1-D Particle-in-Cell electromagnetic code

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Outline

Background

Particle-in-Cell (PIC) codes are well established tools for kinetic simulations in plasma physics and astrophysics. A brief history of PIC simulations is given below.

- In late 1950s John Dawson began 1D electrostatic "charge-sheet" experiments at Princeton, later at UCLA.
- 1965 Hockney and Buneman introduced grids and direct Poisson solver.
- 1970s theory of electrostatic PIC developed (Langdon).
- 1980s-90s First electromagnetic codes. 3D EM PIC takes off[2].
- PIC text books come out in 1988 ad 1990[1, 4].

PIC has been widely used in particles acceleration, instabilities[6], radiation, anomalous resistivity, magnetic reconnection[3, 5], relativistic jets[7].

Maxwell's Equations

We need to solve Maxwell's equations for the electric field $\boldsymbol{E} = (E_x, E_y, E_z)$ and magnetic field $\boldsymbol{B} = (B_y, B_z)$ in a one-dimensional system.

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} \quad (1)$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \quad (2)$$

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0} \quad (3)$$

$$\nabla \cdot \boldsymbol{B} = 0 \quad (4)$$

Two sets of spatial grids are introduced along $x-$axis. One is full-integer grid system, and the other is a half-integer grid system. E_y, B_y, J_y and ρ are defined on full-integer grids, while E_x, E_z, B_z, J_x are defined on the half-integer grids. Spatial and time derivatives in Maxwell's equations are replaced by centred differences by Δx and Δt. Courant condition for the time step and grid spacing is used.

$$c\Delta t < \Delta x \quad (5)$$

Lorentz Equation

We need to push particles in the electromagnetic field by solving Lorentz equation.

$$\frac{d\boldsymbol{v}}{dt} = \frac{q}{m}(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \tag{6}$$

$$\frac{dx}{dt} = v_x \tag{7}$$

It is solved by Buneman-Boris method[4, 1]. The advantage of the method is strict conservation of the kinetic energy in calculation of cyclotron motion[6].

Buneman-Boris

$$\frac{\boldsymbol{v}^{n+1/2} - \boldsymbol{v}^{n-1/2}}{\Delta t} = \frac{q}{m}\left(\boldsymbol{E}^n + \frac{\boldsymbol{v}^{n+1/2} + \boldsymbol{v}^{n-1/2}}{2} \times \boldsymbol{B}^n\right) \quad (8)$$

$$\boldsymbol{v}^- = \boldsymbol{v}^{n-1/2} + \frac{q}{m}\boldsymbol{E}^n \frac{\Delta t}{2} \quad (9)$$

$$\boldsymbol{v}^+ = \boldsymbol{v}^{n+1/2} - \frac{q}{m}\boldsymbol{E}^n \frac{\Delta t}{2} \quad (10)$$

$$\frac{\boldsymbol{v}^+ - \boldsymbol{v}^-}{\Delta t} = \frac{1}{2}\frac{q}{m}(\boldsymbol{v}^+ + \boldsymbol{v}^-) \times \boldsymbol{B}^n \quad (11)$$

$$\boldsymbol{v}^+ = \boldsymbol{v}^- + \frac{2}{1+\boldsymbol{T}^2}(\boldsymbol{v}^- + \boldsymbol{v}^- \times \boldsymbol{T}) \times \boldsymbol{T} \quad (12)$$

where $\boldsymbol{T} = \frac{q}{2m}\Delta t \boldsymbol{B}^n$.

Buneman-Boris

Usually, it takes 5 steps to update the position and velocity information of the particles.

1. $\boldsymbol{v}^{-} = \boldsymbol{v}^{n-1/2} + \frac{q}{m}\boldsymbol{E}^{n}\frac{\Delta t}{2}$
2. $\boldsymbol{v}^{0} = \boldsymbol{v}^{-} + \boldsymbol{v}^{-} \times \boldsymbol{T}$
3. $\boldsymbol{v}^{+} = \boldsymbol{v}^{-} + \boldsymbol{v}^{0} \times \boldsymbol{S}$, where $\boldsymbol{S} = 2\boldsymbol{T}/(1+\boldsymbol{T}^{2})$
4. $\boldsymbol{v}^{n+1/2} = \boldsymbol{v}^{+} + \frac{q}{m}\boldsymbol{E}^{n}\frac{\Delta t}{2}$
5. $\boldsymbol{r}^{n+1} = \boldsymbol{r}^{n} + \boldsymbol{v}^{n+1/2}\Delta t$

Whole process

The whole process can be generalized as the graph below.

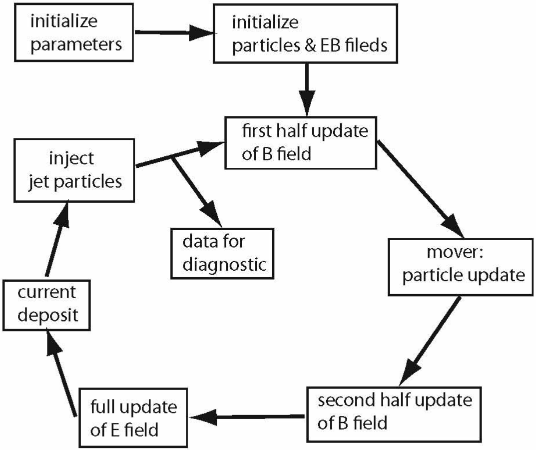
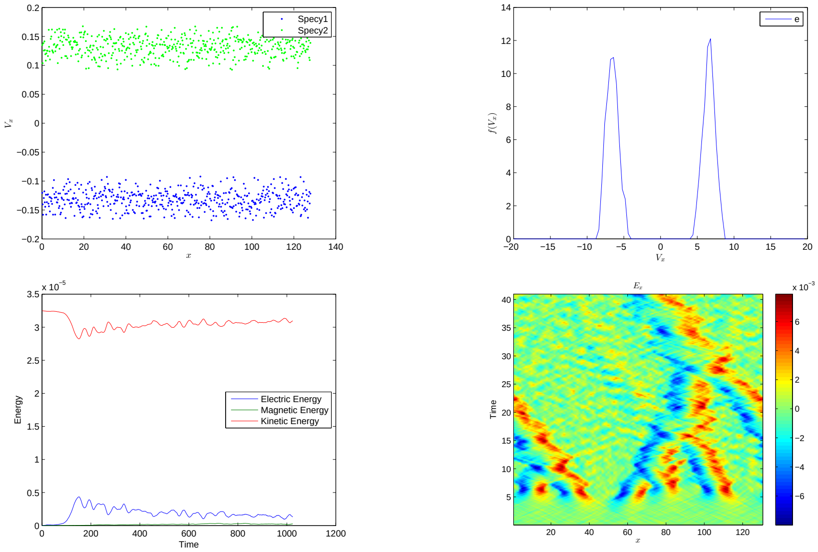


Figure : Processes of one PIC code

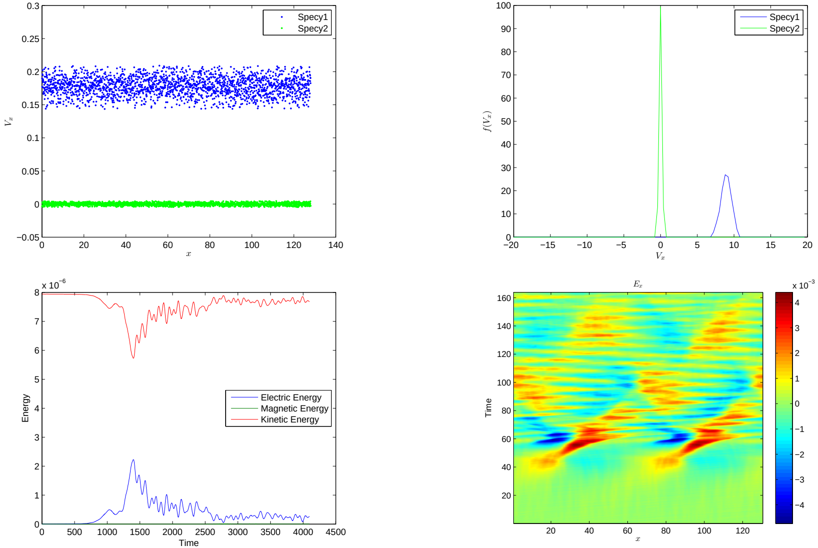
Setup

- Two groups of electrons have different drift velocities parallel to the static magnetic field. The thermal velocity of the electrons is much smaller than the drift velocity. It will arise a strong electrostatic instability.
- When there arises a large scale parallel electric field with a very low frequency, electrons are accelerated along the magnetic field forming a field-aligned current. In the presence of a large relative drift velocity $V_{d\|}$ between the thermal electrons and the thermal ions, a strong electrostatic instability called "Buneman instability" arises.

Two-stream instability



Buneman instability



Summary

The code is based the classical procedures of PIC methods. The simulation shows some results that are expected based on the physics of the problems. So it should be a valid code.

The course helps me in

1. PDE parts help a lot.
2. It helps me to use C in more efficient way.
3. Learning how to debug my code.

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