CAL A: EXTREME VALUES

Relative Extreme Values:

First Derivative Test:

- Local maximum at \( x = c \) if \( f'(c) = 0 \) and the function changes from increasing to decreasing at \( x = c \) and \( f(c) \) is defined
- Local minimum at \( x = c \) if \( f'(c) = 0 \) and the function changes from decreasing to increasing at \( x = c \) and \( f(c) \) is defined

Second Derivative Test:

- Local maximum at \( x = c \) if \( f'(c) = 0 \) and \( f''(c) < 0 \) and \( f(c) \) is defined
- Local minimum at \( x = c \) if \( f'(c) = 0 \) and \( f''(c) > 0 \) and \( f(c) \) is defined

Example: Find relative max./min. of \( f(x) = 3x^2 - 12x + 4 \).

First Derivative Test:

- \( f'(x) = 6x - 12 = 0 \)
  
  Critical Point at \( x = 2 \)

- On \( (-\infty, 2) \), let the test point be \( x = 0 \).

- \( f'(0) = 6(0) - 12 = -12 < 0 \)
  
  \( f(x) \) is decreasing on interval \( (-\infty, 2) \)

- On \( (2, \infty) \), let the test point be \( x = 3 \).

- \( f'(3) = 6(3) - 12 = 18 - 12 = 6 > 0 \)
  
  \( f(x) \) is increasing on interval \( (2, \infty) \)

- Since \( f(x) \) changes from decreasing to increasing at the critical point \( x = 2 \), \( x = 2 \) is a relative minimum.

- Define \( f(c) \)

Second Derivative Test:

- \( f''(x) = 6 \)

- \( f''(2) = 6 > 0 \)
  
  \( f(x) \) is concave up at \( x = 2 \)

- Since \( f(x) \) is concave up at the critical point \( x = 2 \), \( x = 2 \) is a relative minimum

- Define \( f(c) \)
**Absolute Extreme Values:**

1. Find critical points
2. Evaluate $f(x)$ for critical points and endpoints
3. Compare $f(x)$ values to determine maximum and minimum

*Example:* Find absolute max. and min. of $f(x) = x^3 - 3x + 5$ on the interval $[0,2]$.

- $f'(x) = 3x^2 - 3 = 0$
  - Critical points are $x=-1$ and $x=1$

- $f(0) = (0)^3 - 3(0) + 5 = 5$
- $f(1) = (1)^3 - 3(1) + 5 = 3$
- $f(2) = (2)^3 - 3(2) + 5 = 7$

*Critical point $x=-1$ is not included because -1 is not on the interval [0,2]*

- Absolute maximum at $x=2$ and absolute minimum at $x=1$.  

For more information, visit a **tutor**. All appointments are available in-person at the Student Success Center, located in the Library, or online. Adapted from Hass, J., Weir, M.D., & Thomas, G.B. (2012). *University Calculus: Early Transcendentals* (2nd ed.). Boston: Pearson Education.