Intrinsic Thermodynamic Noise in Passive Fiber Systems

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Abstract: A new theory of the thermodynamic noise in passive fibers is developed using the fluctuation-dissipation theorem. It models the spontaneous phase noise in fibers, and agrees well with the previous theory and experimental results.

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1. Introduction

Fiber interferometric instruments and sensors have found applications in a wide breadth of fields owing to their superior phase sensitivity. Fundamentally, however, the phase sensitivities of fiber interferometers are limited by their intrinsic noise, which is caused by the spontaneous fluctuations occurring in the fiber. Such noise is generally referred to as the *thermal noise*, and has been shown to be much greater than the shot noise under common operation conditions [1]. Over the last two decades, several theoretical models have been developed in efforts to understand the physics of the thermal noise in optical fibers [1-4]. Most of these efforts have been given to the thermodynamic noise, which is caused by spontaneous local-temperature fluctuations through thermal expansion and the thermo-optic effect [1-3]. Notably, Wanser gave an elegant formula for the power spectral density of the thermodynamic phase noise in passive fibers without disclosing any derivation [2]. His formula has found excellent agreement with experiments in some cases [5] but discrepancies up to 3dB in other cases [6]. To clarify some of the theoretical questions surrounding the Wanser theory, Foster *et al*. independently developed a thermodynamic model for fiber laser cavities using the Langevin equation [3]. The phase noise spectrum based on this theory has some distinct differences compared to the Wanser theory, especially at the asymptotic frequencies, but nevertheless shows good agreement with experiments for both fiber lasers [3] and passive fibers [7]. In the current work, I present yet another thermodynamic theory for fiber thermal noise. It is intended to provide an independent evaluation of the Wanser and the Foster theories, and address the need for a fully disclosed thermodynamic model for passive fibers.

2. A Thermodynamic Model based on the Fluctuation-Dissipation Theorem

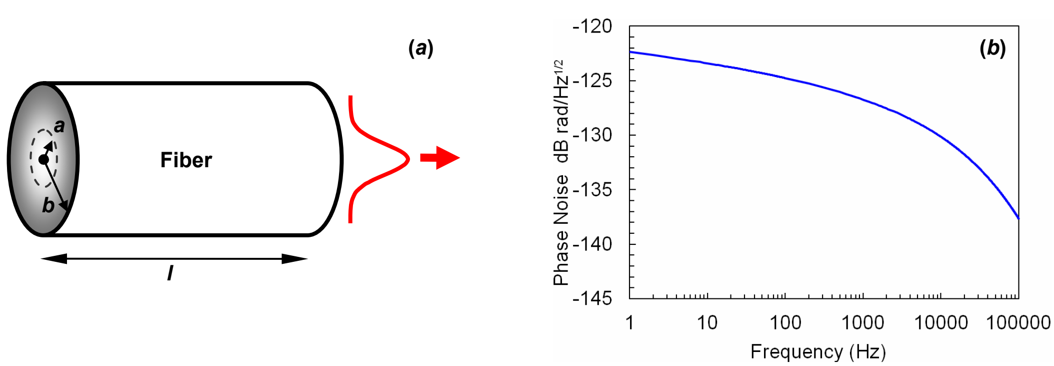

Fig. 1. (*a*) A single-mode fiber of length l carrying a laser beam with a mode-field radius of a. The imagined entropy injection shares the same form factor as the Gaussian beam. (*b*) The calculated thermodynamic phase noise spectrum for $l = 80$ m and $\lambda = 1319$ nm.

The new method follows the thought experiment based on the Fluctuation-Dissipation Theorem (FDT), outlined by Levin for the analysis of the mirror thermal noise in the Laser Interferometer Gravitational-Wave Observatory [8]. In order to find out the thermodynamic dissipation of a single-mode fiber, an imagined external perturbation is introduced in the form of a harmonic entropy modulated by a spatial form factor that closely resembles the Gaussian profile of the fiber mode, as shown in Fig. 1 (a). According to the general theory of thermal conduction, this entropy injection would create a field of temperature variations (the fiber is otherwise in thermal equilibrium at T), which is described by the following nonhomogeneous differential equation,

$$\frac{\partial^2 \delta T}{\partial r^2} + \frac{1}{r}\frac{\partial \delta T}{\partial r} - \frac{1}{D}\frac{\partial \delta T}{\partial t} = -i\frac{2\omega F_0 T}{\pi a^2 l \kappa} e^{-2r^2/a^2} e^{-i\omega t}, \quad (1)$$

where δT is the variation of the temperature induced by the perturbation, D is diffusivity, a is the mode-field radius of the fiber mode, l is the fiber length, κ is thermal conductivity, F_0 is a scale parameter for the entropy injection, and r and t are radial position and time, respectively. This equation can be solved by the Green's function method, yielding a solution for δT as

$$\delta T(r,t) = \frac{i\omega F_0 T}{4\pi l\kappa} e^{-i(\omega t+\psi_0)} \int_1^\infty e^{i\psi_0\zeta - \frac{2r^2}{a^2\zeta}} \frac{d\zeta}{\zeta}, \tag{2}$$

where $\psi_0 \equiv \omega a^2/(8D)$, ζ is the variable of integration, and an "infinite cladding" approximation has been made. The temperature differences along the radial direction breaks the thermal equilibrium. As the system tries to "relax" back to thermal equilibrium, thermal conduction causes energy dissipation, which, according to theory of thermal conduction, can be expressed as

$$W_{diss} = \int \frac{\kappa}{T} \left\langle \left(\nabla \delta T\right)^2 \right\rangle dV = \frac{\omega^2 F_0^2 T}{8\pi l\kappa} \int_1^\infty \int_1^\infty \frac{e^{i\psi_0(\zeta-\zeta')}}{(\zeta+\zeta')^2} d\zeta\, d\zeta' . \tag{3}$$

where $\langle\ \rangle$ denotes time average. It is straightforward to evaluate the double integral, which leads to an expression for the energy dissipation W_{diss}. The spectral density for the temperature fluctuation is related to W_{diss} by the FDT through the relation $S_{\delta T}(\omega) = 8k_B T W_{diss}/(\omega^2 F_0^2)$ [8]. Substituting (3) into this relation yields

$$S_{\delta T}(\omega) = \frac{k_B T^2}{\pi l\kappa} \mathrm{Re}\left[e^{\frac{i\omega a^2}{4D}} E_1\left(\frac{i\omega a^2}{4D}\right)\right]. \tag{4}$$

where k_B is Boltzmann constant, Re denotes real part, and $E_1(x)$ is the special function of the exponential integral. This result is remarkably similar to the Foster theory (i.e. Eq. (27) in [3]). The 4π discrepancy can be accounted for by realizing that Foster used a different Fourier transform convention and (4) is a one-sided spectrum. The spectral density of the thermal phase noise is related to the spontaneous temperature fluctuation through the relation

$$S_\varphi(\omega) = \frac{4\pi^2 l^2}{\lambda^2}\left(\frac{dn}{dT} + n\alpha_L\right)^2 S_{\delta T}(\omega), \tag{5}$$

where λ, n and α_L are wavelength, refractive index and linear thermal expansion coefficient, respectively. The resulted noise spectrum should be able to directly compare with experimental results.

3. Comparison with Experiment

Although comparisons between the Foster theory and experimental data have been made recently by Bartolo *et al.* [7], it is still worthwhile to make an independent confirmation using (4) and (5), given the fact that additional rescaling has been made to take into account the differences in the Fourier transform conventions. Using the same critical parameters listed in [7], i.e. $k_B = 1.38\times10^{-23}$ J/K, $T = 295$ K, $\lambda = 1319$ nm, $a = 2.350\ \mu$m, $l = 80$ m, $n = 1.457$, $dn/dT = 9.520\times10^{-6}$ K^{-1}, $\alpha_L = 5.0\times10^{-7}$ K^{-1}, $\kappa = 1.37$ W/(m·K) and $D = 0.82\times10^{-6}$ m^2/s, the phase noise spectrum $\sqrt{S_\varphi(\omega)}$ can be calculated from 1 Hz to 100 kHz. The result is shown in Fig. 1 (b). It agrees very well with both the experimental data (above 1 kHz) and the numerical calculation given in [7]. This confirms that the Foster theory applies for passive fibers and, when correct convention is used, agrees with experiments.

4. Reference

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