Some extreme value problems arising with ship motions

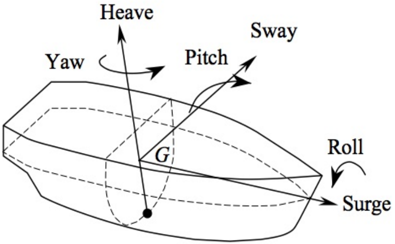
Vladas Pipiras (UNC–Chapel Hill)[1]

NSF/CBMS Conference, August 3, 2021

[1]Based on joint work and papers with V. Belenky, K. Weems and others (NSWCCD), T. Sapsis (MIT), (under)graduate students.

Ship motions

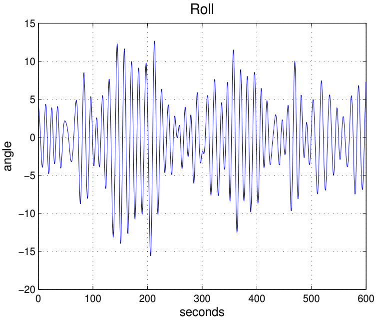
3 rotational (*roll, pitch, yaw*) and 3 linear (*surge, sway, heave*) motions. The focus here will typically be on roll, pitch, heave motions.

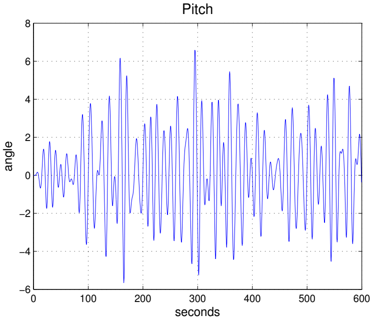


What do motions depend on? *Ship geometry and loading*, *operational parameters* (heading, speed) and *surrounding sea* (spectrum involving significant wave height, modal period, and so on). *Computer code* used as well. Randomness is due to random waves.[2]

[2] Waves are typically generated from a spatio-temporal Gaussian (or thought so) field.

The time series of **roll** (left) and **pitch** (right) obtained by the SimpleCode for 10mins at 0.5s sampling rate. The *ship geometry* is the ONR tumblehome top (THT). The *heading* is at 45°, the *speed* is 6 knots, the waves are modeled using Bretschneider spectrum in open ocean (with the *significant height* of 9m, the *mean zero-crossing period* of 10.65s, etc).





Extremes

Rare (extreme) events: These include:

A. *Extreme ship motion (e.g. roll exceeding a certain large angle)*;
B. *Capsizing of a ship (through pure loss of stability)*;
C. *Surfriding and broaching-to*;
D. *Extreme loads (due to waves and/or slamming)*

Basic problems: These include:

- Estimate the (small) probabilities of these rare events from the data obtained through a ship motion simulator (SimpleCode, LAMP, etc);
- Understand distribution tails of the "metrics" associated with these rare events, through reduced-order models.

This is important in e.g. ship design and concerning ship stability. The talk is also about the interplay of *Statistics*, *Stochastic Dynamics*, *Multifidelity Methods*, *Naval Architecture* in addressing/understanding these problems.

Estimating probabilities of rare events

Direct estimation of probabilities of rare events from long records of ship motion should be avoided for several reasons: *events are rarely observed*, *many conditions to consider*, a desire to have *methodology that can be applied to other data* (e.g. from a basin) where long records are not available. (**Analytic calculation** is not a possibility either. Other approaches though exist.)

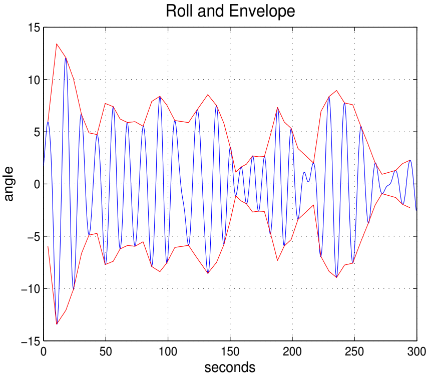
Instead of direct estimation, estimation of the probability of a rare event is first recast as the problem of estimation of the **exceedance probability**

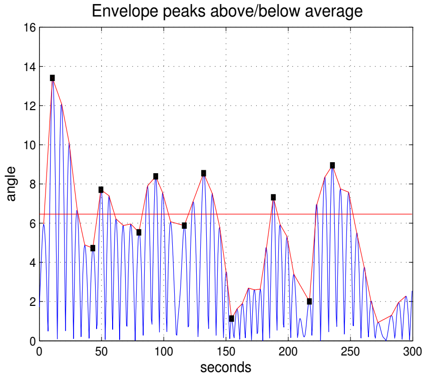
$$\mathbb{P}(Y > y^*),$$

where Y is some variable ("metric") related to the rare event of interest and y^* is some large critical value, given *independent observations* $Y_1, \dots, Y_n$ *of* Y, with no values of Y_i typically larger than y^*. Then, "standard" methods of Extreme Value Theory in Statistics are used to extrapolate data distribution into the tail to estimate the exceedance probability. (This is also about a **distribution tail**.)

Exceedance probability for extreme motion

Extreme motion: For the probability of extreme motion as e.g. roll exceeding a certain large angle y^*, Y is taken as a *suitable envelope peak* of a ship motion.





Statistical solution: peaks-over-threshold (PoT)

Peaks-over-threshold (PoT) approach from Extreme Value Theory: The idea is to fit a *generalized Pareto distribution* (GPD) to the data above an intermediate threshold and use it for extrapolation:

$$\mathbb{P}(Y > y^*) = \mathbb{P}(Y > u)\mathbb{P}(Y > y^* | Y > u) \approx \mathbb{P}(Y > u)\overline{F}_{u,\xi,\sigma}(y^*),$$

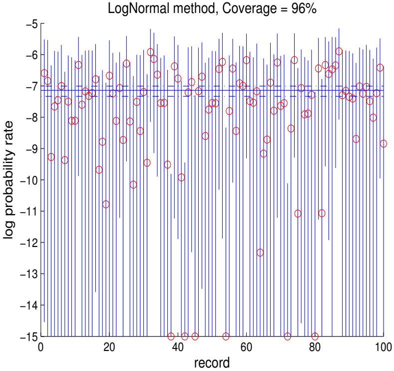
where $\overline{F}_{u,\xi,\sigma}(y)$ is the complementary CDF of the GPD given by

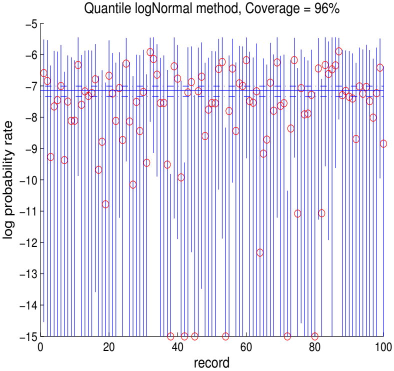
$$\overline{F}_{u,\xi,\sigma}(y) = \begin{cases} \left(1 + \frac{\xi(y-u)}{\sigma}\right)^{-1/\xi}, & u < y, & \text{if } \xi > 0, \\ e^{-\frac{y-u}{\sigma}}, & u < y, & \text{if } \xi = 0, \\ \left(1 + \frac{\xi(y-u)}{\sigma}\right)^{-1/\xi}, & u < y < u + (-\frac{\sigma}{\xi}), & \text{if } \xi < 0, \end{cases}$$

where ξ, σ and u are the *shape*, *scale* and *threshold* parameters. Some issues: (i) estimating GPD parameters, (ii) setting a threshold u, (iii) constructing a confidence interval for exceedance probability, etc.

Application to peaks/extreme motion

THT hull, Heading 45°, etc. **Roll motion**, target 60°. Left: The probability (rate) estimates with confidence intervals for 100 runs of 100hrs each. Right: Same but using different confidence intervals.

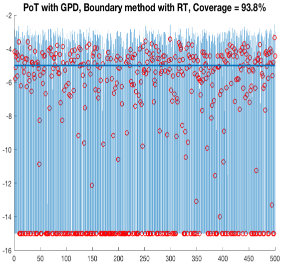




Such plots are used for validation of the proposed methods. The average value of shape parameter estimates is 0.19 (with standard error of 0.13).

Statistical solution: PoT

“Annoying” features: The PoT approach generally works but e.g. (*i*) automatic threshold u selection methods are available but they are not failproof, (*ii*) negative shape parameter estimates can lead to large upper confidence intervals: e.g., the Weibull distribution $\mathbb{P}(Y > y) = e^{-y^2}$ with sample size $N = 2,000$:



Implicit assumption in PoT: Data need to be on the “correct” distribution tail. What about tails with with changing character? Any real-world examples where this *not* the case and the PoT fails/struggles? (In my limited experience, this is the rule rather than the exception.)

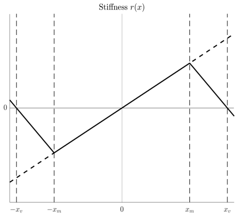
Piecewise linear (PWL) random oscillator

PWL oscillator: As a model for ship roll, the system satisfies the equation

$$\ddot{X} + 2\delta\dot{X} + r(X) = s\dot{W}(t)$$

with white noise excitation $\dot{W}(t)$ and restoring force[3]

$$r(x) = \omega_0^2 x 1_{\{|x|\le x_m\}} - (kw_0^2 x \pm w_0^2 x_m(k+1))1_{\{|x|>x_m\}}.$$



Notes: $|x| \le x_m$: linear regime; $|x| > x_m$: non-linear regime; ω_0: natural frequency; x_m: "knuckle" point; x_v: point of vanishing stability.

[3]For linear restoring, the system is Gaussian.

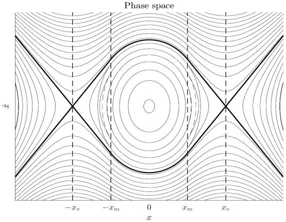
PWL random oscillator

PDF of response/derivative: In the statistically steady state, it is given by

$$f(x,\dot{x}) = Ce^{-\frac{4\delta}{s^2}(\frac{1}{2}\dot{x}^2+V(x))}$$

with normalizing constant C and potential $V(x) = \int_0^x r(y)dy$.

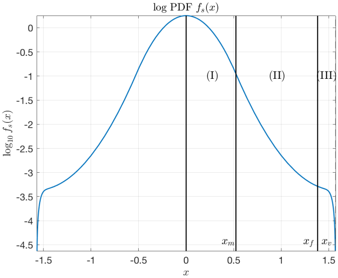
"Capsizing:" Condition on the system to be inside the separatrix, that is, for $|\dot{x}| < x_s(x)$ with $\dot{x}_s(x) = \sqrt{2(V(x_v) - V(x))}$.



PDF of (conditioned) response: $f_s(x) \propto \int_{-\dot{x}_s(x)}^{\dot{x}_s(x)} f(x,\dot{x})d\dot{x}$ for $|x| < x_v$.

PWL random oscillator: PDF tail regimes/Cases of interest

PDF of (conditioned) response:



3 tail regimes:

(I) Gaussian core: $x \in [0, x_m]$

(II) Heavy tail (possibly power-law): $x \in [x_m, x_f]$

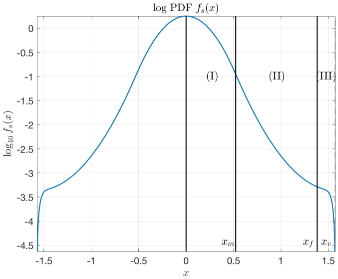
(III) Light (bounded) tail: $x \in [x_f, x_v]$

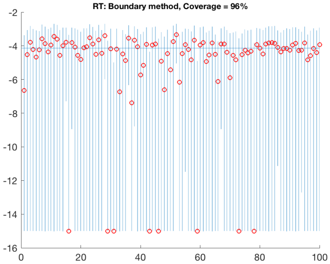
Cases:

- Good: Data in (I)-(II), target in (II)
- Bad 1: Data in (I), no/little data in (II), target in (II)
- Bad 2: Data in (I)-(II), target in (III)

Good case: Data in (I)-(II), target in (II)

Setting: Target $x^* = x_f$, $\mathbb{P}(X > x_f) = 7.22 \times 10^{-5}$.
Sample $N = 3,000$, $\mathbb{P}(X > x_m) = 0.012$, $N \cdot \mathbb{P}(X > x_m) = 36$.

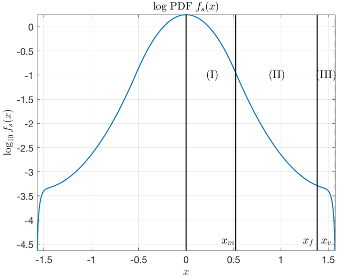


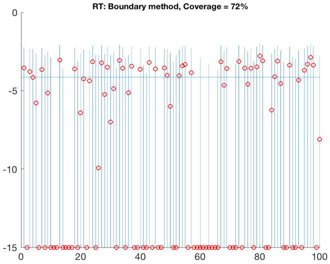


Note: Several cases with small estimated probability: Automatic threshold selection methods had difficulties in finding the “right” threshold.

Bad case 1: Data in (I), no/little data in (II), target in (II)

Setting: Target $x^* = x_f$, $\mathbb{P}(X > x_f) = 7.22 \times 10^{-5}$.
Sample $N = 500$, $\mathbb{P}(X > x_m) = 0.012$, $N \cdot \mathbb{P}(X > x_m) = 6$.

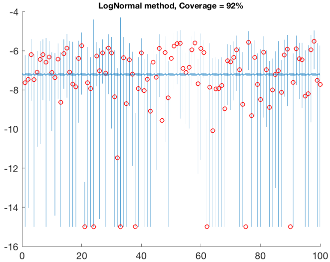


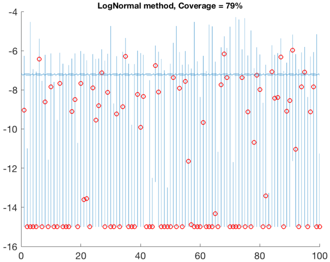


Note: A number of cases with small estimated probability: (a) when no confidence interval is computed, the estimated shape parameter is typically smaller than -0.5, (b) if confidence interval is computed, it is very large on its upper side.

Data example of extreme ship motions

PoT analysis on envelope peaks from roll motion: THT geometry, 9m significant wave height, 15sec modal period, 90 degree heading, etc. Left: 100 hour datasets. Right: 10 hour datasets.





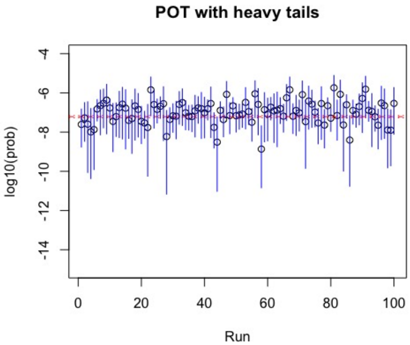
Notes: This is along the same lines as bad case 1. Conclusion: Need to understand the underlying physics and e.g. adjust for sample sizes or use other approaches (such as fitting the exponential tail or the heavy tail).

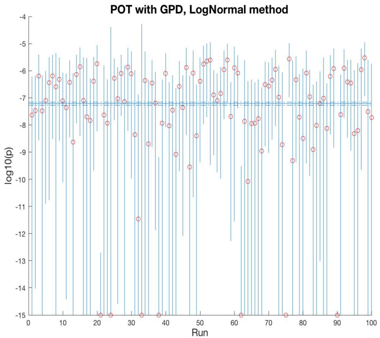
PoT analysis for heavy-tailed distributions

If heavy (power-law) tail is expected as for peaks/extreme motion, a natural possibility is to use PoT with Pareto or heavy tail:

$$\mathbb{P}(Y > y) \approx C y^{-1/\xi}, \quad \text{for large } y,$$

where $\xi > 0$ is a parameter and C is a constant (possibly replaced by a function which varies "slowly"). There are estimators which ensure $\xi > 0$. Comparing POTs with heavy tail and GPD for peaks data:





Ship motion codes: LAMP and SC

Large Amplitude Motion Program (LAMP): The higher-fidelity LAMP is based on a 3-dimensional potential flow panel method integrating pressures over the submerged hull and incorporating a direct calculation of Froude-Krylov and hydrostatic (FKHS), radiation, diffraction and other forces.

LAMP-2	3-D body-nonlinear method
LAMP-1	body-linearized 3-D method
LAMP-0	"Hydrostatics only" formulation

Simple Code (SC) and reduced-order ODE models: The lower-fidelity but computationally more efficient SC and similar ODE models may assume a particular, e.g. linear and separable, form of FKHS forces, and furthermore capture radiation forces through (constant) added mass and damping coefficients. Currently, SC does not incorporate diffraction forces.

LAMP, SC and reduced-order models

Focus on the case of head seas, with the resulting heave ζ_g and pitch θ.[4]

LAMP: The equations of motion can be expressed as

$$\begin{cases} m\ddot{\zeta}_g = F_{3,fkhs} + F_{3,hd} \\ I_Y\ddot{\theta} = F_{5,fkhs} + F_{5,hd} \end{cases} \quad \text{with} \quad F_{i,hd} = F_{i,rad} + F_{i,dif}.$$

SC, etc: Assuming $-F_{i,rad} \simeq A_{i3}\ddot{\zeta}_g + A_{i5}\ddot{\theta} + B_{i3}\dot{\zeta}_g + B_{i5}\dot{\theta}$, the SC program solves

$$\begin{cases} (m + A_{33})\ddot{\zeta}_g + A_{35}\ddot{\theta} + B_{33}\dot{\zeta}_g + B_{35}\dot{\theta} + F_3(\zeta_g, \theta, t) = 0 \\ (I_Y + A_{55})\ddot{\theta} + A_{53}\ddot{\zeta}_g + B_{53}\dot{\zeta}_g + B_{55}\dot{\theta} + F_5(\zeta_g, \theta, t) = 0. \end{cases}$$

An assumption of small motions leads to the separation of restoring and excitation in FKHS forces, for example, as

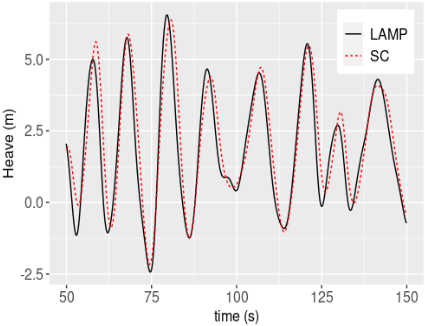
$$\begin{cases} (m + A_{33})\ddot{\zeta}_g + A_{35}\ddot{\theta} + B_{33}\dot{\zeta}_g + B_{35}\dot{\theta} + C_{33}\zeta_g + C_{35}\theta + F_{3k}(t) = 0 \\ (I_Y + A_{55})\ddot{\theta} + A_{53}\ddot{\zeta}_g + B_{53}\dot{\zeta}_g + B_{55}\dot{\theta} + C_{53}\zeta_g + C_{55}\theta + F_{5k}(t) = 0. \end{cases}$$

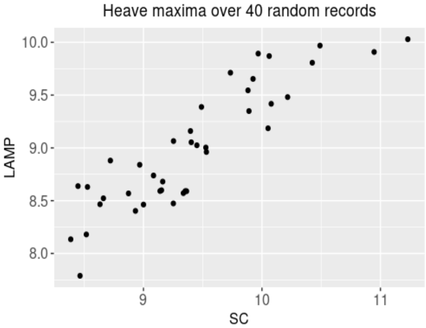
Calibration is another problem of interest.

[4] E.g. the elastic beam equation $M\partial^2 w/\partial t^2 + EI\partial^4 w/\partial x^4 = q(t,x)$ is of interest for slamming loads.

LAMP and SC

Setting: ONR topsides flared hull geometry; 180° heading; Sea state 8; etc.

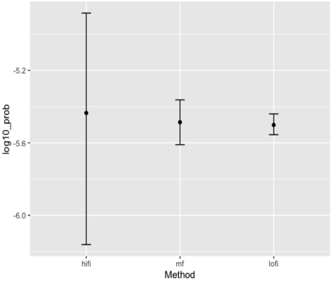




Basic question: Can SC data help answer (e.g. extreme value) questions about LAMP?

MF estimation for extremes: Extrapolation

Extrapolation framework: If parametric distributions (e.g. Weibull, Gumbel, Gaussian, etc.) are used for extrapolation, we have a framework when under suitable conditions, more lower-fidelity data (e.g. SC) can help in estimation of extremal quantities of high-fidelity model (e.g. LAMP).



MFMC estimation: non-rare problem

Setting: Have (independent) samples $(X_1^{(1)}, X_1^{(2)}), \ldots, (X_{n_1}^{(1)}, X_{n_1}^{(2)})$ from $(X^{(1)}, X^{(2)})$, and additional samples $X_{n_1+1}^{(2)}, \ldots, X_{n_2}^{(2)}$ for some $n_2 > n_1$.

MF estimator: To estimate the unknown mean $\mathbb{E}X^{(1)}$, set

$$\widehat{\mu}_{mf} = \overline{X}_{n_1}^{(1)} + \alpha(\overline{X}_{n_2}^{(2)} - \overline{X}_{n_1}^{(2)}) = \alpha\overline{X}_{n_2}^{(2)} + (\overline{X}_{n_1}^{(1)} - \alpha\overline{X}_{n_1}^{(2)}),$$

where $\alpha \in \mathbb{R}$ and $\overline{X}_n = \frac{1}{n}\sum_{i=1}^n X_i$.

Baseline estimator: $\widehat{\mu}_{bl} = \overline{X}_{n_1}^{(1)}$.

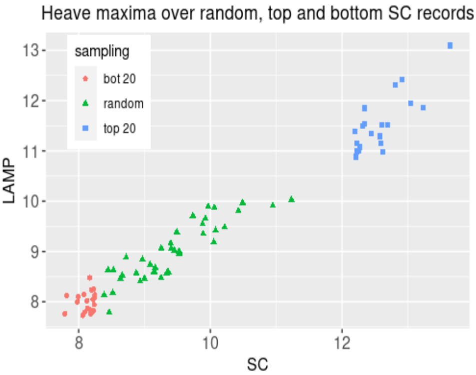
Punchline: If $X^{(1)}$ and $X^{(2)}$ are correlated enough, then

$$\mathrm{Var}(\widehat{\mu}_{mf}) < \mathrm{Var}(\widehat{\mu}_{bl}),$$

so that the resulting confidence intervals are smaller for MF estimator. Naturally, $\alpha = \mathrm{argmin}_a \mathrm{Var}(\widehat{\mu}_{mf})$.

MF estimation for extremes: Selective sampling

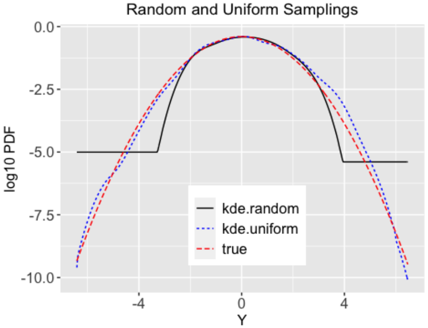
Sampling framework: LAMP could be run only for selective SC records, for example, with largest record maxima/minima among many records.



20 top and 20 bottom SC record maxima are from the total of 2,000 SC records, but many more could easily be considered.

MF estimation for extremes: Selective sampling

Importance sampling: Illustration with 300 points of bivariate standard Gaussian for SC, LAMP and correlation $\rho = 0.8$. SC records selected uniformly over the value range and kernel density estimation with suitable weights for LAMP data.



Notes: No such benefit when e.g. $\rho = 0$. Other methods based on GPR, multi-armed bandits are also being investigated.

Some references

Extreme motions:
- Belenky, Campbell and Pipiras (2016), "Application of the envelope peaks over threshold (EPOT) method for probabilistic assessment of dynamic stability," Ocean Engineering 120, pp. 298–304.
- Belenky, Campbell, Glotzer, Pipiras and Smith (2017), "Confidence intervals for exceedance probabilities with application to extreme ship motions," Revstat 15, pp. 537–563.
- Belenky, Glotzer, Pipiras and Sapsis (2019), "Distribution tail structure and extreme value analysis of constrained piecewise linear oscillators," Probabilistic Engineering Mechanics 57, pp. 1–13.

Capsizing:
- Belenky, Weems, Glotzer, Pipiras and Sapsis (2018), "Tail structure of roll and metric of capsizing in irregular waves," Proceedings of the 32nd Symposium on Naval Hydrodynamics.

Extreme loads:
- Sapsis, Belenky, Weems and Pipiras (2020), "On extreme value properties of vertical bending moment," Proceedings of the 33rd Symposium on Naval Hydrodynamics.
- Sapsis, Belenky, Weems and Pipiras (2021), "Extreme properties of impact-induced vertical bending moments," Proceedings of the 1st International Conference on the Stability and Safety of Ships and Ocean Vehicles.

Multifidelity:
- Brown and Pipiras (2019), "On extending multifidelity uncertainty quantification methods from non-rare to rare problems," Proceedings of the 17th International Ship Stability Workshop.
- Brown and Pipiras (2021), "Multifidelity Monte Carlo estimation for extremes through extrapolation distributions," Preprint.

Calibration:
- Pipiras, Belenky, Weems, Brown, Frommer and Ouimette (2021), "Calibrating multifidelity ship motion codes through regression," Proceedings of the 1st International Conference on the Stability and Safety of Ships and Ocean Vehicles.