This work presents two generalized coloring schemes for graphs. The first involves partitioning the vertex set, $V(G)$, into disjoint color classes, namely $\mathfrak{S} = \{S_1, S_2, \ldots, S_t\}$ where S_i is color class i and $\bigcup_i S_i = V(G)$. When considering a set D for any given parameter, the coloring restriction requires that if $S_i \cap D \neq \emptyset$ then $S_i \subseteq D$ for $1 \leq i \leq t$. In particular, $\beta(G; \mathfrak{S})$ is the maximum cardinality of an independent set with the coloring restriction given partition $\mathfrak{S}$, and $\beta_{PRT}(G)$ is the minimum cardinality of $\beta(G; \mathfrak{S})$ over all partitions. In particular, $\beta_{PRT}(P_n)$ is determined for all paths of order n, and colored-independence is shown to be NP-complete even when G is restricted to be a path.

The second generalization involves considering a collection $\mathcal{R} = \{R_1, R_2, \ldots, R_t\}$ of subsets of the vertex set. These collections can be the edge set $E(G)$, the collection of closed neighborhoods $\mathcal{R} = \{N[v_1], N[v_2], \ldots, N[v_n]\}$, and the collection of open neighborhoods $\mathcal{R} = \{N(v_1), N(v_2), \ldots, N(v_n)\}$, among others. A number of general $\mathcal{R}$-parameters will be defined, many of which are instances of well-studied parameters (such as domination and independence) as well as some instances of previously unstudied parameters. One application of these general parameters involves extending the domination chain as it is defined relative to the edge set. Natural extensions of independence will be considered to give rise to other chains.

$\mathcal{R}$-chromatic problems also involve consideration of these $\mathcal{R}$ collections. The $\mathcal{R}$-proper chromatic number, $\chi_{\mathcal{R}}(G)$ is defined to be the minimum number k of sets $C_1, C_2, \ldots, C_k$ that partition $V(G)$ such that no R_i is a subset of C_j. The $\mathcal{R}$-limited chromatic number, $\chi^+_{\mathcal{R}}(G)$ is defined to be the minimum number k of sets $C_1, C_2, \ldots, C_k$ that partition $V(G)$ such that $|R_i \cap C_j| \leq 1$ for $1 \leq i \leq t$ and $1 \leq j \leq k$.