Thermal Noise-Limited Fiber-Optic Sensing at Infrasonic Frequencies

Lingze Duan, *Senior Member, IEEE*

***Abstract*— The fundamental thermal noise in fiber-optic sensors remains to be an interesting research topic. In particular, its spectral behavior in the infrasonic frequency range has yet to be observed experimentally. In this paper, we assess the feasibility of probing the thermal noise floor at infrasonic frequencies with two sensor configurations: 1) fiber Fabry–Perot strain sensors and 2) Mach–Zehnder–Fabry–Perot hybrid phase sensors. In each case, we compare the theory-predicted thermomechanical noise (the dominant thermal noise in fibers at low frequencies) with other potential system noises such as laser noises and detector noises. Our analysis indicates that the thermal-noise-limited fiber-optic sensing can be very difficult to achieve with the strain-sensing scheme, but much more feasible with the hybrid phase sensors.**

***Index Terms*— 1f noise, Fabry-Perot, low-frequency noise, noise measurement, optical fiber sensors, phase noise, strain measurement, thermal noise.**

I. Introduction

OVER the last few years, there has been a growing interest in the pursuit of the “ultimate limit” of fiber-optic sensors [1]–[5]. Such a limit of sensor sensitivity is generally believed to be set by the fundamental thermal noise in optical fibers [6], [7]. As the contributions of other noise sources, such as the laser noise and the detector noise, continue to decrease thanks to the advances in the relevant technologies, fiber-optic sensors approaching to the thermal noise limit is becoming increasingly attainable.

Direct observation of fiber thermal noise was first achieved in early 1990s with a passive interferometric system: a fiber Mach-Zehnder interferometer (MZI) [8]. The measured phase noise spectrum was found in good agreement with theoretical predictions within the frequency span of 1–100 kHz. The following two decades saw a continued effort to expand the frequency range of thermal noise-limited detection, especially toward the lower frequency end, with both passive fiber sensors [9]–[11] and active fiber sensors (fiber lasers) [12]–[14]. A notable accomplishment was recently made by Bartolo *et al.* [11], who mapped out the thermal noise floor between 20 Hz and 100 kHz with an upgraded version of the MZI used in [8]. In a similar effort, Gagliardi *et al.* demonstrated the highest strain sensitivity below 10 Hz with a fiber Fabry-Perot (FP) cavity and a frequency comb-stabilized diode laser [2]. However, their claim of reaching the thermal noise limit at infrasonic frequencies (below 20 Hz) has been shown to be inconsistent with theoretical predictions [3], [11], [15]. In fact, a thorough analysis recently reported by Skolianos *et al.* shows that the predicted thermal noise is nearly an order of magnitude below the measured noise in [15], indicating the reported strain sensitivity may still be limited by other noises, such as the laser phase noise.

Nevertheless, this debate prompts the question of whether the current technology is ready to attain thermal noise-limited fiber-optic sensing in the infrasonic range [3], [4]. The present work is an attempt by the author to address this question. Specifically, two possible sensor designs are discussed, and in each case, technical feasibility is assessed. Moreover, a new sensing scheme based on a hybrid interferometer configuration is proposed. Preliminary analysis shows that a passive sensor of such kind can enhance the phase sensitivity by two orders of magnitude, thus offering a viable chance of probing down to the thermal noise level at sub-Hz frequencies.

It should be clarified here that the present work focuses only on passive fiber-optic systems. Although a similar low-frequency noise behavior (i.e., a $1/f$ dependence) has also been observed in active fiber-optic sensors, such as distributed-feedback fiber lasers [12]–[14], its physical picture is complicated by the existence of the gain medium [16], [17]. It is unclear at present to what extent the $1/f$ noise observed in fiber lasers can be attributed to the thermal noise of the fiber rather than some effects associated with the gain medium [17]–[19].

The necessity of pursuing thermal noise-limited sensing at very low frequencies is two-fold. From a theoretical point of view, directly probing the thermal noise spectrum at low frequencies can help verify the existing theories of fiber thermal noise. Over the last 20 years, numerous experiments have shown that the thermal noise in optical fibers has two distinctive spectral behaviors. At high frequencies (typically above 1 kHz), the noise spectrum can be very well described by a thermodynamic (or thermoconductive) model based on spontaneous temperature fluctuations [7], [14]. At low frequencies (typically below 1 kHz), the noise deviates from the thermodynamic model and exhibits a strong $1/f$ dependence [11]. The exact mechanism of this $1/f$ noise is still not very well understood. A recent theory developed by the author shows that thermomechanical fluctuations in optical fibers lead to a spontaneous phase noise with a $1/f$ spectral

Manuscript received September 29, 2014; revised November 23, 2014; accepted December 9, 2014. Date of publication December 22, 2014; date of current version January 5, 2015. This work was supported in part by the National Science Foundation under Grant ECCS-1040019 and in part by the National Aeronautics and Space Administration through the Experimental Program to Stimulate Competitive Research under Grant NNX13AB09A.

The author is with the Department of Physics, University of Alabama in Huntsville, Huntsville, AL 35899 USA (e-mail: lingze.duan@uah.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/JQE.2014.2384914

0018-9197 © 2014 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission.
See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

density [19]. Further analysis indicates that combining the thermodynamic and thermomechanical noises results in an overall thermal noise spectrum qualitatively agreeing with the observed noise behaviors [20]. However, direct experimental evidence is needed to verify the thermomechanical theory quantitatively. Recently, Bartolo *et al*. have showed that adding the thermomechanical noise to the thermodynamic noise led to an excellent agreement between theory and experiment within 20–800 Hz, where the thermodynamic model alone failed to match the measured data [11]. Below 20 Hz, however, their measurement was dominated by the residual laser intensity noise. In order to make a more definitive verification, thermal noise probing must be extended further toward lower frequencies, i.e., into the infrasonic region. Meanwhile, from a practical point of view, reaching the thermal noise limit means maximizing sensor sensitivity and resolution. The associated schemes and techniques will certainly serve as blueprints for developing more sensitive fiber-optic sensors.

II. Fiber Thermal Noise Theory

Let us begin by evaluating the expected thermal noise level in the interested frequency range based on the existing theories. Following the discussion in [20], the overall spectral density of thermal phase noise in optical fibers can be written as

$$S_\varphi(f)=\frac{4\pi L}{\lambda^2}\left[\frac{k_B T^2}{\kappa}\left(\frac{dn}{dT}+n\alpha_L\right)^2 F(f)+\frac{2k_B T n^2\phi_0}{3E_0 A}\cdot\frac{1}{f}\right], \tag{1}$$

where L is fiber length, λ is wavelength in vacuum, n is refractive index, k_B is the Boltzmann constant, T is temperature, κ is thermal conductivity, α_L is linear expansion coefficient, A is cross-section area of the fiber, E_0 is Young's modulus without loss, and ϕ_0 is mechanical loss angle. The two terms within the square bracket [ ] represent the thermodynamic and the thermomechanical noises, respectively. $F(f)$ is a unit-less function of frequency, whose exact form depends on the specific thermodynamic model. There are two versions of the thermodynamic model, credited to Wanser [7] and Foster *et al*. [14]. Despite their slight differences in spectral behaviors, both theories appear to agree well with experiments [11]. Two additional assumptions have also been made: *a*) the interested frequency range is below the first longitudinal mechanical resonance of the fiber, and *b*) $S_\varphi(f)$ is a single-sided power spectral density function based on the Fourier transform in terms of frequency f. The first assumption allows for the use of a simplified expression for the thermomechanical noise [20]. It is justified for all the cases discussed in this paper. The second condition leads to an extra factor of 4π compared to the earlier report [20]. It ensures that $S_\varphi(f)$ can be directly compared with experimentally measured noise spectra.

One interesting observation immediately clear from (1) is that the relative scale of the thermodynamic and thermomechanical noises is independent of the fiber length. In other words, no matter how long the fiber is, the thermomechanical noise always intersects the thermodynamic noise at the same frequency. Using typical parameters of the SMF-28 fiber, it is easy to show that this frequency falls within several tens of hertz to several hundred hertz, assuming the loss angle $\varphi_0 = 0.01 \sim 0.1$ [20]. Therefore, for the study of thermal noise at infrasonic frequencies, the thermomechanical noise is the dominant mechanism regardless of the specific sensor dimensions.

To numerically estimate the scale of the thermomechanical phase noise, the following typical values are assumed [11]: $T = 298$ K, $n = 1.457$, $\lambda = 1.55\mu$m, $E_0 = 19$ GPa, $\varphi_0 = 0.01$ and 250-μm fiber diameter. Applying these parameters to the thermomechanical part in (1) and taking a square root yield the following phase noise amplitude

$$\sqrt{S_\varphi(f)} = 0.571\sqrt{\frac{L}{f}}\,\mu\text{rad}/\sqrt{\text{Hz}}. \tag{2}$$

For sensing schemes based on fiber resonators such as fiber FP cavities and fiber Bragg gratings, it is often more convenient to characterize thermal noise in terms of the spontaneous fluctuations of their resonance frequencies. Converting the thermomechanical fluctuation of fiber length to thermal frequency noise is straightforward [19]. Using the same parameters as above, the noise amplitude is

$$\sqrt{S_\nu(f)} = \frac{18.7}{\sqrt{Lf}}\,\text{Hz}/\sqrt{\text{Hz}}. \tag{3}$$

Note that the thermomechanical phase and frequency noises are both $1/f$ noises, but their dependence on fiber length L is opposite. This distinction leads to different strategies in probing these two forms of fiber thermal noise.

III. Strain Sensors

Now that a theoretical prediction of the thermal noise floor is established, let us examine the specific sensing schemes. Here we focus on two types of passive fiber-optic sensors, FP strain sensors [2] and MZI phase sensors [11], and we discuss what the minimum requirements are for each of them to reach the thermal noise limit at infrasonic frequencies. Other promising techniques, e.g., slow-light fiber-Bragg-grating (FBG) strain sensor [5], can be analyzed in a similar fashion.

In a FP strain sensor, strain is read out via the measurement of the relative shifts of resonance frequencies. In the absence of external strains, the thermal frequency noise of the fiber cavity translates into a spontaneous strain noise. However, other noises, especially the laser noise, can mask this thermal noise. Fig. 1(a) illustrates the operating principle of a simple FP strain sensor. The frequency of the interrogation laser is tuned to rest on the side of a cavity resonance peak. A shift in resonance frequency would cause a change in transmitted laser power, which is probed by the photodetector. Since the cavity transmission depends on the *relative* frequency between the laser and the cavity resonance, the laser frequency noise and the cavity frequency noise are indistinguishable. Moreover, for a simple side-locking scheme as shown in Fig. 1(a), the relative intensity noise (RIN) of the laser can also be mistaken for the cavity frequency noise. While the impact of the laser

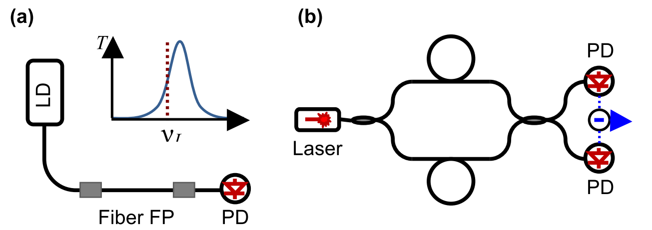


Fig. 1. Conceptual diagrams of (a) a strain sensor based on a fiber Fabry-Perot cavity and (b) a phase sensor based on a fiber Mach-Zehnder interferometer.

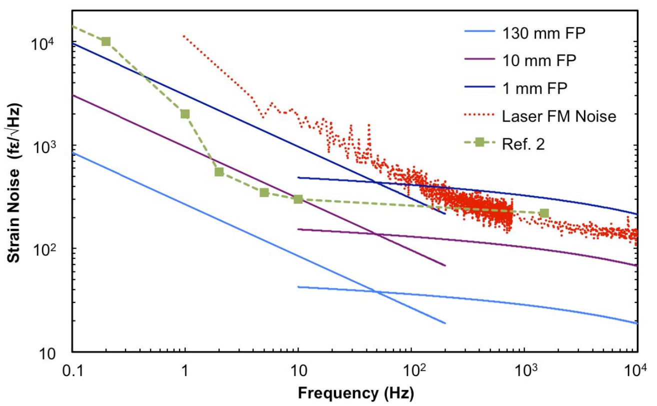


Fig. 2. The thermal strain noise spectra for fiber FP cavities of three different lengths (solid traces), 130 mm, 10 mm and 1 mm, are compared with the equivalent strain noise caused by a typical low-noise diode laser (dotted trace) and the best strain sensitivity reported so far in this frequency range (square dashed trace) [2]. Both the thermomechanical noise (0.1–200 Hz) and the thermodynamic noise (10 Hz–10 kHz) are shown for each FP cavity.

intensity noise can be minimized by using more advanced detection schemes such as the Pound-Drever-Hall (PDH) technique [21]–[23], laser frequency noise remains to be the main challenge for thermal strain noise measurement [2], [5]. Using ultra-low-noise lasers or laser frequency stabilization can partially address this problem. But ultimately, the extent of their effects is limited by the available technology, especially at infrasonic frequencies.

An alternative approach is to increase the thermal noise level instead. From (3), it is clear that the thermal frequency noise (hence the thermal strain noise) of a fiber FP cavity is inversely proportional to the cavity length L. In other words, shorter cavities exhibit higher levels of thermal strain noise and therefore should be preferred for thermal noise-limited sensing. To put this into a practical context, a comparison is made between the predicted thermomechanical noise and some benchmark experimental results in Fig. 2. The straight solid lines are the thermomechanical strain noise for fiber FP cavities of 130 mm (bottom), 10 mm and 1 mm in length, calculated from (3). The dotted trace is a typical frequency noise spectrum measured with a commercial low-noise diode laser (RIO Orion Grade 5 by Redfern Integrated Optics). The square-dashed curve is an outline of the strain noise floor reported in [2], where an optical frequency comb was used to stabilize the frequency of a diode laser. In order to show a complete picture of the predicted thermal noise spectra, the thermodynamic noise is also plotted for the three FP cavities (only within 10 Hz – 10 kHz). Comparing these curves, two observations are evident. First, it is unlikely that the strain sensor used in [2], which is made of a 130-mm fiber FP, is limited by the thermomechanical noise of fiber. Second, it would be very difficult practically to reach the thermomechanical noise limit in the infrasonic range using a fiber FP strain sensor. This is because, even with frequency comb stabilization, the fiber cavity has to be in the scale of 1 mm in order for the thermomechanical noise to dominate over other noise sources below 20 Hz, and this dominance quickly diminishes below 1 Hz. Fiber FP cavities of millimeter or shorter are prohibitively difficult to build with FBG reflectors. Although the new fiber-end coating technology appears to be able to fabricate fiber cavities down to the sub-mm scale [24], using such short cavities in a PDH scheme introduces a number of problems. For instance, a 1-mm fiber cavity has a free-spectral range greater than 100 GHz. Low-noise lasers, on the other hand, usually have very limited frequency tuning ranges (∼tens of GHz). Therefore, extra care has to be taken during the manufacturing stage to match the laser and the cavity. Furthermore, short cavities inevitably have wide resonance linewidths, which are inversely proportional to the PDH frequency discriminant [22]. For a 1-mm fiber FP cavity with a finesse of 1000 and a 1-mW input power, the optimum electrical frequency discriminant is approximately 26.3 pA/Hz (assuming a 1 A/W detector responsivity), which translates the thermomechanical frequency noise in (3) into a $1/f$ current noise of $15.6/\sqrt{f}$ nA/$\sqrt{\text{Hz}}$. Probing a current noise of this level requires special attention to the detector noise because both the dark current (typically 0.1–50 nA) and the flicker noise fall into comparable ranges [25].

IV. Phase Sensors

We now consider the second type of sensing scheme, interferometric phase sensors. Fig. 1(b) shows the basic configuration of a MZI phase sensor. Phase shifts generated in the interferometer are converted into intensity variations at the output, which are detected by a photodetector. The phase sensitivity of such a sensor is limited by the thermal phase noise of the fiber. In principle, this noise can be probed by the detector in the absence of external perturbations. Since the thermal phase noise generated in the two MZI arms can be treated as uncorrelated, the noise amplitude measured by the detector is greater than the noise amplitude in each individual arm by a factor of $\sqrt{2}$ [11]. Such a phase noise measurement scheme is naturally immune from the laser phase noise if the two MZI arms are perfectly matched in length. In addition, when balanced detection is used, the impact of laser intensity noise is also minimized [26]. Since the thermomechanical phase noise has a $\sqrt{L}$ dependence on fiber length, as shown by (2), longer interferometer arms are desirable for the purpose of probing the thermal noise floor. However, a fiber MZI longer than a few tens of meters can be difficult to handle in terms of matching the arm lengths and packaging the fibers [11]. Moreover, the $\sqrt{L}$ scaling is quite slow, i.e. a 10-fold increase of noise amplitude would require a 100-fold increase of fiber length, which presents a considerable challenge to system engineering.

Recently, Skolianos *et al.* have shown that a fiber FP cavity can *enhance* the single-pass phase noise by a factor

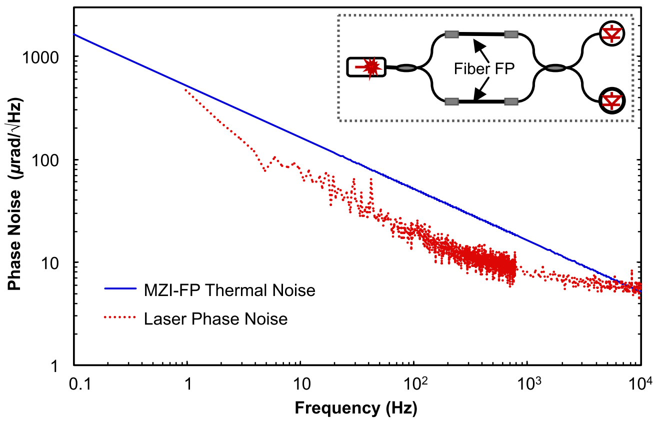


Fig. 3. The thermomechanical phase noise spectrum expected from a MZI-FP hybrid phase sensor (inset) operating on resonance, with the cavity length $L = 1$ m and the finesse $F = 1000$, is compared with a laser phase noise spectrum that would leak through the MZI due to mismatches between the two arms.

of n_g/n when the laser is on resonance with the cavity and the noise frequency is much smaller than the cavity free spectral range [15]. Here n_g refers to the group index of the FP cavity, which is related to the cavity finesse F through the relation $n_g \approx (2/\pi)nF$ when $F \gg 1$. This offers a much more effective way to enhance the phase sensitivity, i.e. using FP cavities as optical path folders in a MZI phase sensor. Fig. 3 inset shows a modified MZI with each arm embedded with a fiber FP cavity. If the laser frequency is maintained on resonance with both cavities and the two cavities are matched in length and finesse, the phase sensitivity of the MZI is a factor of n_g/n greater than a MZI of the same dimensions but without the FP cavities. For example, if the FP cavities are 1 m long with a finesse of 1000, the expected thermomechanical phase noise from each MZI arm is $364/\sqrt{f}\,\mu$rad/$\sqrt{}$Hz according to (2). This is 101 times greater than the expected thermomechanical noise from a 40-m fiber arm similar to the one used in [11]. Yet the actual fiber length is 40 times shorter, making the fiber much more manageable.

To assess the feasibility of this MZI-FP hybrid scheme in attaining the thermal noise limit at low frequencies, we first estimate the relative scales of the various noise sources in such a sensing system in comparison with the thermomechanical phase noise generated by the fiber FP cavities. We use $L = 1$ m and $F = 1000$ as the nominal parameters for the FP cavities, and the expected total thermomechanical phase noise at the output of the MZI is $515/\sqrt{f}\,\mu$rad/$\sqrt{}$Hz.

Any mismatch between the lengths and the finesses of the two FP cavities would allow the laser phase noise to leak through the MZI. When operating on resonance, each FP cavity effectively increases the beam path length (relative to a single pass across the cavity) by a factor of n_g/n, which is approximately $(2/\pi)F$. Since the FP cavity is only 1 m long, it should be easy to control its length to well within 1 mm [26]. The mismatch of cavity finesse is somewhat harder to gauge. The new fiber-end coating technology offers a reliable route toward high-finesse (>1000) fiber FP cavities, and a 1% system error is considered feasible within same coating runs [27]. When both FP cavities in Fig. 3 inset are in resonance with the laser, the worst-case path length mismatch between the two MZI arms is about 7 m. The corresponding laser phase noise present at the output of the MZI is shown in Fig. 3, converted using the same noise spectrum from the RIO Orion module. Also shown in Fig. 3 is the thermomechanical phase noise generated by the two FP cavities. A comparison of the two traces indicates that the fiber thermal noise prevails over the laser phase noise at frequencies above 1 Hz even when the laser is in free-run. A long-term laser frequency stabilization similar to the one reported in [2] can further suppress the laser phase noise at low frequencies (see Fig. 2), making possible thermal noise-limited detection at sub-Hz frequencies.

Laser intensity noise is also a crucial limiting factor. Its potential impact in an MZI phase measurement scheme can be evaluated following the analysis outlined by Dandridge [26]. The laser intensity arriving at the two balanced photodetectors can in general be written as $I_1 = \alpha I_L(A + B\cos\Delta\phi)$ and $I_2 = \alpha I_L(C - B\cos\Delta\phi)$, where α is the optical loss of the MZI (here the two arms are assumed to have the same loss), I_L is the laser output intensity, $\Delta\phi$ is the phase difference between the two arms, and A, B and C are unit-less parameters associated with the power splitting ratios of the two couplers. If both couplers are perfectly 50:50, A, B and C are all equal to 0.5. In practice, small imperfection in coupler power splitting may cause the three parameters to slightly deviate from 0.5. It is straightforward to show that $A - C = \delta k_1 \cdot \delta k_2$, where δk_1 and δk_2 are the relative deviations of the two power splitting ratios from 0.5 [26]. Meanwhile, the phase difference is in general of the form $\Delta\phi = \pi/2 - \phi_q - d\phi$, where the $\pi/2$ is due to the quadrature condition, ϕ_q represents a possible small phase offset from quadrature, and $d\phi$ is the relative phase fluctuation, i.e. phase noise in the current case. This yields $\cos\Delta\phi = \sin(\phi_q + d\phi) \approx \phi_q + d\phi$. When laser intensity noise is considered, we can write $I_L = I_0(1 + dx)$, where I_0 is the nominal laser intensity and dx is the relative intensity noise. Then the differential photo current can be written as

$$i_d = \varepsilon\alpha I_0\left[(\delta k_1\delta k_2 + \phi_q) + d\phi + \delta k_1\delta k_2 dx + \phi_q dx\right] \quad (4)$$

where ε is the responsivity of the photodetectors, $B \approx 1/2$ has been assumed, and only terms up to the first order are kept. The first two terms in (4) represents a small dc current caused by slight imbalances in the power-splitting ratios of the 50:50 couplers and an offset from the quadrature condition. The third term is the phase noise, and the last two terms are due to the laser intensity noise. Clearly, the intensity noise comes into the differential photo current as a result of imbalanced power splitting and an imperfect quadrature condition. To evaluate their respective impact, the last two terms in (4) are *individually* compared with the third term. Taking $d\phi$ to be 515 μrad/$\sqrt{}$Hz at 1 Hz, if $\delta k_1 = \delta k_2 = 0.1$, dx needs to be less than about -26 dB/$\sqrt{}$Hz at 1 Hz in order for the phase noise to stay above the intensity noise. On the other hand, if we take ϕ_q to be 1 degree, then dx must be below -31 dB/$\sqrt{}$Hz at 1 Hz to make the phase noise dominate. All these numbers appear to be achievable, especially if a quatrature lock is used.

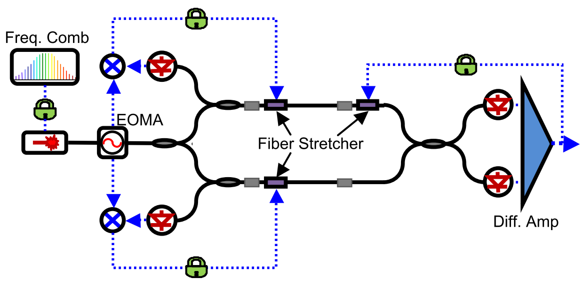


Fig. 4. A proposed system layout for the MZI-FP hybrid phase sensor. Two PDH locking systems keep the FP cavities on resonance with the laser. The laser is slaved to a frequency comb to remove its slow frequency drift. In addition, a quadrature lock ensures the MZI operates in quadrature. EOMA: electro-optic modulator asembly (including the modulator and the driver).

In actual experiments, the requirement for intensity noise is likely more stringent than the predictions given above because *a*) the two factors (i.e. the imbalanced power splitting and the imperfect quadrature) can be present simultaneously in an actual system and *b*) the spectral shape of the intensity noise, which is absent in the simple analysis, can play a critical role in actual measurement as it typically features a very rapid rise at low frequencies [11]. To gauge how the proposed scheme performs under realistic experimental conditions, we make a heuristic estimation by assuming the thermal phase noise in [11] is increased by two orders of magnitude (i.e. 40 dB higher on the dB re rad/$\sqrt{}$Hz scale) and the residual intensity noise remains the same. According to [11, Fig. 4], the residual intensity noise is expected to gain dominance over the thermal phase noise at a frequency below 1 Hz. In other words, the lower frequency limit of thermal noise-limited detection can be extended from 20 Hz down into the sub-Hz region.

The potential impact of detector noise should also be taken into account. To this end, we first estimate the photocurrent associated with the thermomechanical noise. If the optical power incident onto each photodetector is 0.2 mW and the detector responsivity is 1 A/W, the differential photocurrent spectrum due to the thermomechanical noise is $103/\sqrt{f}$ nA/$\sqrt{}$Hz. If the detector pass band is set to 0.1–100 Hz, this current density generates an rms current variation of $\Delta i_{rms} = 271$ nA. This level of signal should be well above the typical ranges of flicker noise [25], [28], which is the dominant detector noise in the baseband, and dark current. Finally, we evaluate the shot noise caused by the photons. With a 0.2-mW optical power and a 1-A/W detector responsivity, the shot noise-induced photocurrent on each detector is a white current noise of about 7 pA/$\sqrt{}$Hz, which is negligible compared to the current caused by thermal noise.

The experimental realization of the above MZI-FP hybrid scheme requires the two FP cavities be kept on resonance with the laser. This can be done by actively locking the laser frequency with one of the transmission peaks of each cavity. A complete system layout is shown in Fig. 4. An electro-optic phase modulator is inserted at the input of the MZI to add side bands to the incident light. A 2×1 fiber coupler in each arm helps extract the PDH error signal from the FP cavity. This error signal is used to drive a fiber stretcher that controls the FP cavity length. To make sure the locking system does not interfere with the phase noise measurement, the bandwidth of the phase locked loop must be set to below the intended noise band (e.g., <0.1 Hz). In other words, the active control systems remove the slow frequency drifts between the laser and the cavities but leave the cavities effectively "free-run" within the interested band. In addition, a quadrature lock may also be necessary since maintaining the MZI in quadrature is critical in minimizing the impact of laser RIN noise. This can be done by actively controlling the length of one MZI arm (outside the FP cavity) as shown in Fig. 4. Finally, a low-noise laser with comparable or lower phase noise floor than the one shown in Fig. 3 must be used. The laser may need to be disciplined by an optical atomic clock or a frequency comb to further suppress its low-frequency noise.

External background noise is another factor one must take into account when demonstrating a thermal noise-limited fiber-optic sensor. To allow the sensor to reach the minuscule thermal noise floor, especially at low frequencies, a good isolation system is needed to reject unwanted external perturbations such as temperature fluctuations, vibrations and acoustic excitations. Unfortunately, most of these background noises are particularly strong in the infrasonic region. Therefore, an isolations system much more sophisticated than what has been employed in [2] and [11] is likely necessary.

V. Conclusion

We have discussed the technical feasibility of thermal noise-limited fiber-optic sensing at infrasonic frequencies using two sensing schemes as examples: strain sensors based on fiber Fabry-Perot cavities and phase sensors based on fiber Mach-Zehnder interferometers. In both cases, the expected thermomechanical noise is compared with potential system noises such as the laser noise and the detector noise. For the strain sensing scheme, it is found that the theoretical margin between the thermomechanical noise and the laser frequency noise is very small even with exceedingly short FP cavities and frequency-comb stabilization. Therefore, reaching the thermal noise limit at very low frequencies appears to be very difficult with fiber FP strain sensors. For the phase sensing scheme, we propose to use a MZI-FP hybrid configuration instead of the conventional long-arm MZI to enhance phase sensitivity. We show that, as an example, replacing a 40-m single-pass MZI arm with a 1-m, $F = 1000$ FP cavity operating on resonance would lead to a 100-fold increase of the thermomechanical phase noise at the output. As a result, the thermal phase noise is able to stay above typical laser phase noise and laser intensity noise within the infrasonic region (into sub-Hz frequencies) without the need for extra laser stabilization. With additional active control of the laser frequency and intensity, the dominance of the thermal noise over laser noises is expected to extend further into the millihertz range. Detector noise and shot noise are also evaluated for this scheme and are found to be negligible. Meanwhile, active stabilization of the cavity lengths is needed to keep the FP cavities on resonance with the laser over long time scales, and some additional factors, such as precise end-mirror coating (which dictates the precision of the cavity finesses) and advanced isolation systems, have been identified as key requirements for this scheme to succeed.

Given these added complexities to the phase measurement scheme due to the inclusion of the FP cavities, it is instructive at this point to revisit the question of whether it is feasible to reach the thermal noise limit at infrasonic frequencies *without* the FP cavities, i.e. by using a simple MZI as in [11]. First of all, regardless of which system configuration is used, a 100-fold enhancement of the thermomechanical phase noise is likely needed in order to overcome the steep rise of the laser intensity noise at low frequencies. Without the FP cavities, this enhancement would have to be achieved by expanding the fiber length. Since the thermomechanical noise scales with length by $\sqrt{L}$, a 100-fold increase in noise amplitude would require the fiber to be 10^4 times longer than the 40 meters used in [11], which would be 400 km! The sheer size of the needed fiber apparently makes this approach highly impractical, let alone potential problems associated with arm-length matching, fiber attenuation, ambient condition control, etc. From this point of view, the MZI-FP hybrid scheme fully benefits from the coherent superposition of the slow thermal phase noise inside the FP cavities and hence offers a much better chance of probing the thermal noise limit at low frequencies.

Overall, our analysis indicates that the MZI-based phase sensing scheme is a more feasible approach to reach the thermal noise limit in fiber-optic sensors at infrasonic frequencies. We do, however, realize that the analysis reported here provides only a few minimal requirements and, therefore, in no way represents a complete study of the topic. Ultimately, the question of whether such sensors are possible will have to be answered by experiments.

ACKNOWLEDGMENT

The authors thank Redfern Integrated Optics Inc. for providing the phase noise test data of the Orion laser module.

REFERENCES

[1] S. Foster, G. A. Cranch, and A. Tikhomirov, "How sensitive is the fibre laser strain sensor?" *Proc. SPIE*, vol. 7004, p. 70043J, May 2008.

[2] G. Gagliardi, M. Salza, S. Avino, P. Ferraro, and P. De Natale, "Probing the ultimate limit of fiber-optic strain sensing," *Science*, vol. 330, pp. 1081–1084, Nov. 2010.

[3] G. A. Cranch and S. Foster, "Comment on 'Probing the ultimate limit of fiber-optic strain sensing,"' *Science*, vol. 335, no. 6066, p. 286, Jan. 2012.

[4] G. Gagliardi, M. Salza, S. Avino, P. Ferraro, and P. De Natale, "Response to Comment on 'Probing the ultimate limit of fiber-optic strain sensing,"' *Science*, vol. 335, no. 6066, p. 286, Jan. 2012.

[5] H. Wen, G. Skolianos, S. Fan, M. Bernier, R. Vallée, and M. J. F. Digonnet, "Slow-light fiber-Bragg-grating strain sensor with a 280-femtostrain/$\sqrt{}$Hz resolution," *J. Lightw. Technol.*, vol. 31, no. 11, pp. 1804–1808, Jun. 1, 2013.

[6] W. Glenn, "Noise in interferometric optical systems: An optical Nyquist theorem," *IEEE J. Quantum Electron*, vol. 25, no. 6, pp. 1218–1224, Jun. 1989.

[7] K. H. Wanser, "Fundamental phase noise limit in optical fibres due to temperature fluctuations," *Electron. Lett.*, vol. 28, no. 1, pp. 53–54, Jan. 1992.

[8] K. H. Wanser, A. D. Kersey, and A. Dandridge, "Measurement of fundamental thermal phase fluctuations in optical fiber," in *Proc. 9th Int. Conf. Opt. Fiber Sensors*, Florence, Italy, 1993, p. 255.

[9] S. Knudsen, A. B. Tveten, and A. Dandridge, "Measurements of fundamental thermal induced phase fluctuations in the fiber of a Sagnac interferometer," *IEEE Phonton. Technol. Lett.*, vol. 7, no. 1, pp. 90–92, Jan. 1995.

[10] A. D. Kersey, "A review of recent developments in fiber optic sensor technology," *Opt. Fiber Technol.*, vol. 2, no. 3, pp. 291–317, Jul. 1996.

[11] R. E. Bartolo, A. B. Tveten, and A. Dandridge, "Thermal phase noise measurements in optical fiber interferometers," *IEEE J. Quantum Electron.*, vol. 48, no. 5, pp. 720–727, May 2012.

[12] E. Rønnekleiv, "Frequency and intensity noise of single frequency fiber Bragg grating lasers," *Opt. Fiber Technol.*, vol. 7, no. 3, pp. 206–235, 2001.

[13] G. A. Cranch, "Frequency noise reduction in erbium-doped fiber distributed-feedback lasers by electronic feedback," *Opt. Lett.*, vol. 27, no. 13, pp. 1114–1116, Jul. 2002.

[14] S. Foster, A. Tikhomirov, and M. Milnes, "Fundamental thermal noise in distributed feedback fiber lasers," *IEEE J. Quantum Electron.*, vol. 43, no. 5, pp. 378–384, May 2007.

[15] G. Skolianos, H. Wen, and M. J. F. Digonnet, "Thermal phase noise in Fabry–Pérot resonators and fiber Bragg gratings," *Phys. Rev. A*, vol. 89, p. 033818, Mar. 2014.

[16] S. Foster, "Fundamental limits on $1/f$ frequency noise in rare-earth-metal-doped fiber lasers due to spontaneous emission," *Phys. Rev. A*, vol. 78, p. 013820, Jul. 2008.

[17] S. Foster, G. A. Cranch, and A. Tikhomirov, "Experimental evidence for the thermal origin of $1/f$ frequency noise in erbium-doped fiber lasers," *Phys. Rev. A*, vol. 79, p. 053802, May 2009.

[18] S. Foster, "Low-frequency thermal noise in optical fiber cavities," *Phys. Rev. A*, vol. 86, p. 043801, Oct. 2012.

[19] L. Z. Duan, "Intrinsic thermal noise of optical fibres due to mechanical dissipation," *Electron. Lett.*, vol. 46, no. 22, pp. 1515–1516, Oct. 2010.

[20] L. Duan, "General treatment of the thermal noises in optical fibers," *Phys. Rev. A*, vol. 86, p. 023817, Aug. 2012.

[21] R. W. P. Drever *et al.*, "Laser phase and frequency stabilization using an optical resonator," *Appl. Phys. B*, vol. 31, no. 2, pp. 97–105, Feb. 1983.

[22] E. D. Black, "An introduction to Pound–Drever–Hall laser frequency stabilization," *Amer. J. Phys.*, vol. 69, no. 1, pp. 79–87, Jan. 2001.

[23] J. H. Chow, D. E. McClelland, M. B. Gray, and I. C. M. Littler, "Demonstration of a passive subpicostrain fiber strain sensor," *Opt. Lett.*, vol. 30, no. 15, pp. 1923–1925, Aug. 2005.

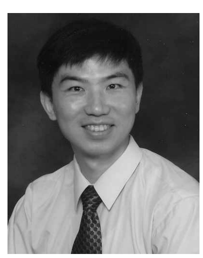
[24] Micron Optics Inc. *Tunable Filter Technology*. [Online]. Available: http://www.micronoptics.com, accessed Jan. 31, 2011.

[25] O. Llopis, S. Azaizia, K. Saleh, A. A. Slimane, and A. Fernandez, "Photodiode 1/f noise and other types of less known baseband noises in optical telecommunications devices," in *Proc. 22nd ICNF*, Jun. 2013, pp. 1–4.

[26] A. Dandridge, "Fiber optic sensors based on the Mach–Zehnder and Michelson interferometers," in *Fiber Optic Sensors: An Introduction for Engineers and Scientists*, E. Udd, and W. B. Spillman, Jr., Eds. New York, NY, USA: Wiley, 1991, pp. 271–323.

[27] Micron Optics Inc., private communication, Oct. 2013.

[28] E. Rubiola, E. Salik, N. Yu, and L. Maleki, "Flicker noise in high-speed p-i-n photodiodes," *IEEE Trans. Microw. Theory Tech.*, vol. 54, no. 2, pp. 816–820, Feb. 2006.



Lingze Duan (M'01–SM'14) was born in Beijing, China. He received the B.S. degree in physics from Tsinghua University, Beijing, China, in 1995, and the Ph.D. degree in electrical engineering from the University of Maryland, College Park, MD, USA, in 2002.

He was a Post-Doctoral Associate with the Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, MA, USA, from 2002 to 2004, where he studied direct detection of the carrier-envelope phase of femtosecond pulses using semiconductor devices. From 2004 to 2007, he conducted post-doctoral research with the Department of Physics, Pennsylvania State University, University Park, PA, USA, where he developed ultrastable diode laser systems directly stabilized by high-Q optical cavities. He joined the Department of Physics, University of Alabama in Huntsville, Huntsville, AL, USA, as a faculty member, in 2007, where he has been an Associate Professor since 2013. His current research interest includes ultrafast nanophotonics, frequency metrology with femtosecond frequency combs, fiber optic sensors, and novel applications of optics in astrophysics.

Dr. Duan was a recipient of the 2013 National Science Foundation Faculty Early Career Development Award. He is a member of the IEEE Photonics Society and the Optical Society of America.