MECHANICS OF MATERIALS: NORMAL & SHEAR STRESS

Normal Stress Caused by Bending:

- Recall shear and moment calculation and graphing techniques:

\[
M = \int V \, dx = \int \int w \, dx
\]

\[
w = \frac{d}{dx} w = \frac{d^2}{dx^2} M
\]

- The Neutral Axis is the axis at which a member in bending experiences no normal stress:

\[
\begin{array}{ccc}
\sigma & \tau \\
\text{max} & 0 \\
0 & \text{max} \\
\text{max} & 0
\end{array}
\]

- When bending, we observe how stress varies as a point moves away from the Neutral Axis.

\[
\varepsilon = -\frac{y}{c} \cdot \varepsilon_{\text{max}} \quad \text{such that} \quad \sigma_x = E \left(-\frac{y}{c}\right) \cdot \varepsilon_{\text{max}} \quad \text{and} \quad \sigma_x = -\frac{y}{c} \cdot \sigma_{\text{max}}
\]

\[
I = \int y^2 \, dA = \frac{Mc}{\sigma_{\text{max}}} \quad \text{or} \quad \sigma_x = -\frac{My}{I} \quad \text{for any} \ y
\]

- The Parallel Axis Theorem is often used for objects that are not strictly rectangular or circular, but rather are comprised of several shapes:

\[
I_t = I' + A_i d_i^2
\]

<table>
<thead>
<tr>
<th>Load (w)</th>
<th>Shear (V)</th>
<th>Moment (M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$-w$ (constant)</td>
<td>$wx$ (linear)</td>
<td>$wx^2$ (parabolic)</td>
</tr>
<tr>
<td>$-wx$ (linear)</td>
<td>$wx^2$ (parabolic)</td>
<td>$wx^3$</td>
</tr>
<tr>
<td>$-wx^2$ (parabolic)</td>
<td></td>
<td>$wx^4$</td>
</tr>
</tbody>
</table>

- The negative sign is necessary because for a positive y point, the beam experiences compression.

<table>
<thead>
<tr>
<th>Rectangle</th>
<th>$I_{\text{rectangle}} = \frac{1}{12}bh^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle</td>
<td>$I_{\text{circle}} = \frac{1}{4}\pi R^4$</td>
</tr>
<tr>
<td>Triangle</td>
<td>$I_{\text{triangle}} = \frac{1}{36}bh^3$</td>
</tr>
<tr>
<td>Semi-circle</td>
<td>$I_{\text{semicircle}} = \frac{1}{8}\pi R^4$</td>
</tr>
</tbody>
</table>

- $b$ is in the direction parallel ($\parallel$) to the NA
- $h$ is in the direction perpendicular ($\perp$) to the NA
Unsymmetrical bending due to moments at angles requires the moment vector be decomposed:

The location of the neutral axis is measured by $\phi$ offset from the positive $z$-axis. By knowing the neutral axis experiences no normal stress,

$$\sigma_x = 0 = \frac{M_x}{I_x} + \frac{M_y}{I_y}$$

$$M_y = \frac{M_x}{\cos \theta} I_y = \tan \theta \frac{I_y}{I_x}$$

Such that

$$\tan \phi = \frac{I_y}{I_x} \cdot \tan \theta$$

The quadrants depend on the angle of the moment.

Shear Stress Caused By Shear Force:

- There are two directions in which shear force acts: longitudinal (along the length of the beam) and transverse (on cut plane)
  1. Horizontal shear (longitudinal): $\Delta H = \frac{VQ}{I} \Delta x$
  2. Transverse shear: $\tau_{avg} = \frac{VQ}{Ic}$ and $\tau_{max} = 1.5 \cdot \frac{V_{max}}{A}$
- Shear flow (shear force per unit length): $q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I}$

How to determine $Q$:

<table>
<thead>
<tr>
<th>$\bar{y}$</th>
<th>Distance from NA to the centroid of $A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Area above the point or above the Shear Plane (opposite side of NA)</td>
</tr>
<tr>
<td>$I$</td>
<td>Moment of inertia of entire object (independent of $A$ or $\bar{y}$)</td>
</tr>
<tr>
<td>$V$</td>
<td>Transverse force applied</td>
</tr>
<tr>
<td>$t$</td>
<td>Thickness of the object at the point observed or Shear Plane</td>
</tr>
</tbody>
</table>

For more information, visit a tutor. All appointments are available in-person at the Student Success Center, located in the Library, or online. Adapted from Hibbeler, R.C. (2014). Mechanics of Materials (9th Edition). Boston, MA: Prentice Hall.