STUDENT SUCCESS CENTER
THE UNIVERSITY OF ALABAMA IN HUNTSVILLE

CAL C: PARAMETERIZATION, TNB FRAME OF REFERENCE, & EXTREME VALUES

Parameterization:

- Curves in space can be parameterized using intermediate variables, such as t (which can be thought of as *time*). When $\boldsymbol{r}(t) = f(t)\boldsymbol{i} + g(t)\boldsymbol{j} + h(t)\boldsymbol{k}$, the head of the vector traces out the path of the curve as a function of *t* over some (time) interval I. All differential rules hold true when a curve is parameterized.
- Introducing $\boldsymbol{r}$ as position and t as time allows for expansion to use $\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt}$ for velocity and $= \frac{d}{dt}\left(\frac{d\boldsymbol{r}}{dt}\right) = \frac{d\boldsymbol{v}}{dt}$. This allows for simplification of formulas such as arc length:

$$L = \int_{t=a}^{t=b} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt \quad = \quad \int_{t=a}^{t=b} \sqrt{|\boldsymbol{v}|}dt$$

TNB Frame of Reference:

- Rather than defining the reference frame from a stationary observer of an airplane, it can be defined from an observer inside the airplane. This frame of reference is completed with 3 descriptions of motion:
  - The unit tangent vector, $\boldsymbol{T}$. This defines forward motion, normalized to be a unit vector:

$$T = \frac{\frac{d\boldsymbol{r}}{ds}}{\left|\frac{d\boldsymbol{r}}{ds}\right|} = \frac{\boldsymbol{v}}{|\boldsymbol{v}|}$$

  - The principal unit normal vector, $\boldsymbol{N}$. This defines the direction which the curve is turning and is, by definition, orthogonal to $\boldsymbol{T}$. It is normalized by dividing the orthogonal vector by length of κ, which defines the intensity/amount of the turn.

$$\boldsymbol{N} = \frac{1}{\kappa} \cdot \frac{d\boldsymbol{T}}{ds} \quad \text{where} \quad \kappa = \frac{1}{|v|} \cdot \left|\frac{d\boldsymbol{T}}{dt}\right| \quad \text{and therefore} \quad N = \frac{\frac{d\boldsymbol{T}}{ds}}{\left|\frac{d\boldsymbol{T}}{ds}\right|}$$

  - The binormal vector, $\boldsymbol{B}$. This is defined as the cross product of T and N and is used in finding the torsion, τ, which describes how the path twists.

$$\tau = -\frac{d\boldsymbol{B}}{ds} \cdot \boldsymbol{N} \quad \text{where } \boldsymbol{B} = \boldsymbol{T} \times \boldsymbol{N}$$

- These are used in finding the acceleration vector, $\boldsymbol{a}$:

Based on Pythagorean's Theorem

$$\boldsymbol{a} = a_T \cdot \boldsymbol{T} + a_N \cdot \boldsymbol{N} \quad \text{where} \quad a_T = \frac{d\boldsymbol{v}}{dt} = \frac{d^2s}{dt^2} \quad \text{and} \quad a_N = \kappa \cdot |\boldsymbol{v}|^2 = \sqrt{|\boldsymbol{a}|^2 - a_T^2}$$

For more information, visit a tutor. All appointments are available in-person at the Student Success Center, located in the Library, or online. Adapted from Hass, J., Weir, M.D., & Thomas, G.B. (2012). *University Calculus: Early Transcendentals* (2nd ed.). Boston: Pearson Education.