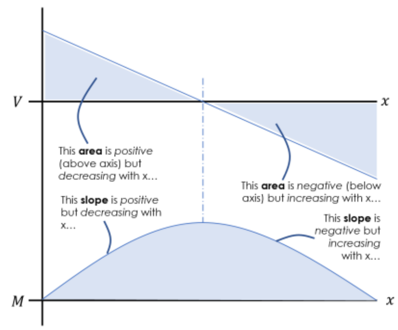
MECHANICS OF MATERIALS: NORMAL & SHEAR STRESS

Normal Stress Caused by Bending:

- Recall shear and moment calculation and graphing techniques:

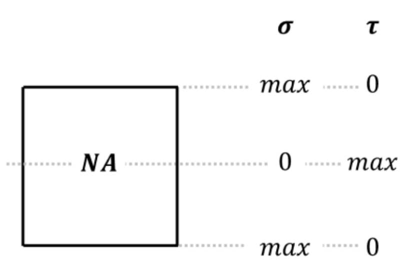
$$M = \int V\,dx = \iint w\,dx$$

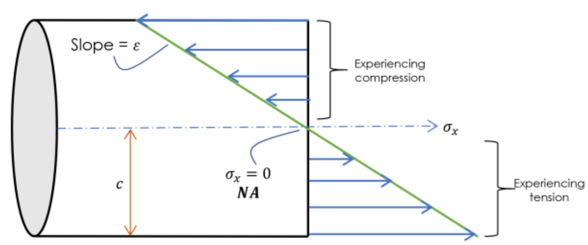
$$w = \frac{d}{dx}w = \frac{d^2}{dx^2}M$$



Load (w)	Shear (V)	Moment (M)
0	0	0
$-w$ (constant)	wx (linear)	wx^2 (parabolic)
$-wx$ (linear)	wx^2 (parabolic)	wx^3
$-wx^2$ (parabolic)	wx^3	wx^4

- The Neutral Axis is the axis at which a member in bending experiences no normal stress:





- When bending, we observe how stress varies as a point moves away from the Neutral Axis.

$$\varepsilon = -\frac{y}{c}\cdot\varepsilon_{max} \quad \text{such that} \quad \sigma_x = E\left(-\frac{y}{c}\right)\cdot\varepsilon_{max} \quad \text{and} \quad \sigma_x = -\frac{y}{c}\cdot\sigma_{max}$$

$$I = \int y^2 dA = \frac{Mc}{\sigma_{max}} \quad \text{or} \quad \sigma_x = -\frac{My}{I} \text{ for any } y$$

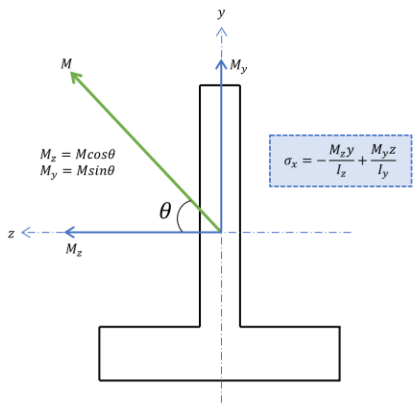
The negative sign is necessary because for a positive y point, the beam experiences compression

- The Parallel Axis Theorem is often used for objects that are not strictly rectangular or circular, but rather are comprised of several shapes:

$$I_i = I' + A_i d_i^2$$

Rectangle	$I_{rectangle} = \frac{1}{12}bh^3$
Circle	$I_{circle} = \frac{1}{4}\pi R^4$
Triangle	$I_{triangle} = \frac{1}{36}bh^3$
Semi-circle	$I_{semicircle} = \frac{1}{8}\pi R^4$
b is in the direction parallel (∥) to the NA h is in the direction perpendicular (⊥) to the NA	

- Unsymmetrical bending due to moments at angles requires the moment vector be decomposed:



The location of the neutral axis is measured by ϕ offset from the positive z-axis.
By knowing the neutral axis experiences no normal stress,

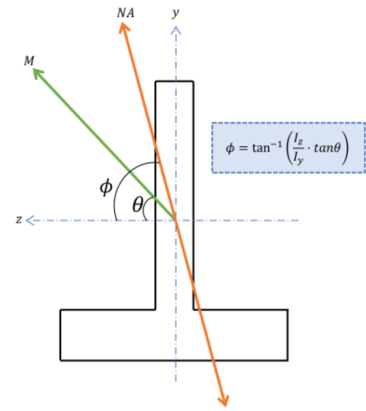
$$\sigma_x = 0 = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y},$$

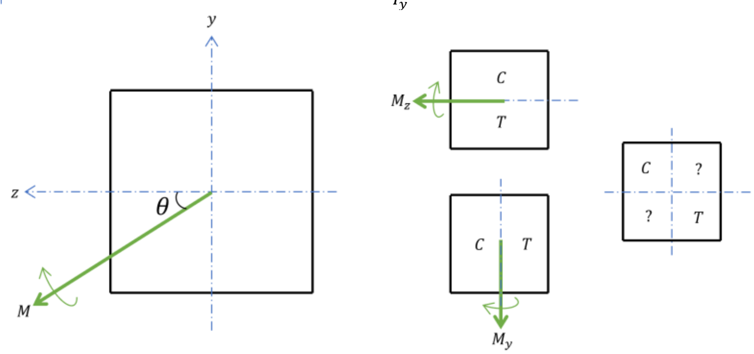
$$\frac{M_z y}{I_z} = \frac{M_y z}{I_y}$$

$$\frac{y}{z} = \frac{M\sin\theta}{M\cos\theta} \cdot \frac{I_z}{I_y} = \tan\theta \cdot \frac{I_z}{I_y} = \frac{y}{z}$$

Such that

$$\tan\phi = \frac{I_z}{I_y} \cdot \tan\theta$$





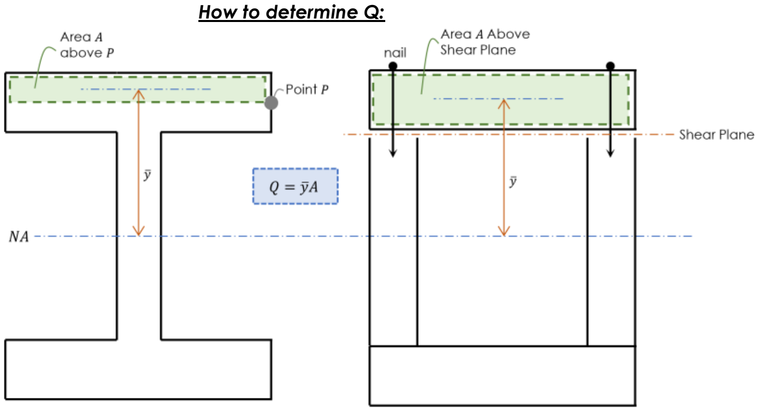
The ? quadrants depend on the angle of the moment

Shear Stress Caused By Shear Force:

- There are two directions in which shear force acts: longitudinal (along the length of the beam) and transverse (on cut plane)
  1. Horizontal shear (longitudinal): $\Delta H = \frac{VQ}{I}\Delta x$
  2. Transverse shear: $\tau_{avg} = \frac{VQ}{It}$ and $\tau_{max} = 1.5 \cdot \frac{V_{max}}{A}$
- Shear flow (shear force per unit length): $q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I}$

$\bar{y}$	Distance from NA to the centroid of A
A	Area above the point or above the Shear Plane (opposite side of NA)
I	Moment of inertia of entire object (independent of A or $\bar{y}$)
V	Transverse force applied
t	Thickness of the object at the point observed or Shear Plane

How to determine Q:



For more information, visit a tutor. All appointments are available in-person at the Student Success Center, located in the Library, or online. Adapted from Hibbeler, R.C. (2014). *Mechanics of Materials* (9th Edition). Boston, MA: Prentice Hall.