Intrinsic thermal noise of optical fibres due to mechanical dissipation

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The spontaneous length fluctuation of optical fibres caused by mechanical dissipation is analysed using a one-dimensional model based on the Fluctuation-Dissipation Theorem. Scale estimate shows evidence that the $1/f$ thermal noise originated from this fluctuation dominates at low frequencies in fibre interferometers and fibre cavities.

Introduction: The intrinsic thermal noise of optical fibres has long been a focus of study in the development of fibre interferometers, fibre-optic sensors and distributed feedback fibre lasers owing to its implications for the fundamental phase noise limit in fibre interferometric systems [1–5]. A widely cited model of fibre thermal noise is based upon the analysis of the random fluctuation of the instantaneous local temperature inside a fibre [1]. It has been shown to have excellent agreement with experiment over a broad frequency range [2]. However, a number of recent reports have pointed out discrepancies between the thermodynamic model and the measured data at low frequencies [3–5]. Below 1 kHz, a $1/f$ frequency noise has been generally observed in fibre-optic cavities while the thermodynamic model predicts a frequency-independent noise behaviour. Such a fundamental disagreement indicates the possible existence of a different type of intrinsic noise that has not been accounted for by the thermodynamic model. Previous effort to model the $1/f$ noise has yet to provide conclusive evidence [6].

In the current work, we attempt to provide an alternative route towards understanding the $1/f$ noise in fibre cavities. The approach is based upon the direct use of the Fluctuation-Dissipation Theorem (FDT) [7], which links spontaneous mechanical thermal fluctuation with mechanical dissipation in solid bodies. Similar approaches have been extensively used in the study of mirror thermal noise for the development of the Laser Interferometer Gravitational-Wave Observatory (LIGO) [8] and have also been applied in the evaluation of frequency stability limit of rigid Fabry-Perot (FP) cavities [9]. To the best of our knowledge, however, such dissipation-induced thermal noise has not been explored in optical fibres. Compared to LIGO mirrors and rigid FP cavities, optical fibres feature a very large aspect ratio between longitudinal and transverse dimensions, which allows the use of a one-dimensional (1D) model that can lead to a closed-form expression of the thermal noise spectrum.

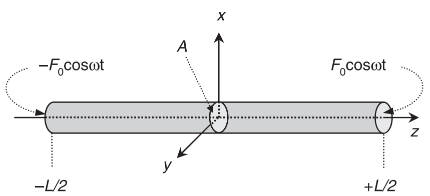

Fig. 1 *One-dimensional model of optical fibres*

Fibre treated as glass rod spanning from $-L/2$ to $L/2$ along z-axis. Cross-section area of fibre is A. To compute longitudinal thermal fluctuation using FDT, imaginary periodic forces $F_0\cos\omega t$ and $-F_0\cos\omega t$ are applied onto two ends of the fibre along the z-axis

Model of fibre thermal fluctuation: We shall first derive a general formula for the spontaneous fluctuation of fibre length. To this end, we directly use the FDT formulated by Levin [8]:

$$S_x(f) = \frac{2k_B T}{\pi^2 f^2}\frac{W_{diss}}{F_0^2} \qquad (1)$$

where $S_x(f)$ is the spectral density of the thermal fluctuation of a spatial variable x (e.g. the displacement of a mirror surface), f is frequency, k_B is Boltzmann's constant, T is the temperature of the body under study, F_0 is the amplitude of an external oscillating force that drives x, and W_{diss} is the energy dissipation rate of the body under this force. In the current case, the longitudinal deformation of the fibre is the variable of interest. If a fibre is treated as a long thin homogeneous glass rod, to the first-order approximation, the only elastic vibration involving the change of total length is the longitudinal vibration [10]. As a result, the fibre can be treated as a 1D object. We can further simplify the model by assuming the fibre is straight and choosing a co-ordinate system as shown in Fig. 1. To evaluate W_{diss}, we artificially apply two harmonic forces uniformly on the two ends of the fibre with the same amplitude but exactly opposite phase. These forces create periodic longitudinal deformation without altering the position of the centre of gravity and hence serve as valid driving forces in the study of fibre length fluctuation [8].

With the above 1D configuration, it is straightforward to solve the wave equation for a longitudinal elastic wave driven by the external harmonic forces in the absence of loss [10]. The maximum density of elastic energy due to the driving forces is

$$\mathcal{E}_{\max}(z) = \frac{F_0^2}{2E_0 A^2}\frac{\sin^2 kz}{\sin^2(kL/2)} \qquad (2)$$

where E_0 is the Young's modulus without loss and $k = 2\pi f\sqrt{\rho/E_0}$ is the wave propagation constant, with ρ being the material density and $v_l = \sqrt{E_0/\rho}$ the speed of sound. When mechanical dissipation is present, the Young's modulus has to be written in a complex form, $E = E_0[1 + i\phi(f, z)]$, where loss angle $\phi(f, z)$ characterises the mechanical dissipation and in general depends on both frequency and position. However, it has been found that the loss angles for most common materials (including glass) have very weak frequency dependence over a wide range of frequencies [7]. We further assume homogeneous dissipation throughout the fibre so that $\phi(f, z)$ can be replaced by a constant ϕ_0. The total energy dissipation rate can be related to the loss angle and the elastic energy by $W_{diss} = 2\pi f\phi_0\int\mathcal{E}_{\max}(z)dV$, where the volumetric integration is over the entire fibre. By using (2) and the low frequency condition (i.e. small kL), the energy dissipation rate is found to be

$$W_{diss} \simeq \frac{\pi f\phi_0 L F_0^2}{3E_0 A} \qquad (3)$$

Substituting (3) into the FDT (1), the spectral density of the spontaneous fibre length fluctuation can be written as

$$S_L(f) = \frac{2k_B T L\phi_0}{3\pi E_0 A}\frac{1}{f} \qquad (4)$$

It must be noted here that (4) is valid only for small kL, which means the fluctuation of concern is much slower than the time it takes for the sound wave to travel across the length of the fibre, i.e. $L/v_l \ll 1/f$.

Scale of thermal noise: According to (4), the thermal fluctuation of fibre length due to mechanical dissipation has a $1/f$ characteristic (at low frequencies). The length fluctuation leads to a $1/f$ phase noise in fibre interferometers or a $1/f$ frequency noise in fibre cavities. The scale of this thermal noise can be estimated by using typical parameters of singlemode (SM) fibres. However, an actual optical fibre is not a homogeneous glass rod. Instead, it consists of a silica inner rod (core and cladding), 125 µm in diameter for a typical SM fibre, and a concentric acrylate buffer layer with a 250 µm outer diameter. Using a lumped-element model, it has been shown that the effective overall Young's modulus of SM fibres is 19.0 GPa [11]. Meanwhile, experimental data for the effective loss angle of SM fibres are available between 75 and 200 kHz, which are approximately 1×10^{-2} [11]. Although there is no direct evidence that this value is valid at frequencies below 1 kHz, since many materials (e.g. glass) display a frequency-independent loss over a broad frequency range [7], we nonetheless assume $\phi_0 \simeq 1 \times 10^{-2}$ for the sake of making a numerical estimate. From (4), we can compute the spectral distribution of the length fluctuation amplitude, $\tilde{\delta}_L(f) \equiv \sqrt{S_L(f)}$, at room temperature. Fig. 2 shows $\tilde{\delta}_L(f)$ from 1 Hz to 1 kHz with fibre lengths of 0.1, 1, and 10 m. The thermal phase noise caused by the length fluctuation can be obtained with $\tilde{\delta}_\phi(f) = 2\pi\tilde{\delta}_L(f)/\lambda$. For instance, with $L = 10$ m and $\lambda = 1300$ nm, $\tilde{\delta}_\varphi \simeq 1.5 \times 10^{-7}\,\mathrm{rad/Hz^{1/2}}$ at 100 Hz, which is comparable to or above the shot noise-limited minimum detectable phase shift [1]. This indicates the possibility that dissipation-induced thermal phase noise becomes the dominant noise in fibre interferometers at low frequencies.

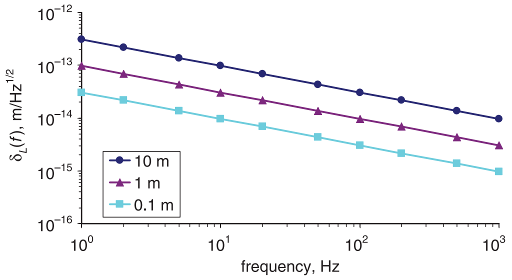

Fig. 2 *Spectra of spontaneous length fluctuation of optical fibres, with various fibre lengths L = 0.1, 1, 10 m*

In the case of fibre cavities, the change of fibre length results in the variation of the resonance frequencies. The spectral density of the thermal frequency noise is related to the length fluctuation by $S_\nu(f) = (\nu/L)^2 \times S_L(f)$, where ν is the optical resonance frequency. In distributed feedback fibre lasers, the laser cavities, which are formed by fibre Bragg gratings, can be approximately viewed as FP cavities with an effective cavity length L_c determined by the grating parameters. Using typical values of $L_c = 1$cm and $\lambda = 1550$nm, we can estimate the thermal frequency noise of such lasers at low frequencies as $\sqrt{S_\nu(f)} \simeq 187 \times f^{-1/2}$ Hz/Hz$^{1/2}$. This appears to fall into a similar scale as the $1/f$ frequency noise discussed in [4] and [5]. Thus, the $1/f$ noise widely observed in distributed feedback fibre lasers may have been caused by the slow fluctuation of fibre cavity length due to mechanical dissipation.

Conclusion: A 1D model offers clear evidence that thermal noise due to mechanical dissipation in optical fibres can be the dominating intrinsic noise at low frequencies in fibre interferometers and fibre cavities. Future work will focus on the development of a 3D model, which takes into account the effects of bending and fibre structure.

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One or more of the Figures in this Letter are available in colour online.

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