Useful Terminology for Understanding Differential Equations

Notation for Derivatives:
The most common notation methods are Lagrange notation (aka prime notation), Newton notation (aka dot notation), and Leibniz's notation (aka dy/dx notation).

Ex 1:
Lagrange Notation: \( y''(x) = 0 \)
Newton Notation: \( \ddot{y} = 0 \)
Leibniz Notation: \( \frac{d^2y}{dx^2} = 0 \)

The example above shows three different ways to write the second derivative of \( y \) is equal to zero. Note that Leibniz notation is the notation used for the rest of the reference sheet.

Independent Variable:
The variable in an equation that can be freely chosen and does not depend on another variable.

Dependent Variable:
The variable that depends on the value of at least one independent variable.

Ex 2:
\[ \frac{dy}{dx} = x + 2 \]

The variable \( y \) is the dependent variable. Variable \( x \) is the independent variable. \( y \) is a function of \( x \) and can be denoted \( y = y(x) \). Note how \( y \) is in the numerator and \( x \) is in the denominator of the derivative.

Differential Equation:
An equation that contains an unknown function and its derivatives.

Ex 3:
\[ \frac{dy}{dx} + y = 0 \]

The example contains the dependent variable \( y \) and its derivative. Again remember that \( y \) is a function of \( x \) and can be denoted \( y = y(x) \).
**Ordinary Differential Equation (ODE):**

A differential equation that is written in terms of one independent variable.  

Ex 4:  
\[ \frac{d^2y}{dx^2} + \frac{dy}{dx} + y = x \]

The example above is written in terms of independent variable x, where y is a function of x. All examples given so far have been ODEs.

**Partial Differential Equation (PDE):**

In contrast to an ODE, a partial differential equation is a differential equation written in terms of more than one independent variable.  

Ex 5:  
\[ \frac{dy}{dx} + \frac{dy}{dv} = x \]

\[ y = f(x,v) \]

The example above is written in terms of independent variables x and v. The dependent variable is y, where y is a function of both x and v.

**Order:**

The value of the highest derivative of an ODE. If given a system of equations, the order of the system is the sum of the order of each equation.  

Ex 6:  
\[ \frac{d^2y}{dx^2} = x \]

The highest derivative in the example is two. Therefore, it is a second-order equation. Examples 1 and 4 also show second order equations. Examples 2, 3 and 5 are first order equations. The term ‘Higher order’ refers to an order of three or more.

**Separable:**

When the variables of an ODE can be rearranged on to opposite sides of the equal sign.  

Ex 7:  
\[ \frac{dy}{dx} = x + xy \]

For more information, visit a tutor. All appointments are available in-person at the Student Success Center, located in the Library, or online.
\[
\frac{dy}{dx} = x(1 + y) \\
\frac{dy}{1 + y} = xdx
\]

The example ODE equation was first factored into \(x\) and \((1+y)\). The term \((1+y)\) was divided to the left hand side. The \(x\) variable was multiplied to the right hand side. Separating equations in this way allows for easy integration.

**Linear:**

An equation that forms a line when plotted. The dependent variable should always be to a power of 1 and should not be multiplied by another dependent variable term.

Ex 8:

\[
a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = 0
\]

Ex 9:

\[
a \frac{d^2y}{dx^2} \frac{dy}{dx} + b \frac{dy}{dx} + cy^2 = 0
\]

Example 8 is the form of a second-order linear equation with coefficients \(a\), \(b\), and \(c\). Example 9 is a non-linear second-order equation with the same coefficients. Note why the equations are different.

**Homogeneous:**

A linear equation that is equal to zero when only the dependent variable terms are on the left-hand side of the equal sign.

Ex 10:

\[
\frac{dy}{dx} + y = 0
\]

The example above is homogenous. Examples 1, 3, and 8 are also homogeneous. Examples 2, 4-7, and 9 are not homogenous.