Computational Methods for Kinetic Processes in Plasma Physics

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* Plasma physics on computer (General description)
* Kinetic Plasma Simulations for high energy particles
* How PIC works (cold plasma dispersion, plasma dispersion function)
* Electrostatic codes (grid quantities, beat heating)
* ES1 test run (dipole scheme, grid noise)
* Electromagnetic codes (hybrid oscillation, unmagnetized plasma)
* EM1 test run (force weighting)
* Finite-difference Time-Domain Maxwell solver on Yee grid
* Particle movers: Boris's algorism
* Conservative charge deposition method
* Boundary conditions (particles and fields)
* Simulations for astrophysical plasmas
* Recent work 1: Weibel instability in relativistic jets (radiation, weighted beam)
* Recent work 2: Reconnection (particle acceleration)

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1. Basic Plasma Physics

- What is plasma?
- Where is plasma found?
- What consists of plasma?
- How do particles move in magnetic field?
- How does plasma evolve as fluid?
- What is plasma instability?
- What we need to know about plasma to understand radiation from moving particles?
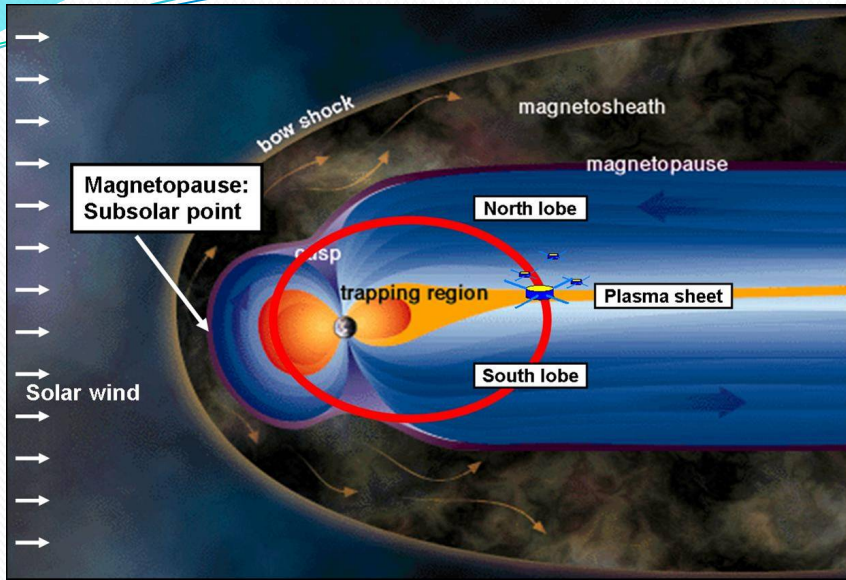- How do we investigate plasma dynamics?
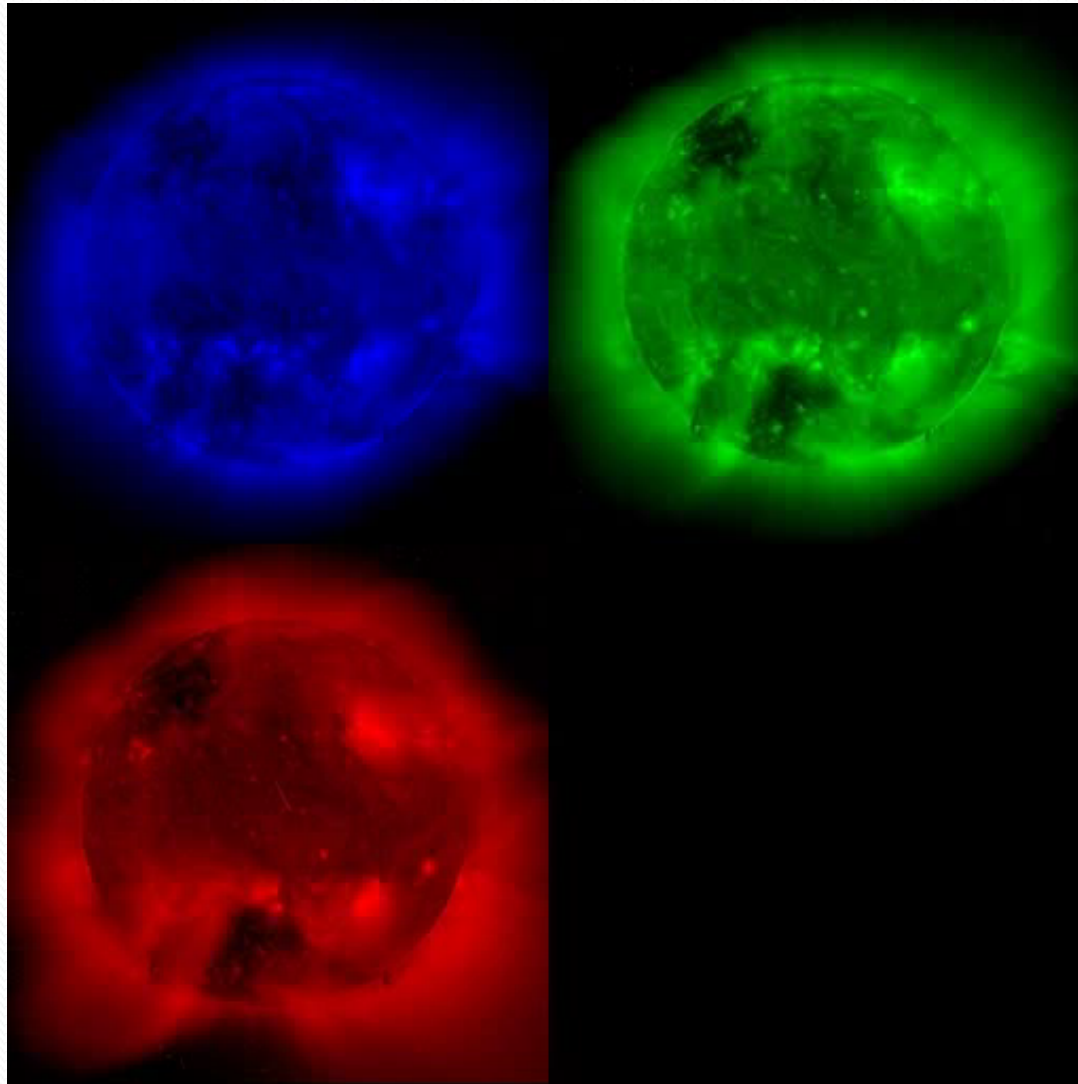
play
trico.mov

Three images (171A, 195A, and 284A) combined into one composite image

Solar wind-magnetosphere interaction: aurora

Geospace: Solar wind with interstellar magnetic fields (IMFs) interacts with Earth magnetosphere and create complicated structures and evolution: one of them is aurora
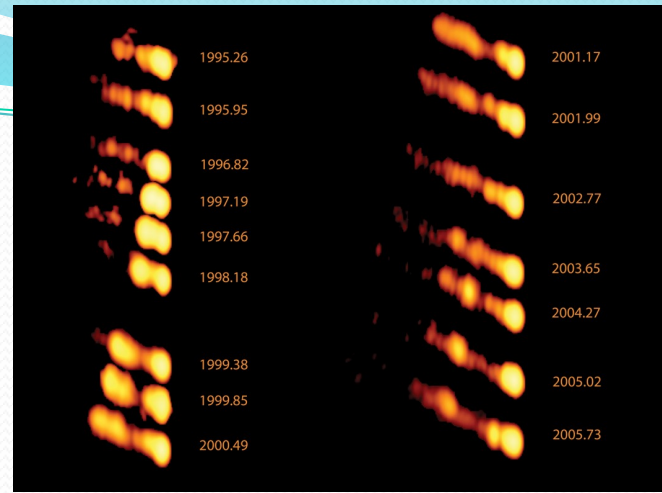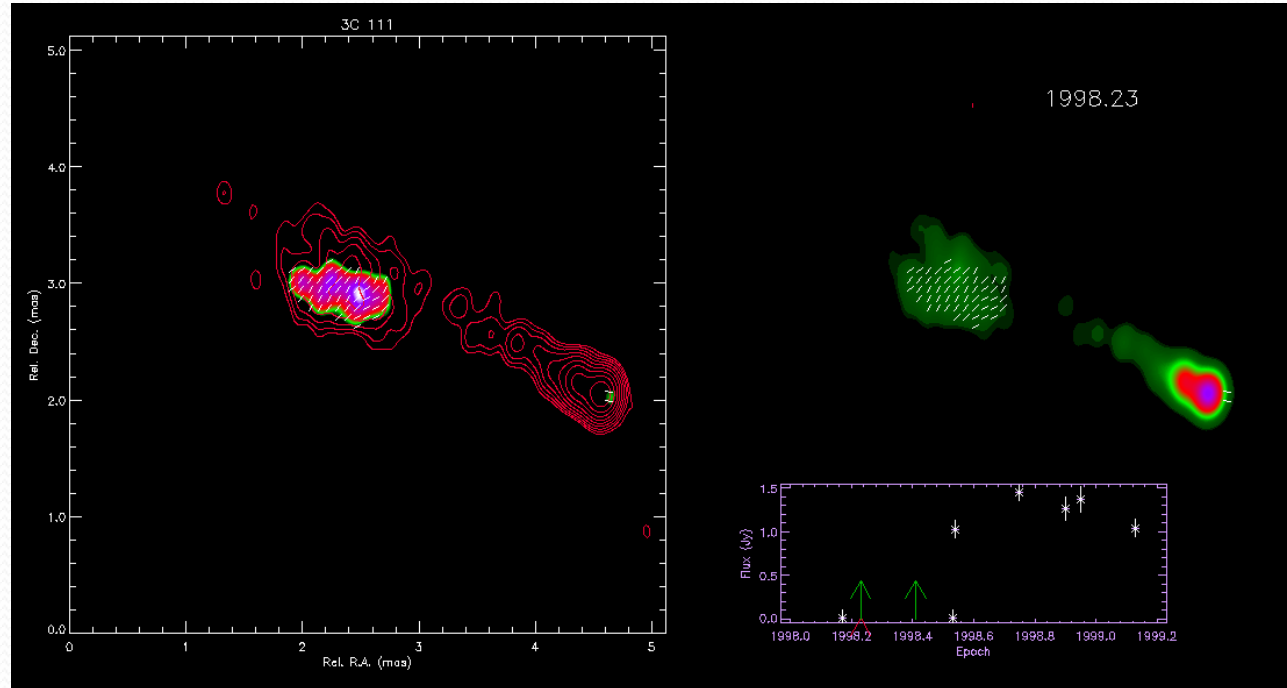
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play yellowkife-combination.mov
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play www-4.iaa.mp4

3C111 (Radio galaxy, z=0.0485) observed by Very Long Baseline Array
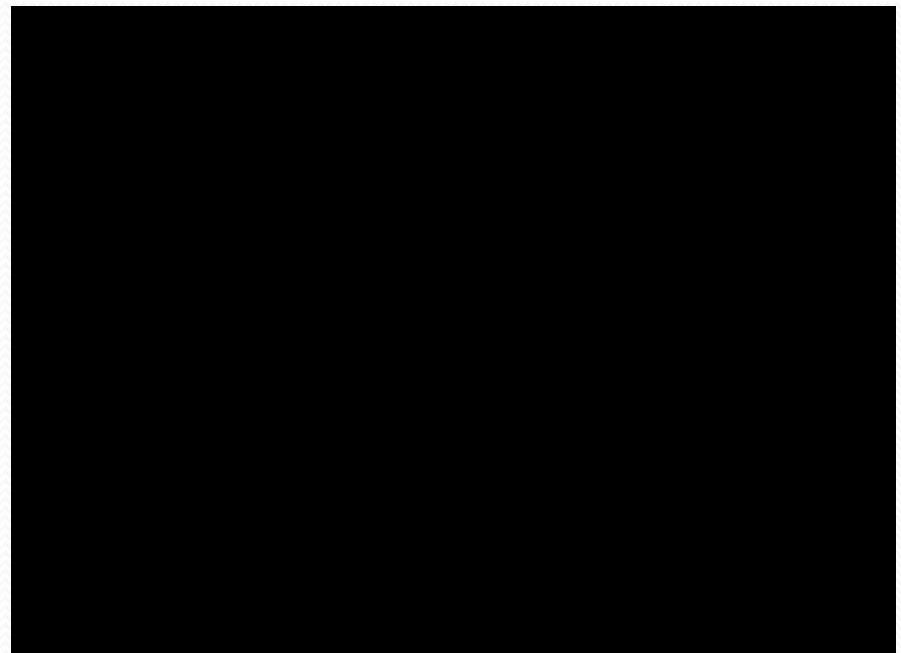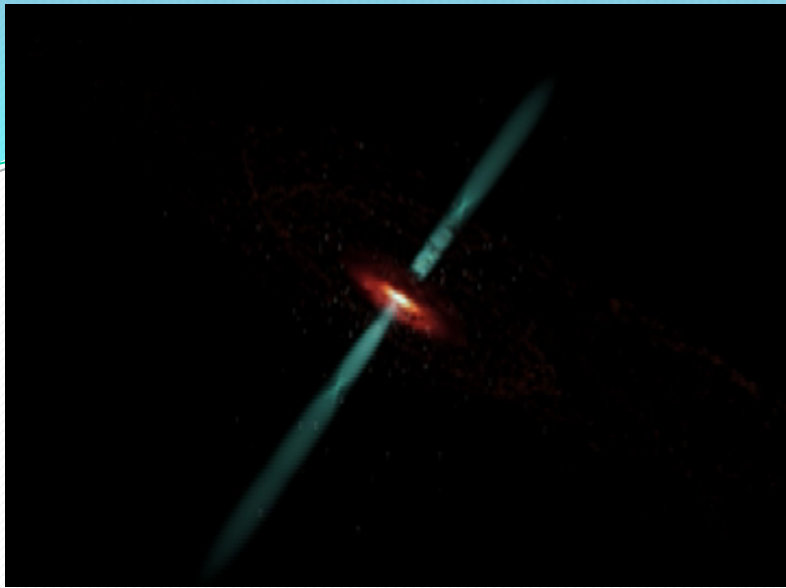
Black hole ejects relativistic jets

Relativistic jets

play 3c111vec.mov

High energy plasmas in astrophysics
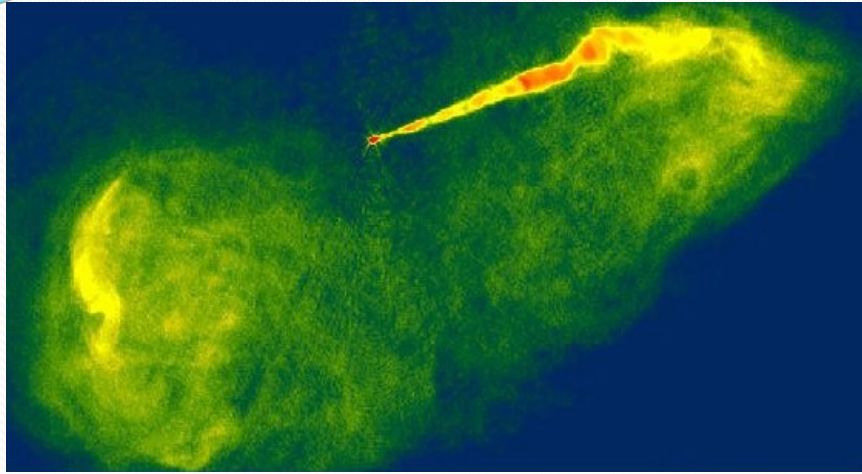
Relativistic jets: M87

Supernova Remnants: Cassiopeia A

Highly accelerated particles (electrons, positrons, and ions) are found in these astrophysical systems

Particle acceleration occurs in shocks, reconnections, and other phenomena

`play combinedmovie.mov`
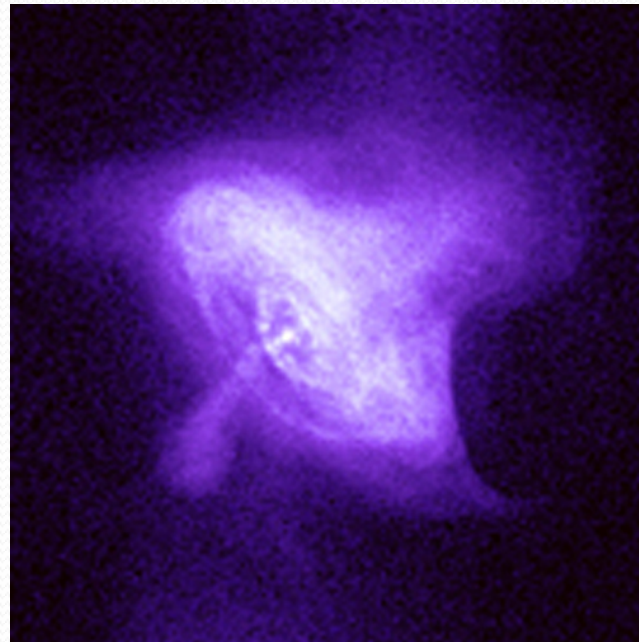
Pulsar Winds: Crab nebula

What is plasma?

- Ionized gas:

  e^-: electrons, e^+: positrons, i^+: ions (p^+: protons)

- Dairy examples: Florescent lamp (partially ionized)

- Controlled Fusion device (Tokamak, Laser Fusion, etc)

- Universe is consist of plasmas (99%)

  Active Galactic Nucleus (AGN) jets,

  Supernova Remnants (SNRs),

  Gamma-ray burst jets

Good text books for plasma physics

- Introduction to Plasma Physics and Controlled Fusion:

    vol. 1 Plasma Physics, by Frances F. Chen

- Principles of Plasma Physics,

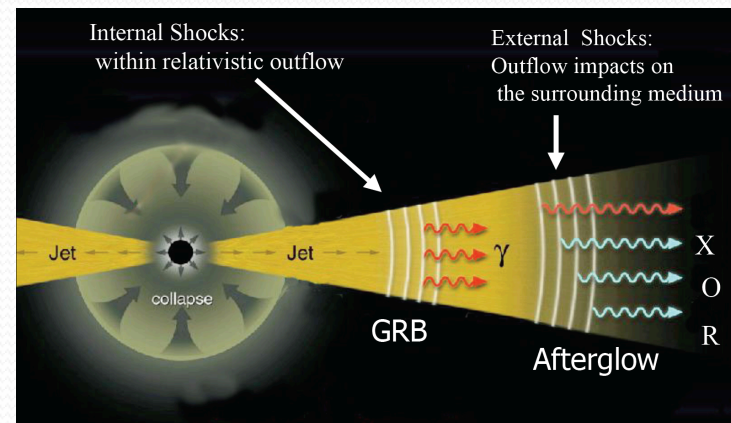   by N. A. Krall, and A. W. Trivelpiece

- Plasma Physics for Astrophysics,

    by Russell M. Kulsrud

- Plasma Physics Via Computer Simulation,

    by C.K. Birdsall, and A. B. Langdon

- Basic Space Plasma Physics and Advanced Space Plasma Physics

    by R. A. Treumann and W. Baumjohann

- many other good text books for instabilities, waves and nonlinear phenomena

Single particle motion

Uniform B and E

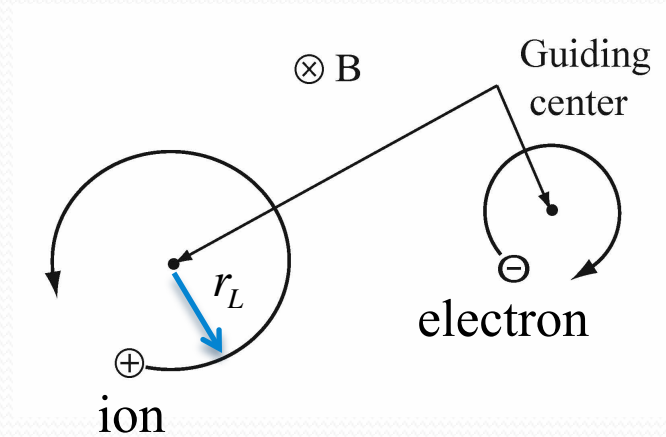$$m\frac{dv}{dt} = q(E + v \times B)$$

if $E = 0$

$$m\dot{v}_x = qBv_y \qquad m\dot{v}_y = -qBv_x$$

$$\ddot{v}_x = \frac{qB}{m}\dot{v}_y = -\left(\frac{qB}{m}\right)^2 v_x \qquad \ddot{v}_y = -\frac{qB}{m}\dot{v}_x = -\left(\frac{qB}{m}\right)^2 v_y$$

Cyclotron frequency $\qquad \omega_c = \frac{|q|B}{m}$ $\qquad$ Larmor radius $\qquad r_L \equiv \frac{v_\perp}{\omega_c} = \frac{mv_\perp}{|q|B}$

If E ≠0

z component

$$\frac{dv_z}{dt} = \frac{q}{m}E_z \qquad v_z = \frac{qE_z}{m}t + v_{t0}$$

Ion   Electron
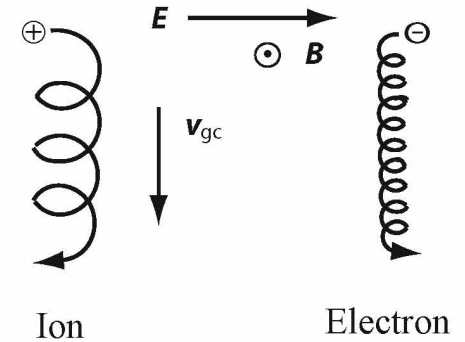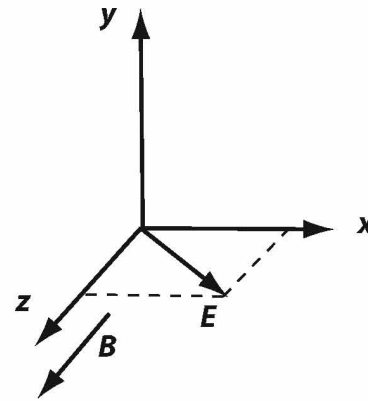
transverse component

$$\frac{dv_y}{dt} = 0 \mp \omega_c v_x$$

if $\quad \dfrac{dv}{dt} = 0$

$$\ddot{v}_x = -\omega_c^2 v_x \qquad \ddot{v}_y = -\omega_c^2 (v_y + \frac{E_x}{B})$$

$$E + v \times B = 0$$

$$E \times B = B \times (v \times B) = vB^2 - B(v \cdot B)$$

$$v_x = v_\perp e^{i\omega_c t}$$

E×B drift

$$v_y = \pm i v_\perp e^{i\omega_c t} - \frac{E_x}{B}$$

$$\mathbf{v}_{\perp gc} = \mathbf{E} \times \mathbf{B} / B^2 \equiv \mathbf{v}_E$$

Nonuniform B field

$$\nabla B \perp \mathbf{B} : \text{Grad-}B$$
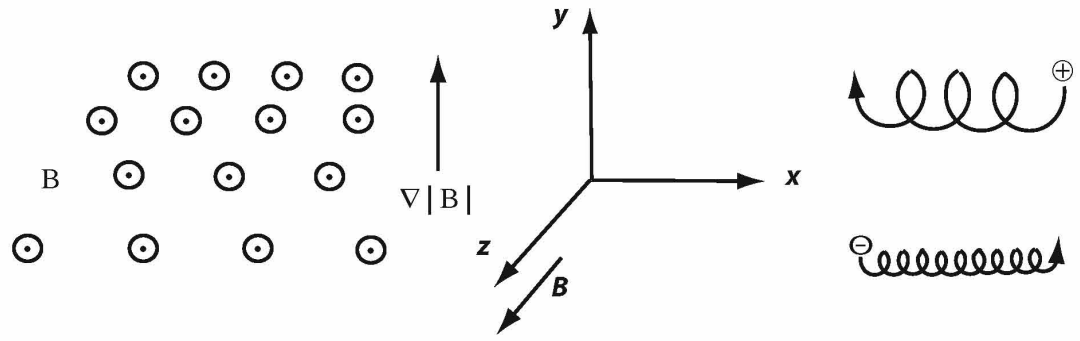
$$F_y = -qv_x B_z(y) = -qv_\perp(\cos\omega_c t)\left[B_0 \pm r_J(\cos\omega_c t)\frac{\delta B}{\delta y}\right] \longrightarrow \bar{F}_y = \mp qv_\perp r_L \frac{1}{2}\left(\frac{\partial B}{\partial y}\right)$$

$$v_{gc} = \frac{1}{q}\frac{\mathbf{F}\times\mathbf{B}}{B^2} = \frac{1}{q}\frac{\bar{F}_y}{|B|}\hat{\mathbf{x}} = \mp\frac{v_\perp r_L}{B}\frac{1}{2}\frac{\partial B}{\partial y}\hat{\mathbf{x}} \qquad v_{\nabla B} = \frac{1}{2}v_\perp r_L \frac{\mathbf{B}\times\nabla B}{B^2}$$

Summary of guiding center drifts

General force **F**:

$$\mathbf{v}_F = \frac{1}{q}\frac{\mathbf{F}\times\mathbf{B}}{B^2}$$

Electric field:

$$\mathbf{v}_E = \frac{\mathbf{E}\times\mathbf{B}}{B^2}$$

Gravitational field:

$$\mathbf{v}_g = \frac{m}{q}\frac{\mathbf{g}\times\mathbf{B}}{B^2}$$

Nonuniform **E** :

$$\mathbf{v}_E = \left(1 + \frac{1}{4}r_L^2\nabla^2\right)\frac{\mathbf{E}\times\mathbf{B}}{B^2}$$

Grad-**B** drift:

$$\mathbf{v}_{\nabla B} = \pm\frac{1}{2}v_\perp r_L\frac{\mathbf{B}\times\nabla\mathbf{B}}{B^2}$$

Curved vacuum field:

$$\mathbf{v}_R + v_{\nabla\mathbf{B}} = \frac{m}{q}\left(v_\parallel^2 + \frac{1}{2}v_\perp^2\right)\frac{\mathbf{R}_c\times\mathbf{B}}{R_c^2 B^2}$$

Polarization drift:

$$\mathbf{v}_p = \pm\frac{1}{\omega_c B}\frac{d\mathbf{E}}{dt}$$

Concept of temperature

One-dimensional Maxwellian distribution

$$f(u) = A\exp(-\frac{1}{2}mu^2/KT)$$
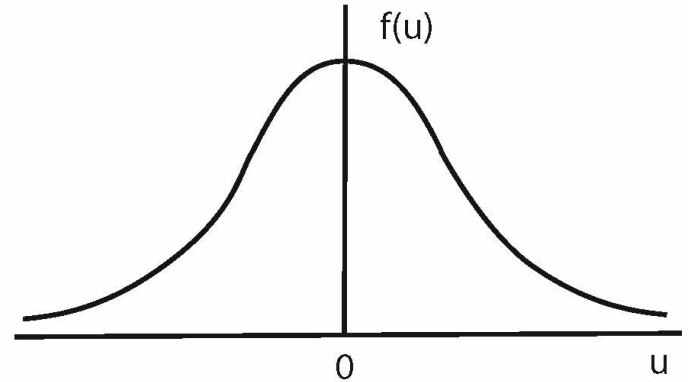
K: Boltzman's constant $\quad K = 1.38 \times 10^{-23} \mathrm{J}/{}^\circ\mathrm{K}$

Density $\quad n/m^2 \qquad n = \int_{-\infty}^{\infty} f(u)\,du$

$$E_{av} = \frac{\int_{-\infty}^{\infty} \frac{1}{2}mu^2 f(u)\,du}{\int_{-\infty}^{\infty} f(u)\,du} \qquad v_{th} = (2KT/m)^{1/2}$$

$$KT = 1\mathrm{eV} = 1.6 \times 10^{-19}\mathrm{J}$$

$$1\mathrm{eV} = 11{,}600{}^\circ\mathrm{K}$$

$$E_{av} = \frac{1}{4}mv_{th}^2 = \frac{1}{2}KT$$

Three-dimensional Maxwell's distribution

$$f(u,v,w) = A_3 \exp[-\frac{1}{2}m(u^2+v^2+w^2)/KT]$$

$$A_3 = n\left(\frac{m}{2\pi KT}\right)^{3/2} \qquad E_{av} = \frac{3}{2}KT$$

Debye shielding

Poisson's equation

$$\varepsilon_0 \nabla^2 \phi = \varepsilon_0 \frac{d^2\phi}{dx^2} = -(n_i - n_e) \quad n_i = n_\infty$$
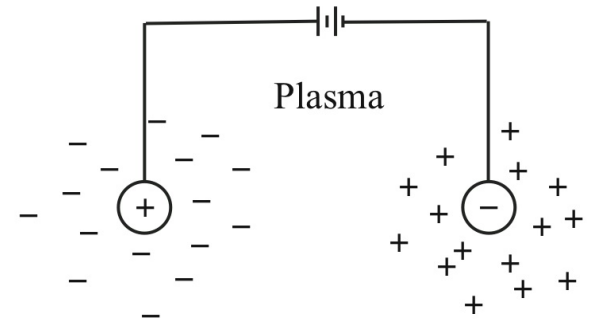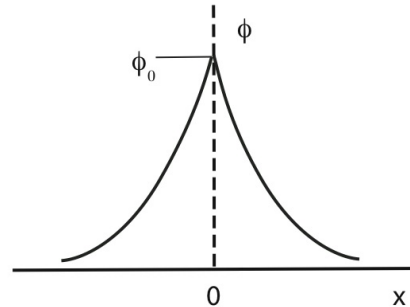
Electron distribution function

$$f(u) = A \exp\left[-(\frac{1}{2}mu^2 + q\phi)/KT\right] \qquad n_e = n_\infty \exp(e\phi/KT_e)$$

$$\varepsilon_0 \frac{d^2\phi}{dx^2} = en_\infty \left\{\left[\exp\left(\frac{e\phi}{KT_e}\right)\right] - 1\right\} \approx en_\infty \left[\frac{e\phi}{KT_e} + \frac{1}{2}\left(\frac{e\phi}{KT_e}\right)^2 + \cdots\right]$$

$$\varepsilon_0 \frac{d^2\phi}{dx^2} = \frac{n_\infty e^2}{KT_e}\phi \qquad \phi = \phi_0 \exp(-|x|/\lambda_D)$$

Debye length $\qquad \lambda_D = \left(\dfrac{\varepsilon_0 KT_e}{n_e^2}\right)^{\frac{1}{2}}$

Maxwell equations

In a medium

$$\nabla \cdot \mathbf{D} = \sigma \qquad\qquad \mathbf{D} = \varepsilon \mathbf{E}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t} \qquad\qquad \mathbf{H} = \mathbf{B}/\mu$$

Fluid equation of motion

$$mn\left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u}\right] = qn(\mathbf{E} + \mathbf{u} \times \mathbf{B}) - \nabla \cdot \mathbf{P} - \frac{mn(\mathbf{u} - \mathbf{u}_0)}{\tau}$$

Complete set of fluid equations

$$\varepsilon_0 \nabla \cdot \mathbf{E} = \sigma = n_i q_i + n_e q_e$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\mu_0^{-1} \nabla \times \mathbf{B} = \mathbf{j} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} = n_i q_i \mathbf{v}_i + n_e q_e \mathbf{v}_e + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$m_j n_j \left[\frac{\partial \mathbf{v}_j}{\partial t} + (\mathbf{v}_j \cdot \nabla) \mathbf{v}_j \right] = q_j n_j (\mathbf{E} + \mathbf{v}_j \times \mathbf{B}) - \nabla \cdot \mathbf{P}_j \qquad j = i, e$$

Plasma oscillations (1-dimensional)

Electron equations of motion and continuity

$$n_e = n_0 + n_1 \qquad \mathbf{v}_e = \mathbf{v}_0 + \mathbf{v}_1 \qquad \mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1$$

Basic equations

$$mn_e\left[\frac{\partial \mathbf{v}_e}{\partial t} + (\mathbf{v}_e \bullet \nabla)\mathbf{v}_e)\right] = -en_e\mathbf{E}$$

$$\frac{\partial n_e}{\partial t} + \nabla \bullet (n_e\mathbf{v}_e) = 0$$

$$\varepsilon_0 \nabla \bullet \mathbf{E} = \varepsilon_0 \partial \mathbf{E}/\partial \mathbf{x} = e(n_i - n_e)$$

Linear analysis

$$m\left[\frac{\partial \mathbf{v}_1}{\partial t} + (\mathbf{v}_1 \bullet \nabla)\mathbf{v}_1\right] = -e\mathbf{E}_1$$

$$\frac{\partial n_1}{\partial t} + \nabla \bullet (n_0\mathbf{v}_1 + n_1\mathbf{v}_1) = \frac{\partial n_1}{\partial t} + n_0\nabla \bullet \mathbf{v}_1 = 0$$

$$\varepsilon_0 \nabla \bullet \mathbf{E}_1 = \varepsilon_0 \partial \mathbf{E}_1/\partial \mathbf{x} = -en_1$$

Plasma oscillations (1-dimensional)

Assumed perturbations (sinusoildal)

$$\mathbf{v}_1 = v_1 e^{i(kx-\omega t)}\hat{\mathbf{x}} \qquad n_1 = n_1 e^{i(kx-\omega t)} \qquad \mathbf{E} = E_1 e^{i(kx-\omega t)}\hat{\mathbf{x}}$$

Linearlization of equations

$$-im\omega v_1 = -eE_1 \qquad -i\omega n_1 = -n_0 ikv_1 \qquad -ik\varepsilon_0 E_1 = -en_1$$

Eliminating $\quad n_1 \text{ and } E_1$

$$-im\omega v_1 = -e\frac{-e}{ik\varepsilon_0}\frac{-n_0 ikv_1}{-i\omega} = i\frac{n_0 e^2}{\varepsilon_0 \omega}v_1 \qquad \omega^2 = n_0 e^2 / m\varepsilon_0$$

Plasma frequency $\quad \omega_p = \left(\dfrac{n_0 e^2}{m\varepsilon_0}\right)^{1/2} \qquad\qquad f_p = \omega_p/2\pi \approx 9\sqrt{n}$

$$n = 10^{18}\,\mathrm{m}^{-3} \qquad B \approx 0.32\mathrm{T} = 0.32\times 10^4\mathrm{G} \ \ (\text{Gauss})$$

$$f_p \approx 9(10^{18})^{1/2} = 9\times 10^9\,\sec^{-1} = 9\mathrm{GHz} \simeq \mathrm{f}_{ce} \simeq 28\mathrm{GHz/Tesla}$$

Single-fluid equations

Quasineutral plasma with singly charged ions

$$Mn\frac{\partial \mathbf{v}_i}{\partial t} = en(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) - \nabla p_i + Mn\mathbf{g} + \mathbf{P}_{ie}$$

$$mn\frac{\partial \mathbf{v}_e}{\partial t} = -en(\mathbf{E} + \mathbf{v}_e \times \mathbf{B}) - \nabla p_e + mn\mathbf{g} + \mathbf{P}_{ei}$$

$$Mmn\frac{\partial}{\partial t}(\mathbf{v}_i - \mathbf{v}_e) = en(M + m)\mathbf{E} + en(m\mathbf{v}_i - M\mathbf{v}_e) \times \mathbf{B} - m\nabla p_i + M\nabla p_e - (M + m)\mathbf{P}_{ei}$$

$$\rho \equiv n_i M + n_e m \approx n(M + m) \qquad p = p_i + p_e$$

$$\mathbf{v} \equiv \frac{1}{\rho}\left(n_i M\mathbf{v}_i + n_e m\mathbf{v}_e\right) \approx \frac{M\mathbf{v}_i + m\mathbf{v}_e}{M + m}$$

$$\mathbf{j} \equiv e\left(n_i\mathbf{v}_i - n_e\mathbf{v}_e\right) \approx ne(\mathbf{v}_i - \mathbf{v}_e)$$

$$\sigma = n_i q_i + n_e q_e$$

Single-fluid equations

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{j} + \frac{1}{en}(\mathbf{j} \times \mathbf{B} - \nabla p_e) \qquad \eta = \frac{m}{ne^2}\nu_{ei} = \frac{e^2}{16\pi\varepsilon_0^2 mv^2} \approx \frac{\pi e^2 m^{1/2}}{(4\pi\varepsilon_0)^2 (KT_e)^{3/3}} \ln\Lambda \qquad \Lambda = \overline{\lambda_D/r_0}$$

$$\frac{Mmn}{e}\frac{\partial}{\partial t}\left(\frac{\mathbf{j}}{n}\right) = e\rho\mathbf{E} - (M+m)ne\eta\mathbf{j} - m\nabla p_i + M\nabla p_e + en(m\mathbf{v}_i + M\mathbf{v}_e) \times \mathbf{B}$$

$$m\mathbf{v}_i + M\mathbf{v}_e = M\mathbf{v}_i + m\mathbf{v}_e + M(\mathbf{v}_e - \mathbf{v}_i) + m(\mathbf{v}_i - \mathbf{v}_e) = \frac{\rho}{n}\mathbf{v} - (M-m)\frac{\mathbf{j}}{ne}$$

Generalized Ohm's law

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} - \eta \mathbf{j} = \frac{1}{e\rho}\left[\frac{Mmn}{e}\frac{\partial}{\partial t}\left(\frac{\mathbf{j}}{n}\right) + (M-m)\mathbf{j} \times \mathbf{B} + m\nabla p_i - M\nabla p_e\right]$$

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{j} + \frac{1}{en}(\mathbf{j} \times \mathbf{B} - \nabla p_e)$$

Set of MHD equations

equation of motion

$$\rho \frac{\partial \mathbf{v}}{\partial t} = \mathbf{j} \times \mathbf{B} - \nabla p + \rho \mathbf{g}$$

simplified Ohm's law

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{j}$$

equation of continuity

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \mathbf{v}) = 0$$

conservation of charge

$$\frac{\partial \sigma}{\partial t} + \nabla \bullet \mathbf{j} = 0$$

Kinetic theory

For some phenomena fluid theory is inadequate

number of particles

$$f(x, y, z, v_x, v_y, v_z, t) dv_x dv_y dv_z$$

$$n(\mathbf{r}, t) = \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z f(\mathbf{r}, \mathbf{v}, t) = \int_{-\infty}^{\infty} f(\mathbf{r}, \mathbf{v}, t) d^3 v = \int_{-\infty}^{\infty} f(\mathbf{r}, \mathbf{v}, t) d\mathbf{v}$$

$$f(\mathbf{r}, \mathbf{v}, t) = n(\mathbf{r}, t) \hat{f}(\mathbf{r}, \mathbf{v}, t) \qquad \hat{f}_m = (m/2\pi KT)^{3/2} \exp(-v^2/v_{th})$$

$$v = (v_x^2 + v_y^2 + v_z^2)^{1/2} \qquad v_{th} = (2KT/m)^{1/2}$$

Vlasov equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{f} + \frac{q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f}{\partial \mathbf{v}} = 0$$

Summary

Plasma

Kinetic picture   Fluid picture

Particle motion   Global dynamics
acceleration   Plasma temperature
radiation

Linear theory

Waves, instability

Nonlinear theory

Linear theory breaks down, saturation

Particle-in-Cell simulations   MHD simulations

Local microscopic phenomena   Global system