CALCULUS B REVIEW

Trig Sub Reference Triangle

Given two functions, \( P(x) \) and \( Q(x) \), when integrating \( \int P(x) \frac{Q(x)}{Q(x)} \, dx \), and \( P(x) \) is smaller in degree than \( Q(x) \), the integral decomposes into:

\[
(ax + b)^k = \frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} + \cdots + \frac{A_k}{(ax+b)^k}
\]

\[
(ax^2 + bx + c)^k = \frac{A_1x+B_1}{ax^2+bx+c} + \cdots + \frac{A_kx+B_k}{(ax^2+bx+c)^k}
\]

Trapezoidal Rule

To approximate \( \int_a^b f(x) \, dx \), use

\[
T = \frac{\Delta x}{2} (y_0 + 2y_1 + \cdots + 2y_{n-1} + y_n)
\]

The y’s are the values of \( f \) at the partition points

\[
x_0 = a, x_1 = a + \Delta x, \ldots, x_{n-1} = a + (n-1)\Delta x
\]

\[
x_n = b, \text{ Where } \Delta x = \frac{(b-a)}{n}
\]

Simpson’s Rule

To approximate \( \int_a^b f(x) \, dx \), use

\[
S = \frac{\Delta x}{3} (y_0 + 4y_1 + 2y_2 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n)
\]

The number \( n \) is even, and \( \Delta x = \frac{(b-a)}{n} \)

Improper Integrals

Type I

1. If \( f(x) \) is continuous on \([a, \infty)\), then

\[
\int_a^\infty f(x) \, dx = \lim_{b \to \infty} \int_a^b f(x) \, dx
\]

2. If \( f(x) \) is continuous on \((-\infty, b]\), then

\[
\int_{-\infty}^b f(x) \, dx = \lim_{a \to -\infty} \int_a^b f(x) \, dx
\]

3. If \( f(x) \) is continuous on \((-\infty, \infty)\), then

\[
\int_{-\infty}^\infty f(x) \, dx = \int_{-\infty}^c f(x) \, dx + \int_c^{\infty} f(x) \, dx
\]

Type II

1. If \( f(x) \) is cont. on \((a, b]\) and discont. at \( a \), then

\[
\int_a^b f(x) \, dx = \lim_{c \to a^+} \int_a^c f(x) \, dx
\]

2. If \( f(x) \) is cont. on \([a, b)\) and discont. at \( b \), then

\[
\int_a^b f(x) \, dx = \lim_{c \to b^-} \int_c^b f(x) \, dx
\]

3. If \( f(x) \) is discont. at \( c \), where \( a < c < b \), and cont. on \([a, c) \cup (c, b]\), then

\[
\int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx
\]

*In each case, if the limit is finite, the improper integrals converges and that the limit is the value of the improper integral. If the limit does not exist, the integral diverges.*
**Polar Coordinates**
Equations relating Polar and Cartesian

\[ x = r \cos \theta, \quad y = r \sin \theta, \]

\[ \tan \theta = \frac{y}{x}, \quad r^2 = x^2 + y^2 \]

**Sequences**
The sequence **converges** to the finite number \( L \) if the limit of the sequence

\[ \lim_{n \to \infty} a_n = L \]

The sequence **diverges** to negative or positive infinity

\[ \lim_{n \to \infty} a_n = \pm \infty \]

**Infinite Series**
Given a sequence of numbers \( \{a_n\} \),

\[ a_1 + a_2 + \cdots + a_n \]

is an **infinite series**. The number \( a_n \) is the **nth term** of the series. The sequence \( \sum_{k=1}^{n} a_k \) is the **sequence of partial sums** of the series. If the sequence of partial sums converges to a limit \( L \), we say that the series **converges** and its sum is \( L \). If the sequence of partial sums does not converge to a number, we say it **diverges**.

**Convergence Tests**

**Comparison Test**
Let \( \sum a_n, \sum c_n \), and \( \sum d_n \) be series with nonnegative terms. Suppose, for some number \( N \), that

\[ d_n \leq a_n \leq c_n \quad \text{for all} \quad n > N \]

(A) If \( \sum c_n \) converges, then \( \sum a_n \) also converges

(B) If \( \sum d_n \) diverges, then \( \sum a_n \) also diverges

**P-Series Test**
The P-Series \( \sum_{n=1}^{\infty} \frac{1}{n^p} \) converges if \( p > 1 \), diverges if \( p \leq 1 \)

**Limit Comparison Test**
Suppose that \( a_n > 0 \) and \( b_n > 0 \) for all \( n \geq N \)

If \( \lim_{n \to \infty} \frac{a_n}{b_n} \) equals:

1) \( c > 0 \), then both series either converge or diverge

2) \( 0 \), and \( \sum b_n \) converges, then \( \sum a_n \) converges

3) \( \infty \), and \( \sum b_n \) diverges, then \( \sum a_n \) diverges
Ratio Test

Let $\sum a_n$ be a series with positive terms, compute

$$\lim_{n \to \infty} \frac{|a_{n+1}|}{|a_n|} = p$$

Converges if $p < 1$, diverges if $p > 1$ or is infinite, inconclusive if $p = 1$

Root Test

Let $\sum a_n$ be a series with $a_n > 0$ for $n \geq N$

$$\lim_{n \to \infty} \sqrt[n]{a_n} = p$$

Converges if $p < 1$, diverges if $p > 1$ or is infinite, inconclusive if $p = 1$

Alternating Series Test

$$\sum_{n=1}^{\infty} (-1)^{n+1} u_n = u_1 - u_2 + u_3 - u_4 ...$$

Converges if the following is satisfied:
all $u_n$'s are positive, the positive $u_n$'s are non-increasing, and $u_n \to 0$

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