Reference: Peter Simons, *Parts: A Study in Ontology* (New York: Clarendon Press, 2000)

Abbreviations

$x \ll y$	x is a proper part of y
$x=y$	x and y designate the same individual
$x \approx y$	x and y either designate the same individual or are both empty
Ex	x exists
!x	there is at most one x
E!x	there is exactly one x

Definitions

SD1 (part): $x<y$ iff $x \ll y$ or $x=y$

SD2 (overlap): $x \circ y$ iff $\exists z \ulcorner z<x$ or $z<y \urcorner$ (x,y have at least one common part)

SD3 (disjoint): $x|y$ iff $\sim(x \circ y)$ (x,y, have no common part)

SD4 (product): $x \cdot y$ iff $\iota z \forall w \ulcorner w<z \equiv (w<x \;\&\; w<y) \urcorner$
(a product is the common part of x,y such that any common part of x,y is a common part of it)

SD7 (sum): $x+y \approx \iota z \forall w \ulcorner w \circ z \equiv (w \circ x \text{ or } w \circ y) \urcorner$

SD9 (F-fusion): $\sigma x \ulcorner Fx \urcorner \approx \iota z \forall y \ulcorner x \circ y \equiv \exists z \ulcorner Fz \;\&\; z \circ y \urcorner \urcorner$

SD10 (F-product): $\pi x \ulcorner Fx \urcorner \approx \sigma x \ulcorner \forall y \ulcorner Fy \supset x<y \urcorner \urcorner$

SD11 (difference): $x-y \approx \sigma z \ulcorner z<x \text{ and } z|y \urcorner$
(a difference is the largest part of x that has a common part with y)

SD12 (universe): $U = \sigma x \ulcorner x=x \urcorner$

SD13 (complement): $x^* \approx U-x$

Ordering Axioms

These constrain the proper parthood relation.

SA1: $x \ll y \supset \sim(y \ll x)$
Proper parthood is not symmetric.

SA2: $(x \ll y \;\&\; y \ll z) \supset x \ll z$
Proper parthood is transitive.

Supplementation Principles

These constrain how to supplement an individual's proper part in order to obtain a whole.

SF2: $x \ll y \supset \exists z \ulcorner z \ll y \ \& \sim(z<x) \urcorner$

Any individual with a proper part has some other part that is not part of its proper part.

SA3: $x \ll y \supset \exists z \ulcorner z \ll y \ \& \ z | y \urcorner$

Weak Supplementation Principle (WSP)

Any individual with a proper part has at least two disjoint proper parts.

SA4: $(\exists z \ulcorner z \ll x \urcorner \ \& \ \forall z \ulcorner z \ll x \supset z \ll y \urcorner) \supset x<y$

Proper Part Principle (PPP)

If an individual has a proper part and shares all of its proper parts with some other individual, then that latter individual is part of the former individual.

SA5: $\sim(x<y) \supset \exists z \ulcorner z<x \ \& \ z | y \urcorner$

Strong Supplementation Principle (SSP)

Anything that is not part of some individual has a part that is disjoint from that individual.

SA6: $x \circ y \supset \exists z \forall w \ \ulcorner w<z \equiv (w<x \ \& \ w<y) \urcorner$

Overlapping individuals have a unique product.

SF3: $x \circ y \supset E!(x \cdot y)$

Overlaps form a unique product.

SA24: $\exists x \ Fx \supset \exists x \ \forall y \ \ulcorner y \circ x \equiv \exists z \ \ulcorner Fz \ \& \ y \circ z \urcorner \urcorner$

General Sum Principle (GSP)

$\exists x \ Fx \supset E! \ \sigma x \ulcorner Fx \urcorner$ (via SD9)

There is a unique fusion of all Fs.

Theorems

SCT7: $x=y \equiv \forall z \ \ulcorner z<x \equiv z<y \urcorner$

Identical individuals have exactly the same parts.

SCT71: $(\exists z \ \ulcorner z \ll x \urcorner \ \& \ \exists z \ \ulcorner z \ll y \urcorner) \supset (\forall z \ \ulcorner z \ll x \equiv z \ll y \urcorner \supset x=y)$

Mereological Extensionality

Individuals with exactly the same proper parts are identical.

Comments

SA5 ├ SA4

SA5 ├ SA3

SD4+SA6 ├ SF3

SD1 ├ SCT7

SA4 ├ SCT71 (SA4 amounts to mereological extensionality.)

SF2 is compatible with a universe all of whose parts overlap.
SA3 rules out this possibility.

SA3 is compatible with distinct individuals having exactly the same proper parts.
SA4 rules out this possibility.

SA1-5 is compatible with overlapping individuals that do not have a unique product.
SA6 rules out this possibility.

Extensional Systems of Mereology

Minimal Extensional Mereology
SA1, SA2, SA3, SA6

Classical Extensional Mereology
SA1, SA2, SA3, SA24

Counterexamples to Extensionality

F.C. Doepke, "Spatially Coinciding Objects," *Ratio* 24 (1982): 45-60

A person and the person's body have exactly the same parts.
A person exists only if their parts are undergoing certain processes.
But the person's body can exists even if its parts are not undergoing certain processes.
So a person and their body are distinct.
Hence, individuals with the same parts need not be identical.

David Wiggins, "Mereological Essentialism: Asymmetrical Essential Dependence and the Nature of Continuants," in Ernest Sosa (ed.), *Essays on the Philosophy of Roderick M. Chisholm* (1979): 297-316.

Tibbles (cat) consists of Tib (cat body) and Tail (cat tail).
Tibbles and Tib+Tail have exactly the same parts.
Tibbles can lose its tail and continue to exist.
Tib+Tail cannot lose its and continue to exist.
[Tacit Principle: $(\Diamond Fx \ \& \ {\sim}\Diamond Fy) \supset x \neq y$]
So Tibbles and Tib+Tail are distinct.
Hence, individuals with the same parts need not be identical.

Non-Extensional Mereology

SD1* (part*): $x \leq y$ iff $\exists z \ulcorner z \ll x \urcorner \supset \forall z \ulcorner z \ll x \supset z \ll y \urcorner$ or $(\sim \exists z \ulcorner z \ll x \urcorner \supset (x \ll y$ or $x = y)$

SD1* entails SA2 (transitivity of proper parthood).

SCT72: $x < y \supset x \ll y$

The converse of SCT72-- $x \ll y \supset x < y$ --holds only with SA4.

Coincidence: $x \lesseqgtr y$ iff $x \leq y$ & $y \leq x$ (x coincides with y)
x,y are coincident iff x,y have exactly the same parts

CTD5: $x \lesseqgtr_t y$ iff $x \leq_t y$ & $y \leq_t x$ (x coincides at time t with y)

CTT22: $x \lesseqgtr_t y \equiv Ex_t a$ & $Ex_t b$ & $\forall x \ulcorner x <_t a \equiv x <_t b \urcorner$

CTD5+SA2 $\vdash$ CTT22

Superposition: x sup_t y iff x and y occupy exactly the same place at time t

Coincident individuals are perceptually indistinguishable.
Coincident individuals are superposed.
If $x \lesseqgtr_t y$, then x sup_t y.

Without SA4, one cannot prove:

SF12: $(x < y$ & $y \leq x) \supset x = y$
SF13: $\forall z \ulcorner z \leq x \equiv z \leq y \urcorner \supset x = y$

This shows that SA4 amounts to mereological extensionality.

Without SA4,

- parthood* ($\leq$) is not antisymmetric
- parthood* ($\leq$) remains reflexive and transitive
- coincidence ($\lesseqgtr$) is not identity (=)

SA4 $\vdash$ coincidence only if identity

Coincidence and Superposition

x,y are coincident only if x,y are superposed.

> *Proof*: Obvious.

not-(x,y are superposed only if x,y are coincident)

> *Proof*: Consider a ring and the gold in the ring. These are not identical, because they have different life histories. They are not coincident (e.g., the ring contains jewels too?). But they are superposed.
>
> *Doepke's Proof*: Caesar's heart is superposed with the mass of matter that constitutes his heart. But while the matter is part of the heart, the heart is not part of the matter, because the laws that apply to the heart do not apply to the matter. So Caesar's heart is not coincident with its matter.

There is a set of principles that imply that superposition entails coincidence:

x is a container for y = x is a portion of space that contains y

x is a receptacle for y = x is a container for y and x is the minimal portion of space that contains y

> $r_t x$: x's receptacle at t

CTA11: $a <_t b \supset r_t a < r_t b$

CTD19: $a \text{ in}_t\, b \equiv r_t a < r_t b$ (a is inside b at t)

CTD20: $a \text{ sup}_t\, b \equiv r_t a = r_t b$ (a is superposed with b at t)

CTD21: $a \text{ ex}_t\, b \equiv r_t a \mid r_t b$ (a is outside b at t)

CTD 22: $a \text{ ov}_t\, b \equiv r_t a \circ r_t b$ (a overlaps b at t)

WP: $a \text{ sup}_t\, b \supset a \text{ ov}_t\, b$ (Wiggin's Principle)

SSP: $\forall x \ulcorner x <_t a \supset x \text{ o}_t\, b \urcorner \supset a <_t b$ (Strong Supplementation Principle)

PP: $a \text{ in}_t\, b \supset \exists x \ulcorner x <_t b \ \&\ a \text{ sup}_t\, b \urcorner$
If a is in b at t, a is superposed at t with a part of b.

WP+SSP+PP+CTA11 $\vdash$ superposition entails coincidence

> Simon's Solution: Reject SSP.

Objections to the Possibility of Superposition

One-Many View

- a plurality of objects is not identical to a single object.

Problem: a ring and its gold, and a person and its body, are not cases involving a one-many relation

Relative Identity

- see (e), (f), (g) criticism from the Flux argument

Dichronic View

- when the ring comes into existence, the gold ceases to exist and is replaced by the ring, so neither exists at the same time as the other

- This rejects the view that a substratum exists that survives change; instead, change is the replacement of one individual by another.

Problem: On this view, it is hard to explain continuity of properties from, say, gold to a ring.

Reductivism

- the only real objects are the ultimate constituents of continuents; all else is a logical construction

Problem: Parts are not always logically prior to their wholes. For example, integrated wholes (e.g., organisms) have properties and obey laws that are relatively independent of their constituents, because they can survive and sustain these properties despite flux in their parts.

Concluding Remarks from Simons

There are four minimal constraints on the proper parthood relation:

Definition 'Part':	$x<y \equiv (x \ll y \;\&\; (E!x \;\&\; x \approx y))$
Definition 'Overlap':	$x \circ y \equiv \exists z \ulcorner z \ll y \;\&\; \sim(z \approx x) \;\&\; \sim(z \approx y) \;\&\; \sim(z \approx x)\urcorner \;\&\; \exists w \ulcorner w \ll z \;\&\; w<<x\urcorner$
Falsehood:	$x \ll y \supset (E!x \;\&\; E!y)$
Asymmetry:	$x \ll y \supset \sim(y \ll x)$
Transitivity:	$(x \ll y \;\&\; y \ll z) \supset x \ll z$
Weak Supplementation:	$x \ll y \supset \exists z \ulcorner z \ll y \;\&\; \sim(z \circ x)\urcorner$

The basis for stronger mereological principles is the nature of the objects to which the part-relation applies.

- This results in *local* mereological systems with the above four *global* mereological properties.

Putative Objection to Transitivity of Parthood (SA2), and Standard Reply

Putative Counterexamples

1 - The handle is part of the door, which is part of the house; but the handle is not part of the house.

2 - The nucleus is part of the cell, which is part of the organism; but the nucleus is not part of the organism.

3 - The arm is part of the musician, who is part of the orchestra; but the arm is not part of the orchestra.

Standard Reply

The objections gain their plausibility by invoking a sense of part that is restricted to a particular kind of part.

Ex: the handle is a functional part of the door, the door is a functional part of the house; but the handle is not a functional part of the house.

Ex: The nucleus is a distinguished part of the cell, the cell is a distinguished part of the organism; but the nucleus is not a distinguished part of the cell.

Ex: The arm is directly part of the musician who is directly part of the orchestra; but the arm is not directly part of the orchestra.

So the examples show that φ-parthood is not transitive.

> Explanation: φ-ness need not distribute over parthood. But this is a feature of φ-ness, not a feature of parthood.

Ontological Dependence

Weak Foundation: NEC(E!a ⊃ E!b)
(NEC(x) : it is necessarily the case that x)

Problem: This entails that everything is ontologically dependent on necessarily existent individuals (such as numbers, God)

Problem: This allows self-dependence (reflexivity of the dependence relation).

DD1: x WRD on y ≡ ~(x=y) & ~NEC(E!y) & NEC(E!x ⊃ E!y)
(Weak Rigid Dependence)
x WRD y : x is weakly rigidly dependent on y

- This avoids the two problems with Weak Foundation.

x *notionally* depends on y : x cannot be described as x unless y exists

NEC ∀x NEC (Hx ⊃ ∃y ⌜Wy & ~(x=y)⌝

Notional dependence entails ontological dependence (weak rigid dependence) only for Fs that are essentially Fs -- that is, only if:

NEX (Fx ⊃ NEC(E!x ⊃ Fx))

Ontological dependence entails notional dependence. (Obvious.)

Symbols: « ≤ ≥ ○ · ≈ ≡ σ π « – ≅ ⊃ ∃ ∀ ⌜⌝ ι ├