Structural Damping-Induced Thermal Noise in Fiber Interferometric Systems

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Abstract: Fiber thermal noise caused by structural damping is analyzed using a 1-D model based on the Fluctuation-Dissipation Theorem. The result provides a new perspective on the precision limit of fiber interferometric systems at low frequencies.

OCIS codes: (060.2400) Fiber properties; (120.5050) Phase measurement; (260.3160) Interference

1. Introduction

Over the last two decades, a number of authors have studied the intrinsic phase noise in fiber interferometric systems due to spontaneous local temperature fluctuation in optical fibers and have shown that such inevitable thermo-dynamic noise imposes a fundamental limit on the minimum detectable phase shift [1]-[3]. Recently, however, it has been discovered that this thermodynamic model fails to account for some of the spontaneous phase fluctuations observed at frequencies below 1 kHz [4]. This may indicate the existence of other types of spontaneous fluctuations in optical fibers. Here we report our model of a new type of fiber thermal noise based on the Fluctuation-Dissipation Theorem (FDT). The work is inspired by the extensive research on structural damping-induced thermal fluctuation in the test mass of the Laser Interferometer Gravitational-Wave Observatory (LIGO) [5]. It may offer a possible explanation to the low-frequency spontaneous phase noise observed in fiber interferometric systems.

2. Longitudinal Thermal Fluctuation of Fiber Length

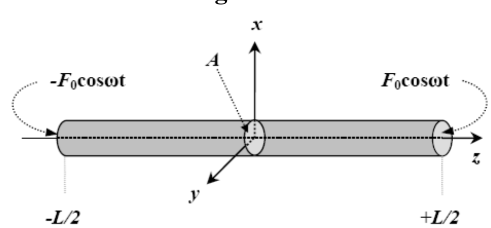

Fig. 1. A one-dimensional model of optical fibers. The fiber is treated as a glass rod spanning from $-L/2$ to $L/2$ along the z-axis. The cross-section area of the fiber is A. To compute the longitudinal thermal fluctuation using the FDT, imaginary periodic forces $F_0\cos\omega t$ and $-F_0\cos\omega t$ are asserted on the ends of the fiber along the z-axis.

We use the method outlined by Levin to analyze the spontaneous longitudinal fluctuation of fiber length under a non-zero temperature [5]. For this purpose, the fiber is treated as a long thin homogeneous glass rod and only the longitudinal deformation is considered. This means a one-dimensional (1-D) model is sufficient for the analysis. To derive the mechanical damping of this system, two artificial periodic forces are introduced to the two ends of the fiber with opposite phases, as shown in Fig. 1. Using basic elasticity theory and considering the boundary conditions set by the two driving forces, it is straightforward to find the amplitude of the longitudinal displacement $U(z)$ as

$$U(z) = \frac{F_0}{E_0 A k \sin(kL/2)} \cos kz \,, \tag{1}$$

where E_0 is the Young's modulus in the absence of loss and k is the longitudinal elastic wave propagation constant. Here we have assumed the elastic oscillation is at a frequency below the first longitudinal resonance frequency of the fiber, which is in the order of kHz for meter-long fibers. Then we can derive the maximum density of elastic energy due to the periodic driving forces,

$$\varepsilon_{\max}(z) = \frac{1}{2} E_0 \left(\frac{dU}{dz} \right)^2 = \frac{F_0^2}{2E_0 A^2} \frac{\sin^2 kz}{\sin^2(kL/2)} \,. \tag{2}$$

If mechanical damping is present, the Young's modulus becomes complex, $E = E_0[1+i\phi(f,z)]$, where the loss angle $\phi(f,z)$ characterizes the mechanical dissipation and in general depends on both frequency and position. For homogeneous structural damping, however, $\phi(f,z)=\phi_0$, where ϕ_0 is a constant. The total energy dissipation rate can be related to the loss angle and the elastic energy density by $W_{diss} = 2\pi f \phi_0 \int \varepsilon_{\max}(z)dV$. Using (2) and the low-frequency assumption, we have $W_{diss} \approx \pi f \phi_0 L F_0^2 / 3E_0 A$. The spectral density of longitudinal thermal fluctuation of the fiber, $S_L(\omega)$, is related to W_{diss} through the FDT in a from formulated by Levin, $S_L(\omega) = 8k_B T W_{diss} / \omega^2 F_0^2$ [5]. Substituting W_{diss} into this formula, the spectral density is finally expressed as a function of frequency f

$$S_L(f) = \frac{2k_B T L \phi_0}{3\pi E_0 A}\frac{1}{f}. \tag{3}$$

3. Thermal Phase Noise of Fiber Interferometers

According to (3), the thermal fluctuation of fiber length due to structural damping has a $1/f$ characteristic (at low frequencies). The length fluctuation leads to a $1/f$ phase noise in fiber interferometers or a $1/f$ frequency noise in fiber Fabry-Perot cavities. The scale of this thermal noise can be estimated by using typical numerical values for single-mode (SM) fibers. However, when choosing the material parameters, one has to realize that an optical fiber is not a homogeneous glass rod. Instead, it is a two-layer rod with a fused-silica inner core, consisting of the fiber core and the cladding, and a concentric plastic (acrylate) buffer layer. A typical SM fiber has an inner core diameter of 125 μm and a total fiber diameter of 250 μm. The Young's modulus for fused silica and acrylate are 66.1 GPa and 3.3 GPa, respectively. With a lumped-element model, it has been shown that the effective overall Young's modulus of SM fibers is 19.0 GPa [6]. In addition, the effective loss angle of SM fibers was measured to be approximately 1×10^{-2} between 75 kHz and 200 kHz [6]. Since the loss angle has been found to be frequency independent across a broad spectral range for a wide variety of materials [7], it is not unreasonable to assume $\phi_0 \approx 1\times10^{-2}$ even at low frequencies. Then the spectral distribution of length fluctuation amplitude at room temperature, normalized to the total length of the fiber, is found by using (3) to be

$$\sqrt{S_L(f)/L} \approx 0.1/\sqrt{f}\ \text{pm/(m·Hz)}^{1/2}. \tag{4}$$

Fig. 2 shows the spectrum of $\tilde{\delta}_L(f) \equiv \sqrt{S_L(f)}$ between 1 Hz and 1 kHz with L = 0.1 m, 1 m, and 10 m. The corresponding thermal phase noise spectrum can be obtained with $\tilde{\delta}_\varphi(f) = 2\pi\tilde{\delta}_L(f)/\lambda$. For instance, with 10-m fiber and a wavelength of 1.3 μm, $\tilde{\delta}_\varphi \approx 1.5\times10^{-7}$ rad/Hz$^{1/2}$ at 100 Hz, which is comparable or above the shot noise-limited minimum detectable phase shift [2]. This confirms that thermal phase noise due to structural damping can indeed become the dominant noise source in fiber interferometers at low frequencies.

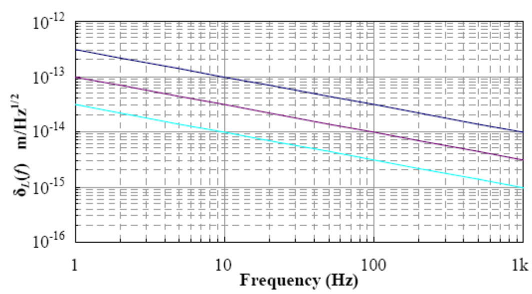

Fig. 2. The spectral density of fiber length thermal fluctuation with L = 0.1 m, 1 m and 10 m (top).

4. Reference

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