EXPERIMENTAL AND NUMERICAL INVESTIGATIONS OF THE DYNAMIC RESPONSE OF HIGHLY COMPLIANT, POLYMER-ENHANCED, GRAPHITE REINFORCED CEMENTITIOUS COMPOSITES

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ABSTRACT

This paper demonstrates how numerical and experimental methods were used to determine the dynamic response of “SURVIVOR,” UAH’s winning entry in the 2001 ASCE/MBT National Concrete Canoe Competition.

This boat has some unique structural and material properties and represents a new class of highly compliant, **P**olymer-**E**nhanced, **G**raphite **R**einforced **C**ementitious **C**omposite (**PEGRCC**) structures designed based on the strength, stiffness, and the position of the component materials in the composite section.

INTRODUCTION

The concrete canoe team at the University of Alabama in Huntsville (Team UAH) has proudly represented the Southeast Region twelve times at the national level in the ASCE/MBT (American Society of Civil Engineers/Master Builders) National Concrete Canoe Competition and they have five national titles and three, second place finishes to their credit.

In 2001, Team UAH built a boat called “SURVIVOR.” They strategically tuned their hull by lowering its natural frequency so that the forcing function created by the paddlers drove the boat toward resonance. When the flexible hull deformed in response to the applied torsional and bending moments, very large stresses and strains developed. The team strived to keep all of the materials elastic so that the structure was absolutely resilient, enabling the strain energy stored in the deformed shape to be recovered. As the crew pulled their paddles from the water, this energy was converted into forward momentum, thereby forcing the boat to surge forward between strokes and swim.

The structural design for “SURVIVOR” was based on prior research conducted to determine the material properties of graphite reinforced cementitious composites [1,2]. The team relied on the large difference in stiffness between the constituents in their composite section to drive the internal stress from a flexible cementitious matrix to three layers of relatively stiff graphite reinforcement. They placed materials symmetrically to form an adaptive section optimized to resist stress reversals and positioned fiber layers to tune the modal response.

This paper demonstrates how numerical and experimental methods were used to determine the dynamic response of this concrete canoe. A finite element model consisting of quadrilateral elements was developed based on classical laminated plate theory and experimental tests were conducted by suspending the structure in a free-hanging configuration. Standard impact hammer tests were conducted to determine the modal parameters.

The experimental test results were compared to those obtained from finite element model to validate the finite element approach. Results show that PEGRCC structures can be modeled and tested using methods and tools similar to those applied to study classical composite structures.

CANOE CONSTRUCTION

"SURVIVOR" was fabricated by placing the cementitious mixture described in Figure 1 over three layers of graphite. According to the manufacturer, each layer of reinforcement consisted of a non-impregnated graphite mesh with 3,000 fibers per tow, spaced at 3.18 mm intervals. Each tow is 0.19 mm thick by 1.07 mm wide; the elastic modulus and tensile strength of the graphite are 231 GPa and 3.65 GPa, respectively.

The team began construction by coating a male mold with a thin sheet of plastic that served as a mold release. The first layer of graphite mesh [90°, 90°] was draped over the mold, and 2.8 mm diameter wires were positioned transversely at 7.6 cm intervals down the length.

The mixture was prepared by first mixing the cement and micro-bubbles. Then the acrylic fortifier, latex, and water were added to produce a mixture having a smooth texture.

Ingredient	% Mass weight
K-25 micro-bubbles	11.75
Portland cement	30.06
Latex	16.70
Acrylic fortifier	8.08
Water	33.41

Figure 1. Mix proportions.

The mixture was placed over the mold and the team used drywall knives to level the mixture to the upper surface of the wires. Once the layer had hardened, they removed the speaker wires and filled the grooves. This construction process was repeated for the second and third layers of mesh.

Since the water required for cement hydration was held in the latex-modified system, the canoe was simply left to dry. After three days, the outer layer of concrete was hand-sanded smooth. The team filled voids with the same mixture used during the main construction and then removed the canoe from the mold and repeated the process on the inner surface.

Using temporary wooden forms located around the upper rim of the canoe, the team placed a gunwale. Since the concrete canoe was inherently buoyant, no flotation was required.

"SURVIVOR" is 6.8 m long and has a mass of 34 kg, a maximum width of 81.3 cm, and a maximum depth of 27.9 cm. The canoe's nominal wall thickness is 0.74 cm.

Figure 2 is a photograph taken of the canoe while suspended during a modal test.

Figure 2. "SURVIVOR" being modal tested.

CONSTITUTIVE EQUATIONS

The constitutive equations that relate the force and moment resultants to the strains for PEGRCC anisotropic laminates can be derived based on classical laminated plate theory and may be written as:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{13} \\ A_{21} & A_{22} & A_{23} & B_{21} & B_{22} & B_{23} \\ A_{31} & A_{32} & A_{33} & B_{31} & B_{32} & B_{33} \\ B_{11} & B_{12} & B_{13} & D_{11} & D_{12} & D_{13} \\ B_{21} & B_{22} & B_{23} & D_{21} & D_{22} & D_{23} \\ B_{31} & B_{32} & B_{33} & D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{0x} \\ \varepsilon_{0y} \\ \gamma_{0xy} \\ K_x \\ K_y \\ K_{xy} \end{bmatrix} \qquad (1)$$

$$\text{where } (A_{ij}, B_{ij}, D_{ij}) = \sum_{r=1}^{N} \int_{z_r}^{z_{r+1}} Q_{ij}^{(r)} (1, z, z^2) dz \text{ and}$$

N_x, N_y, N_{xy} = resultant force and shear force in x-axis, y-axis and x-y plane, respectively

M_x, M_y, M_{xy} = resultant moment and twisting about x-axis, y-axis, and x-y plane, respectively

Z_r, Z_{r+1} = thickness coordinates of the lower and the upper surface of the r-th ply

$Q_{ij}^{(r)}$ = material stiffnesses of the r-th ply

Z = laminate transverse direction, normal to x-y plane

N = number of plies in the laminate

ε_{0x} = midplane strain in x-axis

ε_{0xy} = midplane strain in y-axis

γ_{0xy} = midplane shear strain in x-y plane

K_x = plate bending curvature in the x-z plane

K_y = plate bending curvature in the y-z plane

K_{xy} = plate twisting curvature in the x-y plane

A_{ij} = extensional stiffnesses

B_{ij} = bending-extension coupling stiffnesses

D_{ij} = bending stiffnesses

The *A*, *B*, and *D* matrices in Equation (1) are computed based on the laminate material properties, geometry, and stacking sequence. For a three-ply specially orthotropic laminate (laminate whose principal material axes are aligned with the natural body axes), the matrices were computed based on previously obtained material properties [1,2]. For this configuration, the bending-extensional coupling coefficients, B_{ij}, the bending-twisting coefficients, D_{13}, D_{23}, and the shear-extensional coupling coefficients, A_{13}, A_{23}, are all zero.

The material properties are 3.48 GPa for the elastic modulus, 517 MPa for the shear modulus, 0.137 for the Poisson's ratio, and 757 kg/m^3 for the mass density. Each ply is assumed to have a thickness of 2.54 mm.

FINITE ELEMENT ANALYSIS

The finite element model for "SURVIVOR" was developed based on classical laminated plate theory using thin shell elements. The model was generated using physical dimensions and the material properties mentioned above. MSC/Nastran was selected as the finite element code because of its multi-layered composite element capabilities for normal mode analysis.

The finite element model consisted of equally spaced quadrilateral membrane-bending plate elements with uniform thickness. Damping was neglected in the analysis and refinements in element sizes were made until the natural frequencies converged. Only the lower natural frequencies and their associated mode shapes in a free-free boundary condition were calculated because they adequately describe the dynamic behavior of the canoe.

Figure 3 shows exaggerated plots of the first six mode shapes predicted by finite element analysis; the corresponding natural frequencies are listed in the figure caption. The first mode shape corresponds to an anti-symmetrical torsional deformation while the second mode is a bending mode in which the canoe flutters like a butterfly. The higher modes are much more complex.

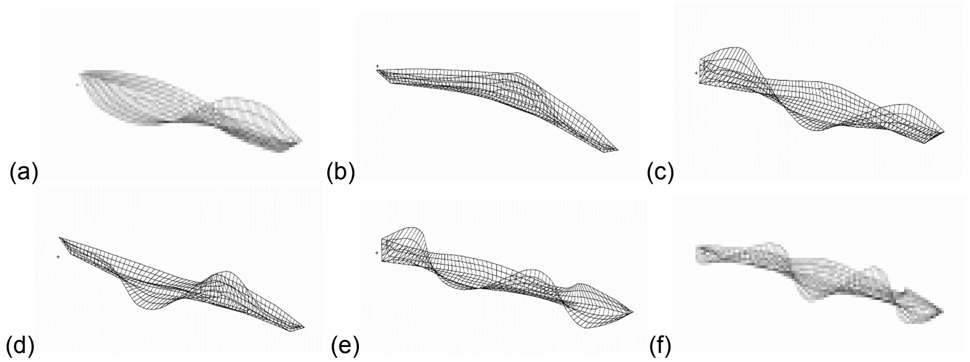

Figure 3. Finite element results showing the first six modes of "SURVIVOR," (a) 1st mode, frequency =7.31 Hz, (b) 2nd mode, frequency = 10.76 Hz, (c) 3rd mode, frequency = 12.06 Hz, (d) 4th mode, frequency = 15.38 Hz, (e) 5th mode, frequency = 15.95 Hz, (f) 6th mode, frequency = 18.52 Hz.

EXPERIMENTAL TESTING

Standard impact hammer tests were conducted on the canoe to determine its modal parameters (natural frequencies and modes shapes) and to validate the finite element model. As illustrated in Figure 4, the impact hammer was used to strike test specimen ("SURVIVOR") suspended by elastic cords in the free-hanging configuration (see Figure 2).

A load cell, located in the tip of the hammer, was used to measure the force input while a tri-axial accelerometer was employed to measure the acceleration in G's along three perpendicular directions. Frequency response functions were collected for 35 different locations (both laterally and tangentially) and they were combined to produce the mode shapes.

Rigorously, a frequency response function (FRF) is the direct linear relationship between the mechanical force input and the measured response output of a test specimen. It is described by the formula,

$$H(\omega) = \frac{y(t)}{F_o e^{i\omega t}} \qquad (2)$$

where y*(t)* is the output function, F_o is the input force, and $e^{i\omega t}$ is the exponential harmonic frequency function.

Figure 5 shows a typical FRF plot taken from the canoe at one of the measurement points. FRF data were combined by the Rational Fraction Polynomial method to determine the modal parameters.

The procedure is to first estimate how many modes there are in the bandwidth of interest and then estimate a pole for each mode. A residue is estimated for each mode. Mode shapes are curve fitted and computed based on these estimates using commercially available software.

Figure 6 shows wire frame plots of the first six mode shapes measured experimentally by impact hammer testing "SURVIVOR;" the corresponding natural frequencies are listed in the figure caption.

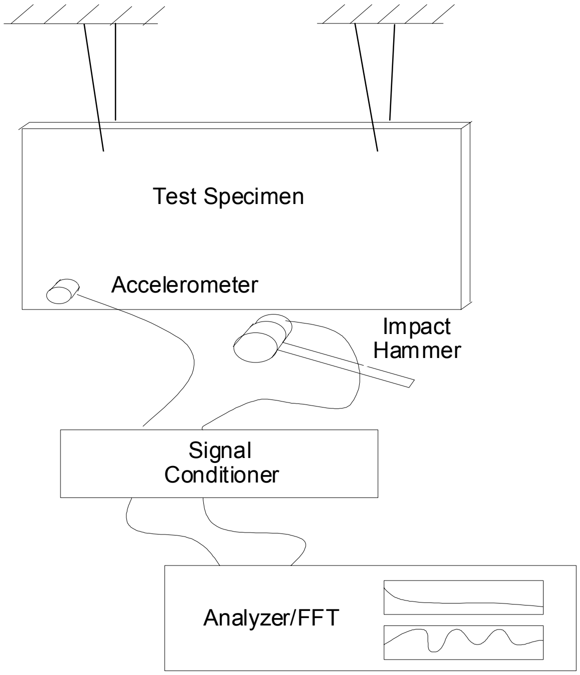

Figure 4. Schematic diagram of a vibration measurement system for modal impact testing.

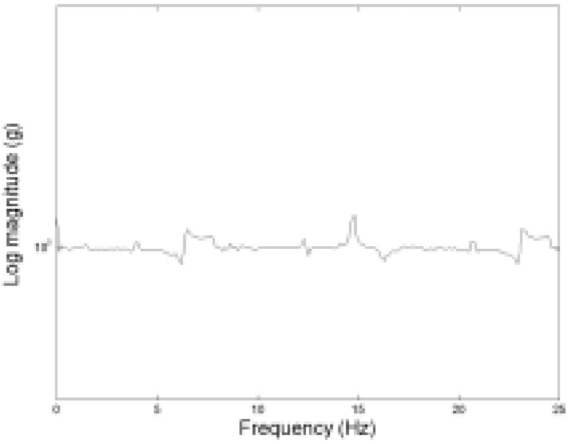

Figure 5. A sample FRF plot from "SURVIVOR."

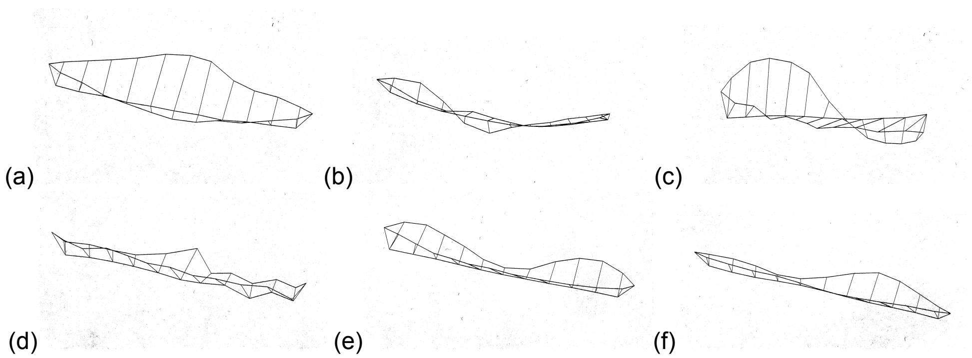

Figure 6. Impact hammer test results showing the first six modes of “SURVIVOR,” (a) 1st mode, frequency = 4.59 Hz, (b) 2nd mode, frequency = 11.54 Hz, (c) 3rd mode, frequency = 12.31 Hz, (d) 4th mode, frequency = 13.64 Hz, (e) 5th mode, frequency = 17.54 Hz, (f) 6th mode, frequency = 18.98 Hz.

Even though the number of points at which measurements were made is small, the experimental mode shapes (see Figure 6) are practically identical to those predicted by the finite element analysis (see Figure 3). Figure 7 compares the finite element predictions of the natural frequencies with the experimentally measured values.

Mode Shape Number	Finite Element Frequency, Hz	Experimental Frequency, Hz
1	7.31	4.59
2	10.76	11.54
3	12.06	12.31
4	15.38	13.64
5	15.95	17.54
6	18.52	18.98

Figure 7. Comparison of finite element predictions with modal test results.

With the exception of the frequencies obtained for the first mode, the results agree quite well. The small discrepancies between the measured and calculated frequencies can be partially attributed to two problems in signal processing. The first is the noise present in the force or response signal as a result of a long time record. The second is the leakage present in the response signal as a result of a short time record.

Other reasons for these frequency differences may include inaccuracies in the constitutive equations; impact signals may be poorly suited for the frequency response function measurements; resolution bias errors may be present in the spectral estimates; and, the system relating the output and input may not be linear.

CONCLUSIONS

This research establishes the procedures for constructing, instrumenting, and testing a PEGRCC structure to determine its natural frequencies and mode shapes. Standard impact modal testing is used to measure its modal parameters. The modal parameters can be used to validate PEGRCC material properties and for future theoretical developments.

Good agreement between the finite element predictions and the experimentally determined natural frequencies and mode shapes show that the standard impact hammer test can be adopted to test PEGRCC materials. Results also indicate that PEGRCC structures can be analyzed using finite element models developed based on the classical laminated plate theory.

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