MECHANICS OF MATERIALS: AXIAL LOADS & TORSION

Axial Loads:

\[ \varepsilon = \frac{\delta}{L} = \frac{\sigma}{E} = \frac{1}{EA} \cdot \frac{P}{A} \]

such that \( \delta = \frac{PL}{AE} \) for homogenous and uniform cross-sections

\[ \delta = \int_{0}^{L} \frac{P(x)}{A(x) \cdot E(x)} \, dx = \sum_{i=0}^{n} \frac{P_{i}L_{i}}{A_{i}E_{i}} \]

- **Superposition** allows the problem to be worked by evaluating how much a body would move if it could. Then, the reaction force must be equal and opposite.
  1. Remove one wall support
  2. Break the body into sections where \( P, L, A, \) or \( E \) changes
  3. Find \( \delta \) for each section and the resulting \( \delta_{\text{total}} = \sum \delta_{i} \) (careful to mind signs)
  4. Find \( \delta_{\text{reaction}} \) in terms of the reaction force \( R_{\text{wall}} \)
  5. Equate the deformations due to the true deflection being 0 (\( \delta = 0 = \delta_{\text{no wall}} + \delta_{\text{reaction}} \))
  6. Solve for reaction force at the wall, \( R_{\text{wall}} \), and consequently other reaction forces.
  7. Solve for the stress in each section (from step 2), where \( P = R_{\text{wall}} \) for all sections and \( A \) varies.

- **Thermal Stress** is a result of heating or cooling and is dependent on the material’s coefficient of thermal expansion, \( \alpha \) (found in tables, measured in \( ^{\circ}\text{F}^{-1} \) or \( ^{\circ}\text{C}^{-1} \)):

\[ \delta_{T} = \alpha L \Delta T \quad \varepsilon_{T} = \alpha \Delta T \]

- Stress concentrations arise when stress flow is abruptly "pinched" around a corner. Two common instances are holes and fillets in a flat plate. This is the reason sidewalk corners crack first.

Given \( D, d, \) and \( r/d, \) \( K \) can be found using provided charts. \( K \) tells the ratio of the maximum stress observed to the average.
**Torsion:**

- Twisting engenders a shear stress:
  \[ T = \int r dF \text{ where } \tau = \frac{dF}{dA} \text{ thus } T = \int r \cdot \tau dA \]

- Angle of twist measures deformation and is dependent on the length and diameter. Shear strain is independent of both diameter and length:
  \[ \gamma_{\text{max}} = \frac{c\phi}{L} = \frac{\tau_{\text{max}}}{G} = \frac{TC}{JG} \text{ and } \phi = \frac{TL}{JG} \]

- Power transmissions can be analyzed to understand the minimum shaft diameter for a given power requirement (or max power for given shaft diameter):
  \[ P = t \cdot \omega = 2\pi \text{ such that } T = \frac{P}{2\pi f} = \frac{\tau_{\text{max}} \cdot J}{c} \]
  \[ \frac{J}{C} = \frac{P}{2\pi f} \cdot \frac{1}{\tau_{\text{max}}} \]
  \[ J = \frac{TL}{G\phi} = \frac{P}{2\pi f} \cdot \frac{L}{G\phi} \]

For more information, visit a [tutor](#). All appointments are available in-person at the Student Success Center, located in the Library, or online. Adapted from Hibbeler, R.C. (2014). Mechanics of Materials (9th Edition). Boston, MA: Prentice Hall.