NSF-CBMS Follow-Up Conference
Gaussian Random Fields, Fractals, SPDEs and Extremes
August 12-13, 2022, University of Alabama in Huntsville (In Person and Remotely)
Tentative Schedule and Abstracts

TENTATIVE SCHEDULE

All talks, including coffee breaks, will be in Room 105, Shelby Center for Science and Technology (STT), delivered in person or remotely.

Friday, August 12 (Chair: Toka Diagana)

8:30-8:50 Refreshments and Coffee

8:50-9:00 Opening, **Toka Diagana**, Chair, Department of Mathematical Sciences, UAH

9:00-9:45 **Yimin Xiao**, *Local times and geometric properties of Gaussian random fields*

9:45-10:30 **Carl Mueller**, *Polymer survival among random obstacles*

10:30-11:00 Coffee Break

11:00-11:45 **Chunsheng Ma**, *Power-law Lévy processes, power-law vector random fields, and some extensions*

11:45-1:30 Lunch

1:30-2:15 **Xia Chen**, *Intermittency for hyperbolic Anderson equations with time-independent Gaussian noise: Stratonovich regime*

2:15-3:00 **Yaozhong Hu**, *Asymptotics of the density of parabolic Anderson random fields*

3:00-3:30 Coffee Break

3:30-4:15 **Xiong Wang**, *Necessary and sufficient conditions to solve parabolic Anderson model with rough noise*

4:15-5:00 **Davar Khoshnevisan**, *Optimal regularity of SPDEs with additive noise*

Saturday, August 13 (Chair: Dongsheng Wu)

8:30-9:00 Refreshments and Coffee

9:00-9:45 **Renming Song**, *Heat kernel estimates for Dirichlet forms degenerate at the boundary*

9:45-10:30 **Vladas Pipiras**, *A physical model with cyclical long-range dependence*

10:30-11:00 Coffee Break

11:00-11:45 **Erkan Nane**, *Level of noise and long time behavior of space-time fractional SPDEs in bounded domains*

11:45-12:30 **Mozhgan Entekhabi**, *Inverse problems for wave propagation in 2 and 3 dimensions*

ABSTRACTS

Intermittency for hyperbolic Anderson equations with time-independent Gaussian noise: Stratonovich regime, *Xia Chen* (University of Tennessee)

Abstract: Recently, a precise intermittency for the hyperbolic Anderson model

$$\frac{\partial^2 u}{\partial t^2}(t,x) = \Delta u(t,x) + \dot{W}(x)u(t,x)$$

has been established in Ito-Skorohod regime. In this talk, we discuss the same problem in Stratonovich regime. Our approach provides new ingridients on representation and computation for Stratanovich moments.

The work is based on a collaborative project with Hu, Yaozhong.

Inverse problems for wave propagation in 2 and 3 dimensions, *Mozhgan Nora Entekhabi* (Florida A and M University)

Abstract: Inverse source scattering problem and uniqueness of the source arises in many areas of science. It has numerous applications to surface vibrations, elasticity and acoustical and bio-medical industries and medical imaging. In particular, inverse source problem seeks the radiating source which produces the measured wave field. The study aims to provide a technique for recovering the source function of the classical elasticity system and the Helmholtz equation from boundary data at multiple wave numbers when the source is compactly supported in an arbitrary bounded C^2 boundary domain, establish uniqueness for the source from the Cauchy data on any open non-empty part of the boundary for arbitrary positive K, and increasing stability when wave number K is getting large. Various studies showed that the uniqueness can be regained by taking multifrequency boundary measurement in a non-empty frequency interval $(0, K)$ noticing the analyticity of wave-field on the frequency. This type of inverse source problem is also motivated by the wide applications in antenna synthesis, medical imaging and geophysics.

Asymptotics of the density of parabolic Anderson random fields, *Yaozhong Hu* (University of Alberta)

Abstract: Parabolic Anderson model is a very simple stochastic heat equation with multiplicative Gaussian noise. The solution $u(t,x)$ of this equation can be represented by the Wiener-Itô chaos expansion. It is related to the Anderson localization and is also

related to the so-called KPZ equation describing the physical growth phenomena. We investigate the shape of the density (t,x; y) of the solution u(t,x) to the stochastic partial differential equation $\frac{\partial}{\partial t}u(t,x) = (1/2)\Delta u(t,x) + u \diamond \dot{W}(t,x)$, where $\dot{W}(t,x)$ is a general Gaussian noise and $\diamond$ denotes the Wick product. We mainly concern with the asymptotic behavior of $\rho(t,x;y)$, the density of the random variable $u(t,x)$, when $y \to \infty$ or when $t \to 0+$. Both upper and lower bounds are obtained and these two bounds match each other modulo some multiplicative constants. If the initial condition is positive, then $\rho(t,x;y)$ is supported on the positive half-line $y \in [0,\infty)$ and in this case we show that $\rho(t,x;0+) = 0$ and obtain an upper bound for $\rho(t,x;y)$ when $y \to 0+$. Our tool is Malliavin calculus and I will also present a very brief and heuristic introduction. This is joint work with Khoa Le.

Optimal regularity of SPDEs with additive noise, *Davar Khoshnevisan* (University of Utah)

Abstract: The regularity of the sample paths of random field solutions to SPDEs depend naturally on the external noise and the type of differential or integral operator that appears in the equation. In this paper, we consider nonlinear heat and wave type SPDEs that are driven by Gaussian noises that are white in time and correlated in space, and by operators that are generators of a non-degenerate Lévy processes. In a fairly generic sense, we find optimal conditions for the Hölder regularity of the solutions to these SPDEs. Our conditions are stated in terms of fractal-type indices that are motivated by the earlier work of Sanz Solé and Sarrá (2002, 2002). This is based on joint work with Marta Sanz Solé.

Power-law Lévy processes, power-law vector random fields, and some extensions, *Chunsheng Ma* (Wichita State University)

Abstract: This talk introduces a power-law subordinator and a power-law Lévy process whose Laplace transform and characteristic function are simply made up of power functions or the ratio of power functions, respectively, and proposes a power-law vector random field whose finite-dimensional characteristic functions consist merely of a power function or the ratio of two power functions. They may or may not have first-order moment, and contain Linnik, variance Gamma, and Laplace Lévy processes (vector random fields) as special cases. For a second-order power-law vector random field, it is fully characterized by its mean vector function and its covariance matrix function, just like a Gaussian vector random field. An important feature of the power-law Lévy processes (random fields) is that they can be used as the building blocks to construct other Lévy processes (random fields), such as hyperbolic secant, cosine ratio, and sine ratio Lévy processes (random fields).

Polymer survival among random obstacles, *Carl Mueller* (University of Rochester)

Abstract: This is joint work with Siva Athreya and Mathew Joseph.

Random obstacle problems have a long history in the context of Brownian motion. To describe the setting, we we start with a Poisson point process in $\mathbb{R}^d$ with constant intensity λ. For each point p of the process, we place a ball of radius α and center p. This gives us a set of obstacles. Finally we start a Brownian motion B_t at the origin of $\mathbb{R}^d$, and kill

B_t when it first hits any of the obstacles. Let $P(t)$ be the probability that B_t is still alive at time t. There are many precise results in the literature about the asymptotics of $P(t)$ as $t \to \infty$.

In this talk, we will replace the Brownian motion B_t by a moving polymer $u(t,x) \in \mathbb{R}^d$ for $t \geq 0$ and $x \in [0, J]$. Our model for the polymer involves the following $SPDE$,

$$\partial_t u = \partial_x^2 u + \dot{W}(t,x)$$

where $\dot{W}(t,x)$ is a vector-valued white noise. This model for a moving polymer was studied by Funaki and also by polymer scientists. Once again, we kill the polymer at the first time t that $u(t,x)$ hits any of the obstacles, for any value of x. Our goal is again to estimate the survival probability $P(t)$. Almost all of the methods used in the Brownian case no longer apply. For the lower bound, we use some of our previous results about small ball estimates for the moving polymer.

Level of noise and long time behavior of space-time fractional SPDEs in bounded domains, *Erkan Nane* (Auburn University)

Abstract: In this talk we present long time behavior of the solution to a certain class of space-time fractional stochastic equations with respect to the level λ of a noise and show how the choice of the order $\beta \in (0, 1)$ of the fractional time derivative affects the growth and decay behavior of their solution. We also present continuity of the solutions in the time fractional parameter. Our results extend the main results in "M. Foondun, Remarks on a fractional-time stochastic equation, Proc. Amer. Math. Soc. 149 (2021), 2235-2247" to fractional Laplacian as well as higher dimensional cases.
These results are our recent joint work with Alemayehu G. Negash, Jebessa Mijena and Nguyen Huy Tuan.

A physical model with cyclical long-range dependence , *Vladas Pipiras* (University of North Carolina at Chapel Hill)

Abstract: I will describe my recent encounter of a curious, physics-based model with the so-called cyclical long-range dependence. While the usual long-range dependence can be characterized by a divergent spectrum around the zero frequency, the spectrum of its cyclical counterpart diverges at non-zero frequency(ies). I will also discuss some implications of this phenomenon for the application in question.

Heat kernel estimates for Dirichlet forms degenerate at the boundary, *Renming Song* (University of Illinois at Urbana-Champaign)

Abstract: Let X be a purely discontinuous symmetric Markov process on $\mathbf{R}^d_+$ with jump kernel of the form $|x-y|^{-d-\alpha}B(x,y)$ and killing function $cx_d^{-\alpha}$, $\alpha \in (0,2)$. The boundary term $B(x,y)$ is comparable to the product of 4 terms with parameters $beta_1, \beta_2, \beta_3, \beta_4$ appearing as exponents in these terms, and $B(x,y)$ is allowed to decay at the boundary. I will present some recent results on sharp two-sided estimates on the transition density of X. This talk is based on a preprint with Soobin Cho, Panki Kim and Zoran Vondracek.

Necessary and sufficient conditions to solve parabolic Anderson model with rough noise, *Xiong Wang* (Johns Hopkins University)

Abstract: We obtain necessary and sufficient conditions for the existence of n-th chaos of the solution to the parabolic Anderson model $\frac{\partial}{\partial t}u(t,x) = \frac{1}{2}\Delta u(t,x) + u(t,x)\dot{W}(t,x)$, where $\dot{W}(t,x)$ is a fractional Brownian field with temporal Hurst parameter $H_0 \geq 1/2$ and spatial parameters $H = (H_1, \cdots, H_d) \in (0,1)^d$. When $d = 1$, we extend the condition on the parameters under which the chaos expansion of the solution is convergent in the mean square sense, which is both sufficient and necessary under some circumstances.

Local times and geometric properties of Gaussian random fields , *Yimin Xiao* (Michigan State University)

Abstract: We study the local times of anisotropic Gaussian random fields satisfying strong local nondeterminism with respect to an anisotropic metric. By applying moment estimates for local times, we prove optimal local and global Hölder conditions for the local times for these Gaussian random fields and deduce related sample path properties. These results are closely related to Chung's law of the iterated logarithm and the modulus of nondifferentiability of the Gaussian random fields.

We apply the results to systems of stochastic heat equations with additive Gaussian noise and determine the exact Hausdorff measure function for the level sets of the solution.

This talk is based on a joint paper with Davar Khoshnevisan and Cheuk Yin Lee.