The Transmission of Moving Optical Cavities

Nazar Pyvovar and Lingze Duan*

Department of Physics and Astronomy, The University of Alabama in Huntsville, Huntsville, AL 35899, USA.

**lingze.duan@uah.edu*

Abstract: We report a theoretical analysis of the transmission properties of an optical cavity, which moves uniformly against the observer. Velocity dependence of the amplitude and phase is examined and application prospects are discussed.

1. Introduction

Optical cavities (OC), also known as Fabry-Perot (FP) resonators, are one of the most widely used optical instruments and have been extensively studied for well over a century [1]. However, one important aspect that has been largely overlooked is the fact that OCs have always been treated in the past as stationary devices relative to the observers. Recent development in hybrid interferometers has created a new way of utilizing the OCs, where one or more FP resonators are nested in a ”host” interferometer, such as a Michelson interferometer [2, 3] or a Mach-Zehnder interferometer [4]. This creates a scenario where the outputs of the OCs are interrogated by the host interferometer and hence invokes an interesting question: what if the OCs are in relative motion against the host interferometer? Answering this question requires the understanding of the behaviors of an OC in a moving frame relative to the (stationary) lab frame. In this report, we analyze the optical transmission of an OC moving along its longitudinal direction and investigate the impact of velocity on the transmittance and transmission phase.

2. Theoretical Analysis

Under current consideration is a rigid OC formed by two lossless plane mirrors separated by an optical medium of refractive index n. The OC is situated in a moving frame, which travels uniformly along the optical axis as illustrated in Fig. 1(a). A collimated light beam interrogates the OC under normal incidence, and the transmitted light is probed by an observer in the lab frame.

We begin by first making the observation in the co-moving frame, where the OC is at rest and the light source is moving toward the opposite direction. The transmission coefficient of the OC is given by

$$\tilde{T} = \frac{(1-r^2)e^{-in\tilde{k}d}}{1-r^2e^{-2in\tilde{k}d}}, \tag{1}$$

where r is the reflection coefficient of the mirrors, n is refractive index, k is the wave number in vacuum, d is the length of the cavity, and the tildes denote quantities in the co-moving frame. To transfer the observation point back to the lab frame, we apply the Lorentz transformation of k, $\tilde{k} = \sqrt{(1-\beta)/(1+\beta)} \cdot k \equiv \zeta k$, where $\beta = v/c$, v is the traveling speed of the OC, and c is the speed of light. This leads to the transmission coefficient of the OC as observed in the lab frame,

$$T = \frac{(1-r^2)e^{-i\zeta(v)nkd}}{1-r^2e^{-2i\zeta(v)nkd}}. \tag{2}$$

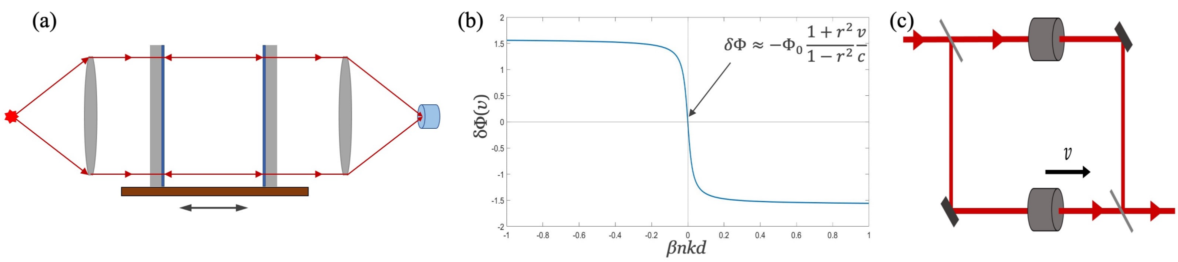

Fig. 1. (a) The concept of a uniformly moving OC. (b) The transmission phase has a sharp velocity dependence near the resonance point. (c) A hybrid MZI scheme with an embedded moving OC.

Evidently, a uniformly moving OC relative to the observer has a similar transmission coefficient as a stationary OC, but with an extra phase scaling factor $\zeta(v)$. The impact of $\zeta(v)$ becomes clear when we examine the transmittance of the OC, which is the well-known Airy function with an added velocity dependence

$$|T|^2 = \frac{1}{1+(4\mathscr{F}^2/\pi^2)\sin^2(\zeta(v)nkd)}, \tag{3}$$

where $\mathscr{F} \equiv \pi r/(1-r^2)$ is the finesse of the OC. Clearly, this is a generalization of the transmittance for a stationary OC, which now becomes a special case of (3) with $\zeta(v) = 1$ $(v = 0)$. The resonance condition of a uniformly moving OC is given by $\zeta(v)nkd = m\pi$, where m is the index of the resonance mode. The condition can be rewritten in terms of wavelength as $m\lambda = 2\zeta(v)nd$ or in terms of optical frequency as $\nu_m = m/[\zeta(v)\Delta\tau_0]$, where ν_m is the frequency of the mth mode and $\Delta\tau_0 = 2nd/c$ is the cavity round-trip time when the OC is at rest. It is also interesting to examine the transmission phase, which is defined by the relation $T = |T|\exp(-i\Phi)$. From (2), one can easily show that Φ is given by

$$\Phi = \zeta(v)nkd + \arctan\left[\frac{r^2\sin(2\zeta(v)nkd)}{1-r^2\cos(2\zeta(v)nkd)}\right]. \tag{4}$$

Eqs. (3) and (4) indicate that both the magnitude and the phase of the transmission coefficient of a moving OC are dependent of velocity.

Since most of the practical cases are nonrelativistic, we will only focus our attention on those scenarios where $\beta \ll 1$. Under this condition, $\zeta \approx 1-\beta$ and $1/\zeta \approx 1+\beta$. The resonance frequencies become

$$\nu_m = \frac{m}{\Delta\tau_0}(1+\frac{v}{c}). \tag{5}$$

This resonance condition is similar to that of a stationary cavity except for the scaling factor $1+v/c$. With a positive velocity, i.e., when the OC travels in the same direction as the incident light, this scaling factor is greater than 1, which means the mode spacing grows larger than a stationary OC. The actual frequency shift for the mth mode is $mv/\Delta\tau_0 c$. Such a frequency shift can be well within the detectable range even for small velocities. For example, at $v = 1$ cm/s, a glass etalon of 1-cm thickness experiences a velocity-induced resonance-frequency shift of about 17 kHz at $\lambda = 600$ nm. Moreover, the motion also causes a rescaling of the transmission linewidth. For example, a positive nonrelativistic velocity would broaden the transmission line by a factor of $1+v/c$.

Similarly, taking the nonrelativistic approximation and using the resonance condition that requires nkd to be an integer multiple of π, the transmission phase can be rewritten as $\Phi = \Phi_0 + \delta\Phi(v)$, where $\Phi_0 = nkd$ is the steady-state transmission phase when the OC is at rest. $\delta\Phi(v)$ is a small velocity-dependent phase change caused by the motion and is plotted against βnkd in Fig. 1(b). Apparently, when $|\beta nkd|$ is small, $\delta\Phi(v)$ has a linear dependence over v. This linear relation can be found by taking the first-order approximation of $\delta\Phi(v)$, which yields

$$\delta\Phi(v) \approx -\Phi_0\frac{1+r^2}{1-r^2}\frac{v}{c}. \tag{6}$$

Such a velocity-dependent phase change can be utilized to develop highly sensitive velocity sensors. One conceivable scheme is shown in Fig. 1(c), where a Mach-Zehnder Fabry-Perot hybrid interferometer is proposed. Two identical OCs operating in resonance are embedded in the two arms of a Mach-Zehnder interferometer (MZI). If one of the OCs begins to move longitudinally while the other remains stationary, an extra phase difference is generated between the two arms according to (6), causing a detectable signal at the output of the MZI.

3. Conclusion

In conclusion, we have established a general relation between the transmission coefficient and the velocity for a uniformly moving OC. The impacts of the motion on the transmittance and the transmission phase are discussed, and potential applications of these effects in velocimetry are proposed.

References

1. J. M. Vaughan, *The Fabry-Perot Inteferometer: History, Theory, Practice and Applications* (Routledge, 1989).
2. B. P. Abbott, "LIGO: the laser interferometer gravitational-wave observatory," Reports on Progress in Physics **72**, 076901 (2009).
3. T. A. Al-Saeed and D. A. Khalil, "Characteristics of a refractometer based on Michelson interferometer integrated with a Fabry-Perot interferometer," Optik **242**, 167170 (2021).
4. N. M. R. Hoque and L. Duan, "A Mach-Zehnder Fabry-Perot hybrid fiber-optic interferometer operating at the thermal noise limit," Scientific Reports **12**, 12130 (2022).