Combined Loadings:

- Thin-walled pressure vessels:
  1. Hoop stress is circumferential direction: \( \sigma_1 = \frac{pr}{t} \)  
     Where \( r \) = inner radius \( t \) = wall thickness
  2. Longitudinal stress is lengthwise: \( \sigma_2 = \frac{pr}{2t} \)

- Combined loading due to normal force and bending:
  \[ \sigma_x = \frac{P}{A} - \frac{M_{zy}}{I_z} + \frac{M_{yz}}{I_y} \]

- Combined loading due to shear forces:
  \[ \tau = \frac{T_{pr}}{J} + \frac{V_y Q_y}{It} + \frac{V_z Q_z}{It} \]

Stress Transformations:

- By rotating the coordinate axis we see all possible combinations of normal and shear stress. They are plotted as Mohr’s Circle. This allows identification of coordinates in which all stress is normal and potentially points where all stress is shear.

\[
\sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cdot \cos(2\theta) + \tau_{xy} \sin(2\theta) \\
\sigma'_y = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cdot \cos(2\theta) - \tau_{xy} \sin(2\theta) \\
\tau'_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \cdot \sin(2\theta) + \tau_{xy} \cdot \cos(2\theta)
\]

To rotate by \( \theta_p \) into the principal stress plane (where \( \tau_{xy} = 0 \) and \( \sigma_x \) is max):

\[ \tan(2\theta_p) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \]

To rotate by \( \theta_s \) into the principal shear plane (where \( \tau_{xy} \) is max and \( \sigma_x \) is min):

\[ \tan(2\theta_s) = \frac{\sigma_x - \sigma_y}{2\tau_{xy}} \]

\( \sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} \)  
Circle of Radius \( R \) centered at \( \sigma_{avg} \) with no height

\[ R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \]

such that \( R^2 = \left(\sigma_{avg} - \sigma_x\right)^2 + \tau_{xy}^2 \)

\( \sigma \) is maximized and minimized at

\[ \sigma_x + \sigma_y \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \]

\( \sigma_{avg} \pm R \)
Beam Design:

- Determine all beams with \( S \geq S_{\text{min}} \) and select the one with the lowest weight to length ratio (less weight is more desirable so long as it meets the minimum criterion set by \( S_{\text{min}} \)).
- \( S = \frac{l}{c} \) and \( \sigma = \frac{Mc}{l} \) therefore \( S = \frac{l}{M} = \frac{M}{\sigma} \)

\[
S_{\text{min}} = \frac{|M_{\text{max}}|}{\sigma_{\text{allow}}}
\]

Max moment is often found via shear and moment diagrams while allowable stress is usually supplied as a design criterion.

For more information, visit a tutor. All appointments are available in-person at the Student Success Center, located in the Library, or online. Adapted from Hibbeler, R.C. (2014). Mechanics of Materials (9th Edition). Boston, MA: Prentice Hall.