

## ABSTRACT

We demonstrate experimentally that the electronic frequency divider (EFD) can be used as a simple tool for measuring phase/frequency modulation noise in the beat note and also provide theoretical explanation for that.

## INTRODUCTION

- Frequency and phase noise (FPN) is an important property of electromagnetic oscillators. Characterizing the FPN of optical oscillators, i.e. lasers, is of particular significance in such emerging fields as optical atomic clocks, coherent optical communication and microwave photonics where laser frequency stabilization and phase locking are universally needed.
- Even though various methods have been proposed to characterize FPN of lasers, the problem of determining the constitution of FPN spectrum persists. Direct measurement of power spectral density (PSD) of FPN is fundamentally incapable of drawing distinction between constitutive frequency modulation (FM) and phase modulation (PM). Neither can it provide any quantitative assessment about the often useful relevant parameters such as modulation frequency and modulation depth.
- We present a simple but elegant method to decode FPN in general, with the ability to precisely predict the modulation frequencies and modulation depths.

## THEORY

- A classical approach to describe output of an oscillator mutilated with FPN is to replace the phase term by a deterministic sinusoidal modulation [1]. A sinusoidal angle modulation of a sinusoidal signal with modulation index  $\beta$  and modulation frequency  $f_m$  is described by

$$x(t) = A \cos(2\pi f_c t + \beta \sin(2\pi f_m t)) \quad (1)$$

where  $f_c$  is carrier frequency and  $A$  is the amplitude of the signal being modulated.

- The function of EFD is to divide the phase of an input signal by its division ratio ( $N$ ). So division of the phase of Eq. (1) yields

$$y(t) = \exp(2\pi f_c / N t) \exp(\beta / N \sin(2\pi f_m t)) \quad (2)$$

- By expanding  $\exp(\beta / N \sin(2\pi f_m t))$  into infinite series, Eq. (2) can be written as

$$y(t) = A \cos \sum_{n=-\infty}^{+\infty} J_n(\beta / N) \exp(j 2\pi f_c / N t + 2\pi n f_m t) \quad (3)$$

- Eq. (3) shows that EFD causes two changes on a signal represented by Eq. (1): it divides the carrier frequency  $f_c$  by  $N$  and it divides the modulation index  $\beta$  by  $N$ . It should be noted that EFD does not change the rate of modulation  $f_m$  so sideband spacing remains the same.
- Empirically, the bandwidth of an angle-modulated signal can be approximately predicted by Carson's rule [1], i.e.  $BW \approx 2(\Delta f + f_m)$  where  $\Delta f = \beta f_m$ . Qualitatively, when  $\beta < 1$ , the bandwidth can be approximated to  $BW \approx 2f_m$  and when  $\beta \gg 1$ ,  $BW \approx 2\Delta f$ .
- Physically,  $\beta < 1$  represents the regime of PM where single-tone modulation produces discrete sidebands at the harmonics of  $f_m$ . An EFD reduces the  $\beta$  by the factor of  $N$ , causing all the sidebands to reduce roughly by same proportion. As a result, the effect of EFD on pure PM noise is simple "lowering" of sidebands without significant change in spectral profile as shown in Fig. 1(a).
- Whereas  $\beta \gg 1$  corresponds to the regime of strong FM. Since  $\Delta f > f_m$ , the carrier frequency sweeps back and forth within the range of  $2\Delta f$  at a rate of  $f_m$ , generating a "flattop" spectrum with  $BW \approx 2\Delta f$ . When such FM signal is applied to an EFD, bandwidth "narrowing" occurs since  $\beta$  is divided. However, as long as  $\beta / N > 1$  remains satisfied, the spectrum maintains its distinctive "flattop" shape while its bandwidth reduces by a factor of  $N$  as shown in Fig 1(b). But if  $\beta / N \leq 1$  then the divided signal becomes PM dominated and discrete sidebands at harmonics of  $f_m$  start to show up.
- For intermediate values of  $\beta$ , one would have to rely on Eq. (3) to understand the effect of EFD. Also the effect of EFD on an input containing PM and FM simultaneously is mere superposition.

## EXPERIMENT

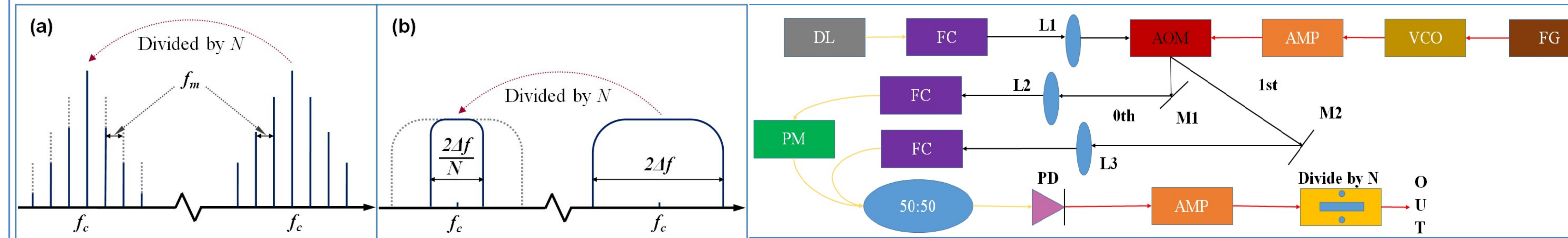


Fig. 1. (a) The effect of EFD on PM sideband is reduction of sideband amplitude (b) The effect of EFD on deep FM spectrum is reduction of spectral width.

## EXPERIMENT

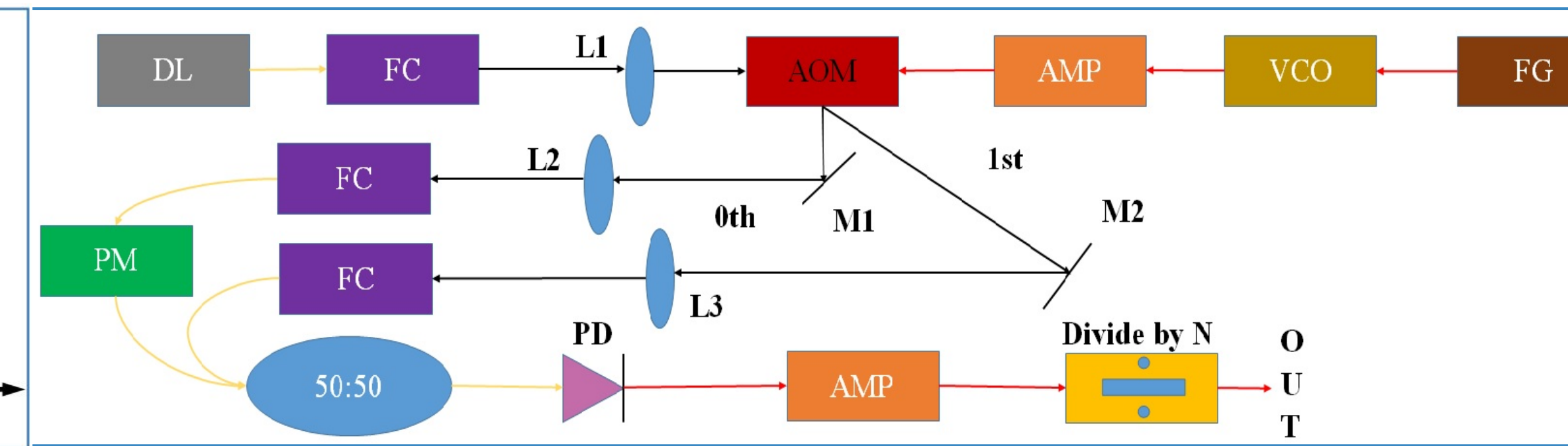


Fig. 2. Schematic of experimental setup. DL: Diode laser; FC: Fiber Collimator; M1, M2: Mirrors; L1, L2, L3: Collimating lenses; AOM: Acousto-optic modulator; Amp: Amplifier; VCO: Voltage-controlled oscillator; FG: Function generator; PM: Electro-optic phase modulator; 50:50: Fiber coupler; PD: Photodiode; Divide by N: EFD; Yellow arrow: single mode fiber; Black arrow: free space path; Red arrow: Electrical cable.

## RESULTS

### Output of EFD to a Frequency Modulated input

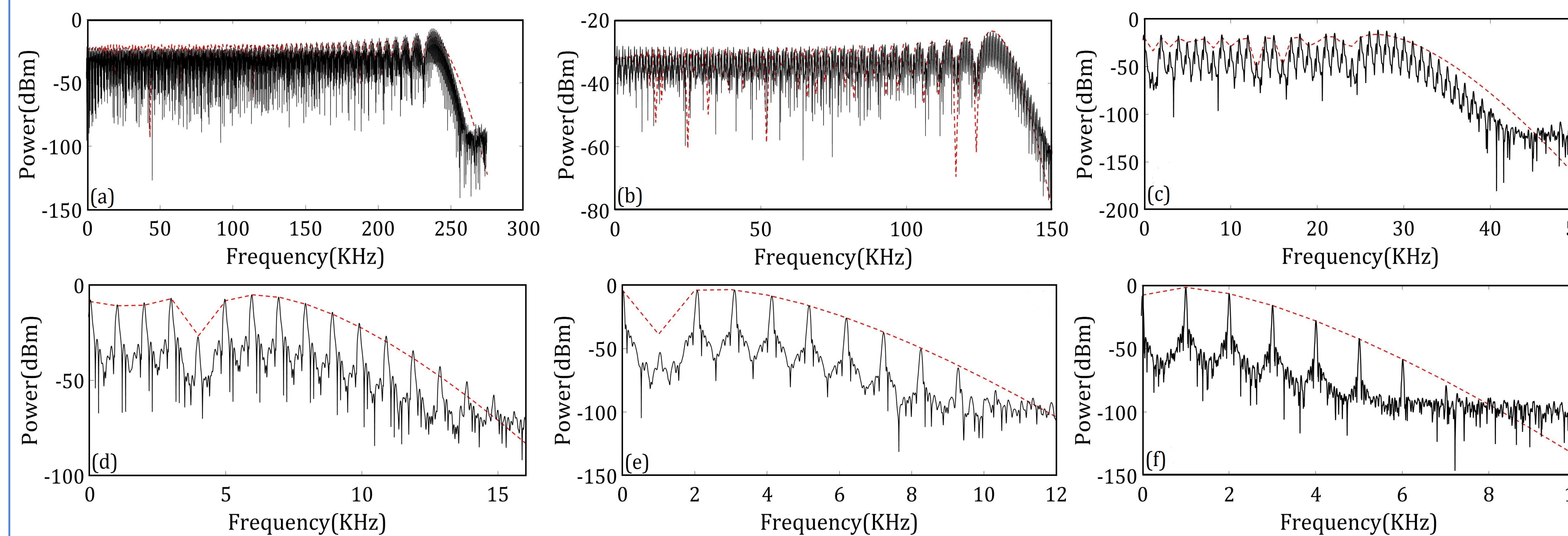


Fig. 2. Comparison of theoretical simulations (red trace) and experimental data (black trace) of output of electronic frequency divider for different division ratio for frequency modulated input. (a) divide by 1 (undivided), (b) divide by 2, (c) divide by 8, (d) divide by 32, (e) divide by 64, (f) divide by 128.

### Output of EFD to a Phase Modulated input

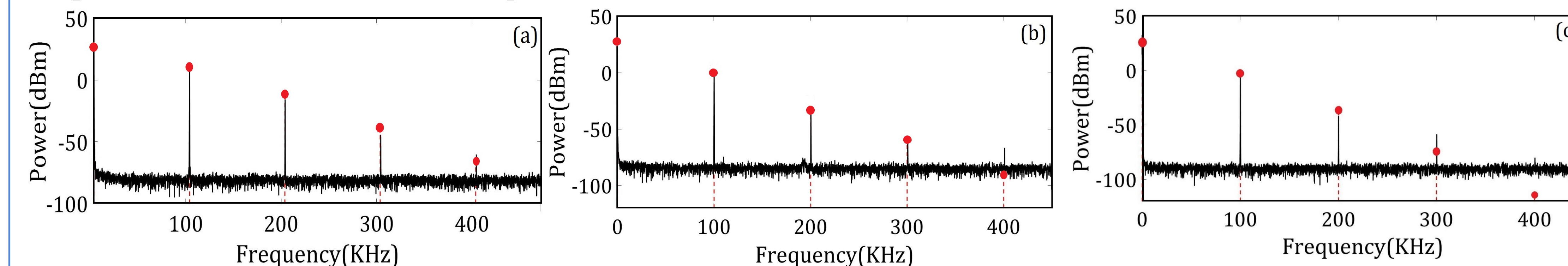


Fig. 3. Comparison of theoretical simulations (red trace) and experimental data (black trace) of output of electronic frequency divider for different division ratio for phase modulated input. (a) divide by 1 (undivided), (b) divide by 2 (c) divide by 4.

### Output of EFD to a simultaneously Phase Modulated and Frequency Modulated input

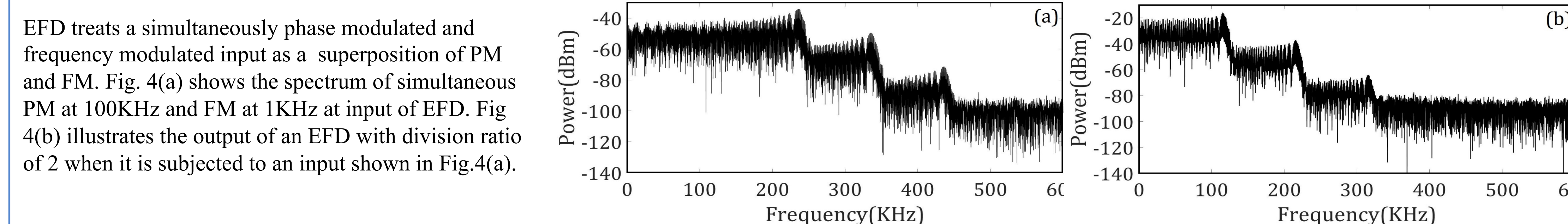


Fig. 4. Experimental data of output of electronic frequency divider for different division ratio for simultaneous PM and FM. (a) divide by 1 (undivided), (b) divide by 2.

## APPLICATION: Beat signal analysis

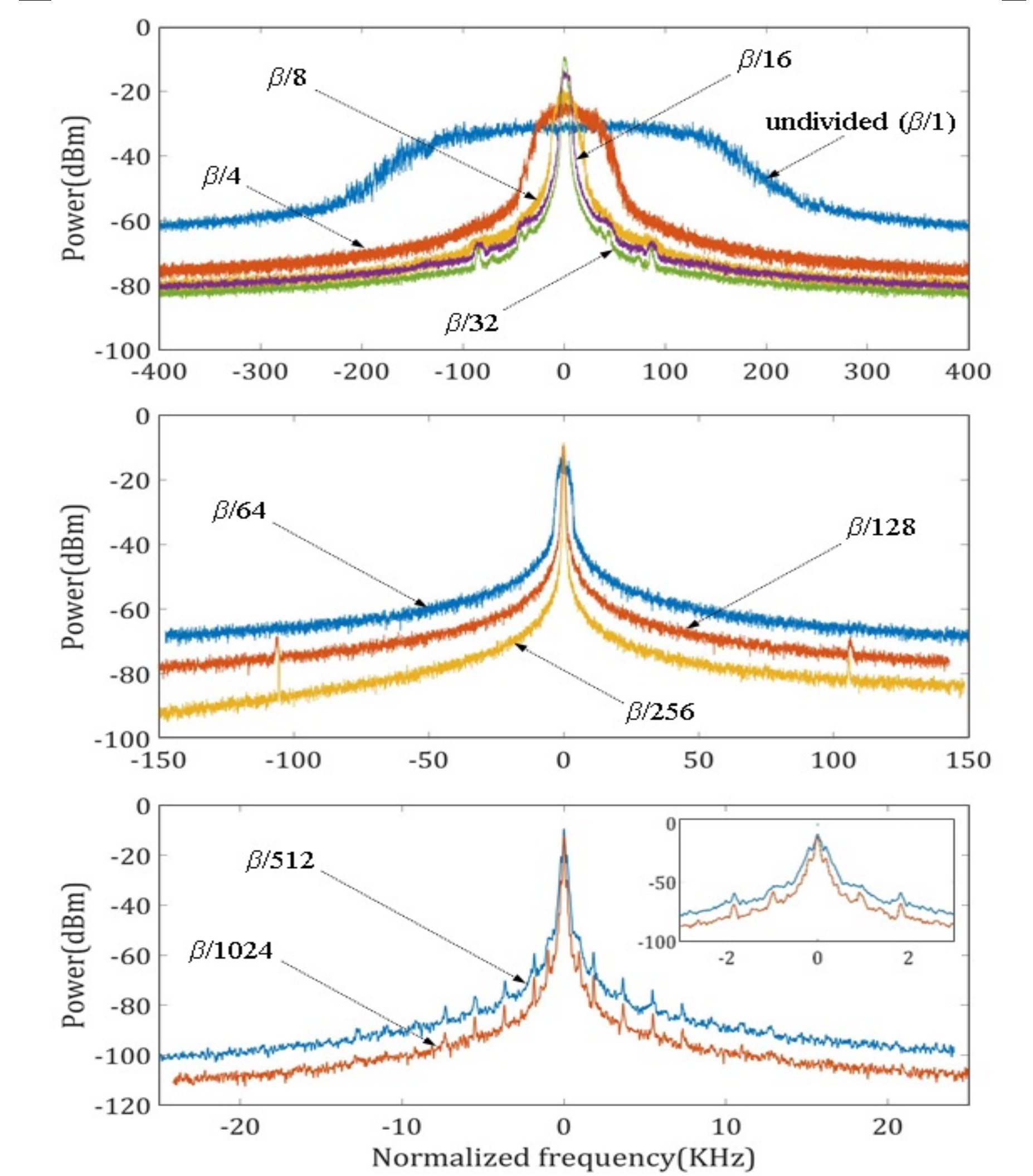


Fig. 5. Frequency-divided beat note between the diode laser (DL) and the Optical frequency comb (OFC) for division ratios of (a) 1 (undivided), 4, 8, 16 and 32; (b) 64, 128 and 256; (c) 512 and 1024.

## CONCLUSION

A method based on the use of EFDs for the analysis of laser FPN has been demonstrated. Unlike conventional spectral analysis techniques, this new method is able to not only distinguish an FM-broadened spectrum from a PM-broadened spectrum but also quantitatively determine the modulation parameters such as modulation frequency, modulation depth and modulation index, leading to a much more comprehensive understanding of the original spectrum. Such a capability can be of great benefit to the development of laser frequency locking systems. An application of this method for the analysis of a beat signal between DL and OFC has also been demonstrated. It can also find applications in a breadth of fields including RF photonics, optical sensing, radar/lidar, etc.

## REFERENCES

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