STUDENT SUCCESS CENTER

THE UNIVERSITY OF ALABAMA IN HUNTSVILLE

CAL A: DERIVATIVES

Definition of the Derivative:

- $\frac{d}{dx}(f(x)) = f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

Derivative Properties:

- $(c \cdot f(x))' = c \cdot f'(x)$
- $(f(x) \pm g(x))' = f'(x) \pm g'(x)$
- $\frac{d}{dx}(c) = 0$

Power Rule:

- $\frac{d}{dx}(x^n) = n \cdot x^{n-1}$

Product Rule:

Used to take the derivative of a product of two functions:

- $\frac{d}{dx}[f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

Example: $\frac{d}{dx}[(3 - x^2)(\sin x)]$

$$= \frac{d}{dx}(3 - x^2) \cdot (\sin x) + (3 - x^2) \cdot \frac{d}{dx}(\sin x)$$

$$= (-2x) \cdot (\sin x) + (3 - x^2) \cdot (\cos x)$$

Quotient Rule:

Used to take the derivative of a quotient of two functions:

- $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$

Example: $\frac{d}{dx}\left(\frac{x^3-x+1}{\ln x}\right)$

$$= \frac{\frac{d}{dx}(x^3 - x + 1) \cdot (\ln x) - (x^3 - x + 1) \cdot \frac{d}{dx}(\ln x)}{(\ln x)^2}$$

$$= \frac{(3x^2 - 1) \cdot \ln x - (x^3 - x + 1) \cdot \frac{1}{x}}{(\ln x)^2}$$

Chain Rule:

Used to take the derivative of a composite function (a function within another function:

- $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$

Example: $\frac{d}{dx}[(2x - 7)^3]$

$$= 3(2x - 7)^2 \cdot \frac{d}{dx}(2x - 7) = 3(2x - 7)^2 \cdot 2$$

$$= 6(2x - 7)^2$$

L'Hopital's Rule:

Used when the limit of a fraction when direct substitution gives and indeterminate form:

- $If \lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{0}{0} \; or \; \frac{\pm\infty}{\pm\infty},$
  $then \lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \lim_{x\to a}\left[\frac{f'(x)}{g'(x)}\right]$

Curve Sketching Definitions:

- Critical Point at x = c if f '(c) = 0 or is undefined
- Function is increasing if f '(x) > 0
- Function is decreasing if f '(x) < 0
- Function is concave up if f "(x) > 0
- Function is concave down if f "(x) < 0
- Possible Inflection Point at x = c if f "(c) = 0 or is undefined and concavity changes

Newton's Method:

- *A technique for finding solutionsto the equation f(x)=0.*
- $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Common Derivatives:

- $\frac{d}{dx}(x) = 1$
- $\frac{d}{dx}(e^x) = e^x$
- $\frac{d}{dx}(a^x) = a^x \cdot ln(a)$
- $\frac{d}{dx}[ln(x)] = \frac{1}{x}$
- $\frac{d}{dx}[log_a(x)] = \frac{1}{x \cdot ln(a)}$
- $\frac{d}{dx}[sin(x)] = cos(x)$
- $\frac{d}{dx}[cos(x)] = -sin(x)$
- $\frac{d}{dx}[tan(x)] = sec^2(x)$
- $\frac{d}{dx}[cot(x)] = -csc^2(x)$
- $\frac{d}{dx}[sec(x)] = sec(x) \cdot tan(x)$
- $\frac{d}{dx}[csc(x)] = -csc(x) \cdot cot(x)$
- $\frac{d}{dx}[sin^{-1}(x)] = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}[cos^{-1}(x)] = -\frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}[tan^{-1}(x)] = \frac{1}{1+x^2}$
- $\frac{d}{dx}[cot^{-1}(x)] = -\frac{1}{1+x^2}$
- $\frac{d}{dx}[sec^{-1}(x)] = \frac{1}{|x|\sqrt{x^2-1}}$
- $\frac{d}{dx}[csc^{-1}(x)] = -\frac{1}{|x|\sqrt{x^2-1}}$

Optimization:

Steps to find the maximum or minimum of a function:

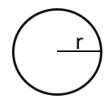
1. Draw a picture and assign variables
2. Write an equation for the variable being optimized *(rewrite the unknown as a function of a single variable if applicable)*
3. Test critical points and endpoints to determine absolute maximum or minimum
4. L=6 is a minimum and not a maximum – use the 2nd derivative test.

Related Rates:

Steps to find the rate of change of a variable using other known rates of change:

1. Draw a picture and assign variables
2. Write an equation relating the variables
3. Differentiate with respect to t
4. Solve for the unknown

Example: Find the rate of change of the area of a circle when the radius is 4 in. and the radius is increasing at a rate of 3 in/min.



$\longrightarrow \frac{dr}{dt} = 3$

- $A = \pi r^2$
- $\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt} = 2\pi(4)(3) = 24\pi$
- The rate of change of the area of the circle is 24π in^2/min.