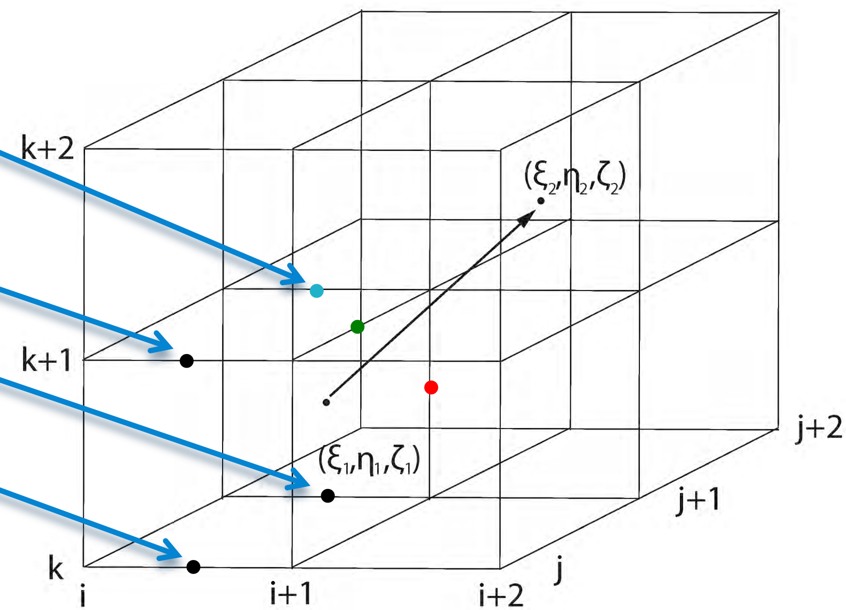
$$= \Delta x \bar{\eta}\bar{\zeta} + \Delta x \Delta y \Delta z / 12$$

- J_y at (i+1, j+1/2, k+1)
- J_z at (i+1, j+1, k+1/2)

$$\Delta x(1-\bar{\eta})\bar{\zeta} - \Delta x \Delta y \Delta z / 12$$

$$\Delta x \bar{\eta}(1-\bar{\zeta}) - \Delta x \Delta y \Delta z / 12$$

$$\Delta x(1-\bar{\eta})(1-\bar{\zeta}) + \Delta x \Delta y \Delta z / 12$$



$$i, \Delta x, \bar{\eta} \Rightarrow j, \Delta y, \bar{\xi} \Rightarrow k, \Delta z, \bar{\zeta} \Rightarrow i, \Delta x, \bar{\eta}$$

The total fluxes into the cell indexed *i*+1, *j*+1, *k*+1

$$\Delta x\bar{\eta}\bar{\zeta} + \Delta y\bar{\zeta}\,\bar{\xi} + \Delta z\bar{\xi}\bar{\eta} + \Delta x\Delta y\Delta z / 4$$
$$= (\bar{\xi} + \tfrac{1}{2}\Delta x)(\bar{\eta} + \tfrac{1}{2}\Delta y)(\bar{\zeta} + \tfrac{1}{2}\Delta z)$$
$$- (\bar{\xi} - \tfrac{1}{2}\Delta x)(\bar{\eta} - \tfrac{1}{2}\Delta y)(\bar{\zeta} - \tfrac{1}{2}\Delta z)$$

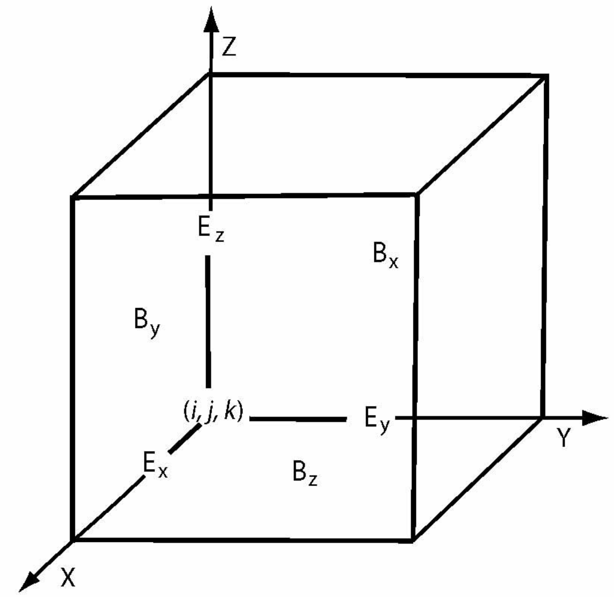
The difference between the particle fractions protruding into the cell before and after the move.

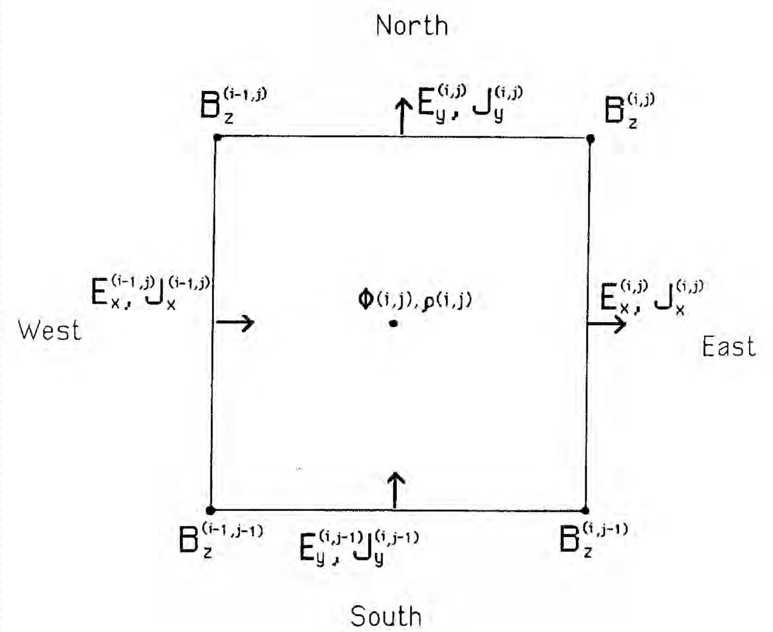
Staggered mesh with E and B

$$B_z^{new} - B_z^{old} = \frac{\delta t}{\delta x}\Big((E_x^{east} - E_x^{west}) - (E_y^{north} - E_y^{south})\Big),$$

$$E_x^{new} - E_x^{old} = \frac{\delta t}{\delta x}(B_z^{north} - B_z^{south}) - \delta t J_x,$$

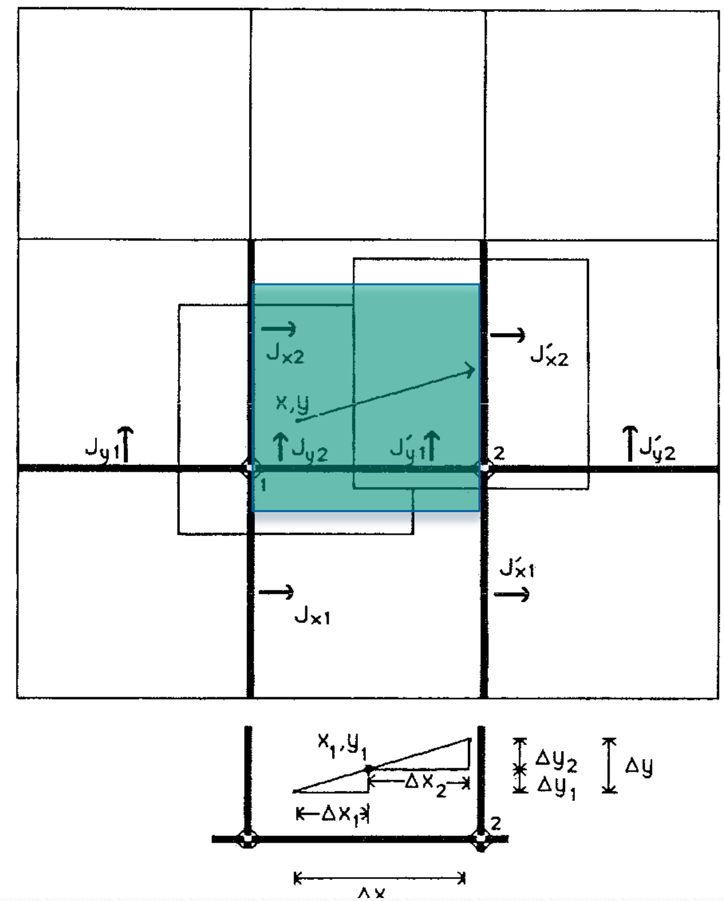
$$E_y^{new} - E_y^{old} = \frac{\delta t}{\delta x}(B_z^{east} - B_z^{west}) - \delta t J_y$$





Current deposition seven-boundary move

$\Delta x_1 = 0.5 - x$,
$\Delta y_1 = (\Delta y/\Delta x)\Delta x_1$,
$x_1 = -0.5$,
$y_1 = y + \Delta y_1$,
$\Delta x_2 = \Delta x - \Delta x_1$,
$\Delta y_2 = \Delta y - \Delta y_1$



$$J_{x1} = q\Delta x_1\left(\tfrac{1}{2} - y_1 - \tfrac{1}{2}\Delta y_1\right)$$

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$$J'_{x1} = q\Delta x_2\left(\tfrac{1}{2} - y - \tfrac{1}{2}\Delta y_2\right)$$

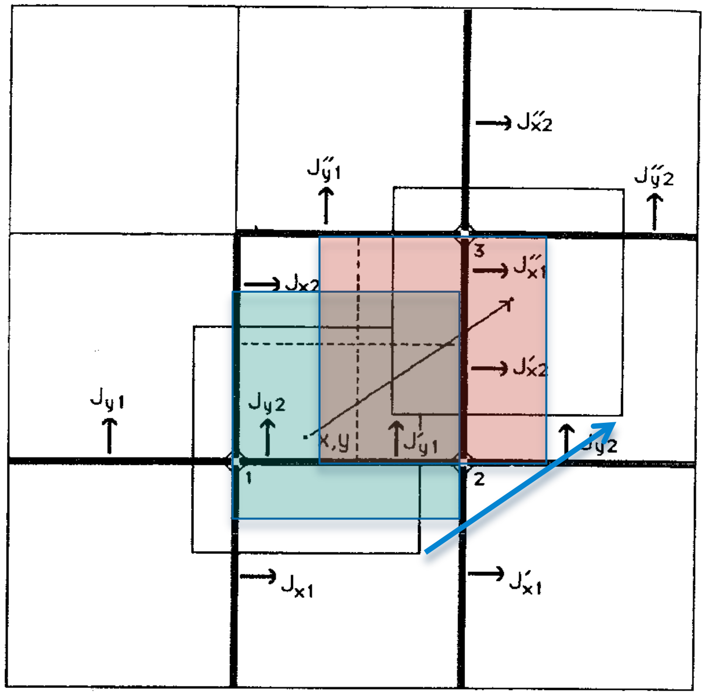
$$J'_{x2} = q\Delta x_2\left(\tfrac{1}{2} + y + \tfrac{1}{2}\Delta y_2\right)$$

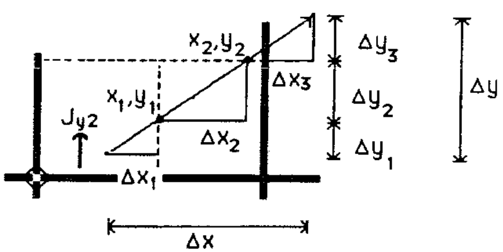
$$J'_{y1} = q\Delta y_2\left(\tfrac{1}{2} - x - \tfrac{1}{2}\Delta x_2\right)$$

$$J'_{y2} = q\Delta y_2\left(\tfrac{1}{2} + x + \tfrac{1}{2}\Delta x_2\right)$$

Current deposition ten-boundary move

$\Delta x_1=0.5-x$,
$\Delta y_1=(\Delta y/\Delta x)\Delta x_1$,
$x_1=-0.5$,
$y_1=y+\Delta y_{1,}$
$\Delta y_2=0.5y-y-\Delta y_1$,
$\Delta x_2=(\Delta x/\Delta y)\Delta y_2$,
$x_2=\Delta x_2-0.5$,
$y_2=0.5$,
$\Delta x_3=\Delta x-\Delta x_1-\Delta x_2$,
$\Delta y_3=\Delta y-\Delta y_1-\Delta y_2$





Current deposit scheme (2-D)

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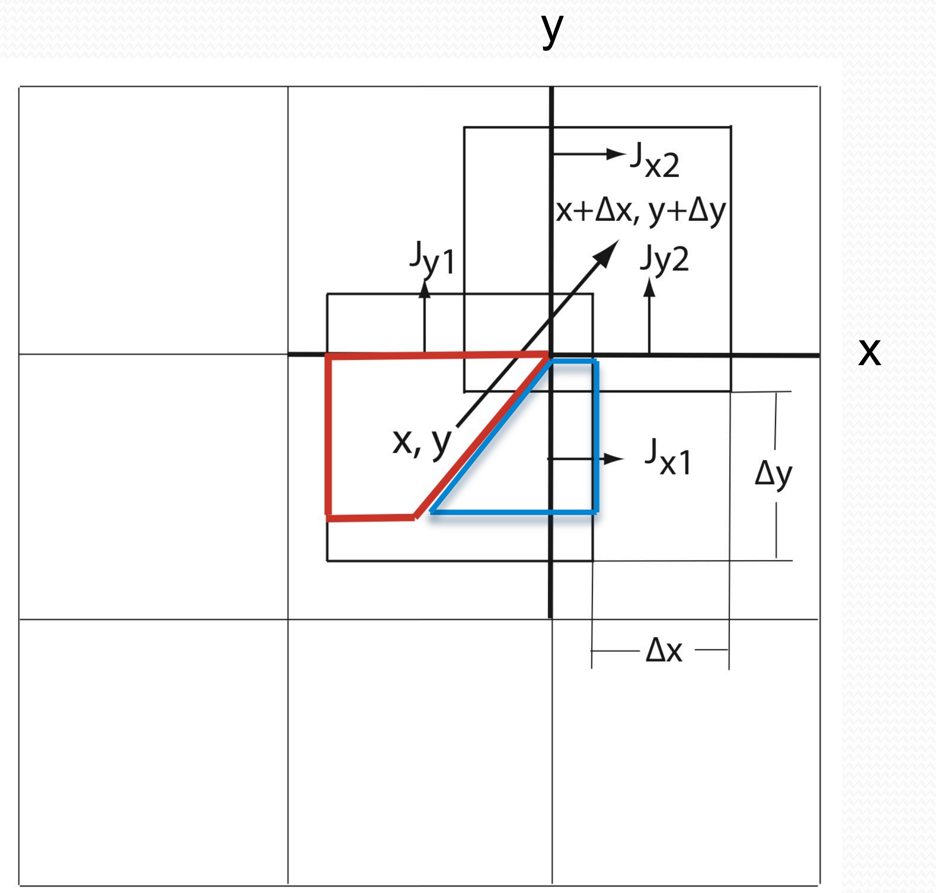
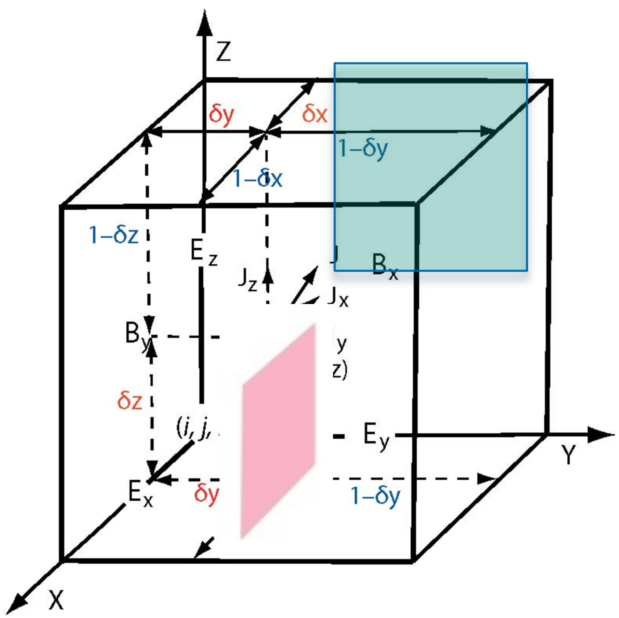
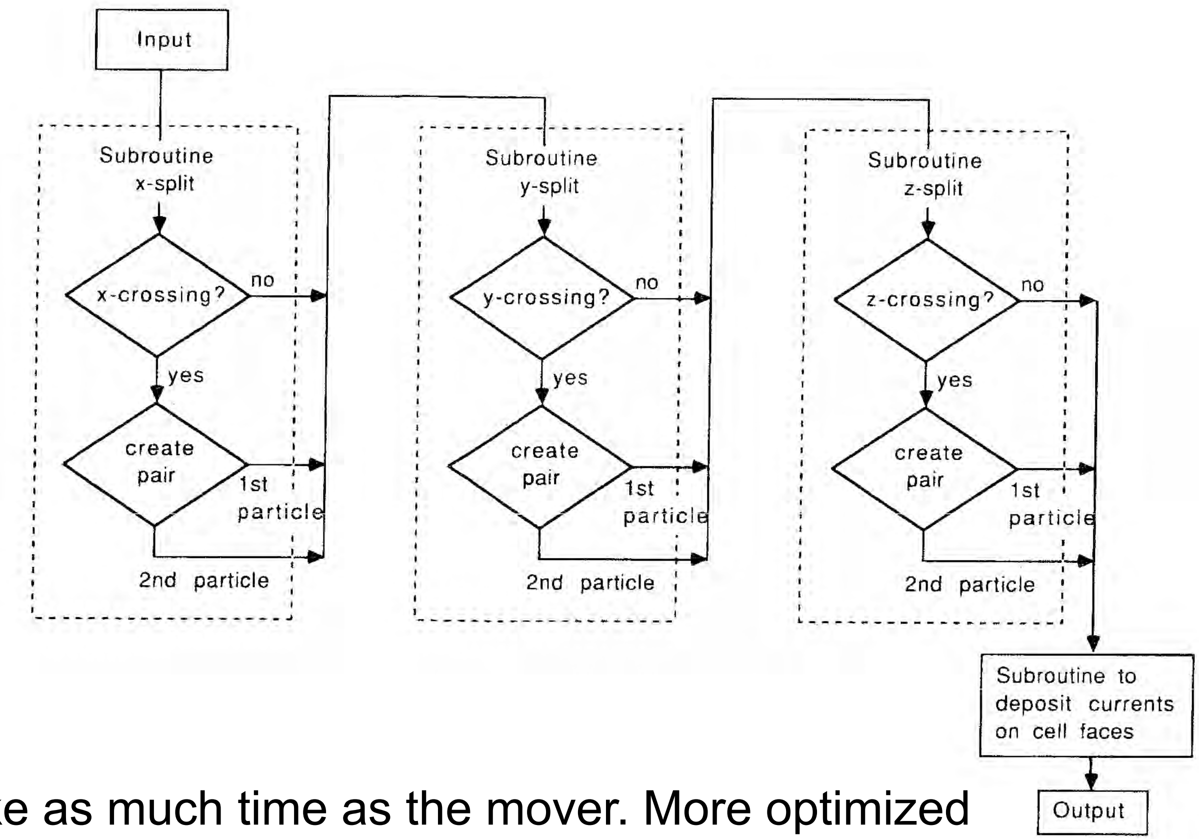


Charge and current deposition





Current deposition can take as much time as the mover. More optimized deposits exist (Umeda 2003).

Charge conservation makes the whole Maxwell solver local and hyperbolic. Static fields can be established dynamically.

Zigzag scheme in two-dimensional systems

$$\frac{J_x^{t+\Delta t/2}(i+\frac{1}{2},j) - J_x^{t+\Delta t/2}(i-\frac{1}{2},j)}{\Delta x} + \frac{J_y^{t+\frac{\Delta t}{2}}(i,j+\frac{1}{2}) - J_y^{t+\Delta t/2}(i,j-\frac{1}{2})}{\Delta y}$$
$$= \frac{\rho^t(i,j) - \rho^{t+\Delta t}(i,j)}{\Delta t}.$$

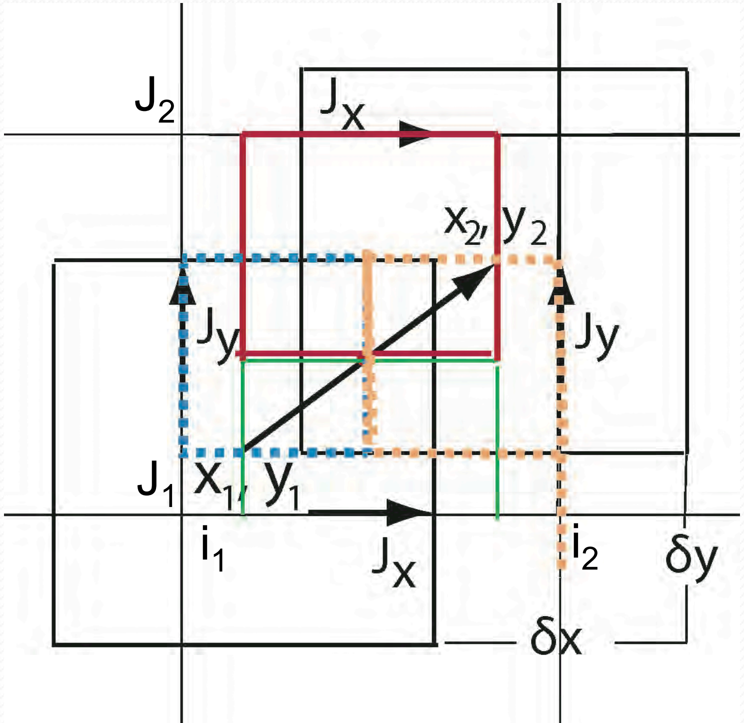
$$J_x\left(i_1+\tfrac{1}{2}, j_1\right) = \frac{1}{\Delta x \Delta y} F_x(1-W_y), \qquad J_x\left(i_1+\tfrac{1}{2}, j_1+1\right) = \frac{1}{\Delta x \Delta y} F_x W_y,$$
$$J_y\left(i_1, j_1+\tfrac{1}{2}\right) = \frac{1}{\Delta x \Delta y} F_y(1-W_x), \qquad J_y\left(i_1+1, j_1+\tfrac{1}{2}\right) = \frac{1}{\Delta x \Delta y} F_y W_x,$$

$i_1 = i_2$ and $j_1 = j_2$

$$i_1 \equiv \mathrm{floor}(x_1/\Delta x), \qquad i_2 \equiv \mathrm{floor}(x_2/\Delta x),$$
$$j_1 \equiv \mathrm{floor}(y_1/\Delta y), \qquad j_2 \equiv \mathrm{floor}(y_2/\Delta y),$$

$$F_x = q\frac{x_2 - x_1}{\Delta t}, \qquad F_y = q\frac{y_2 - y_1}{\Delta t},$$

$$W_x = \frac{x_1 + x_2}{2\Delta x} - i_1, \qquad W_y = \frac{y_1 + y_2}{2\Delta y} - j_1.$$



see Umeda (2003) for detailed numerical method