PH 662 Project
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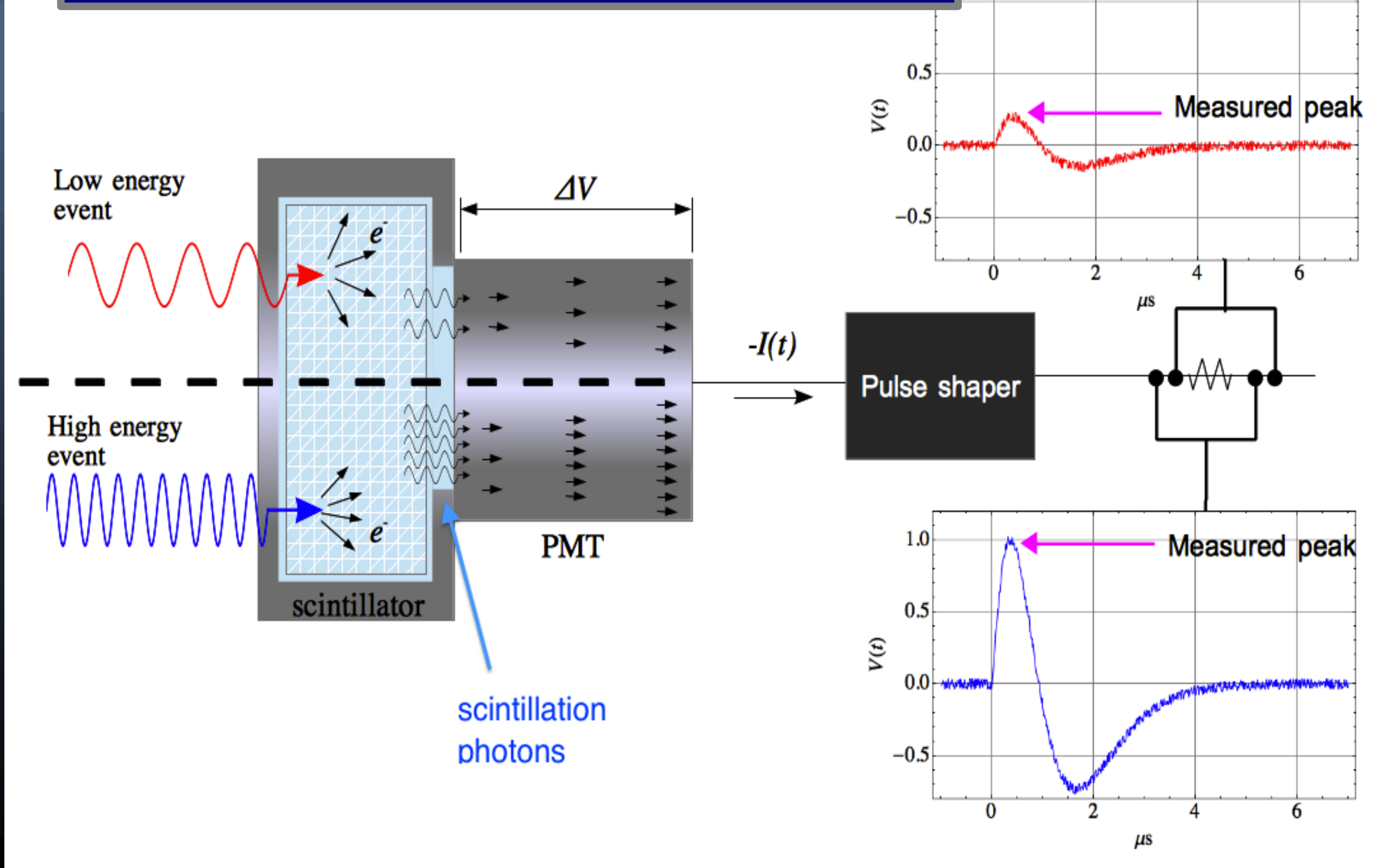
- Part 1: Pulse-pileup: Fitting a non-linear empirical model using Gauss-Newton algorithm

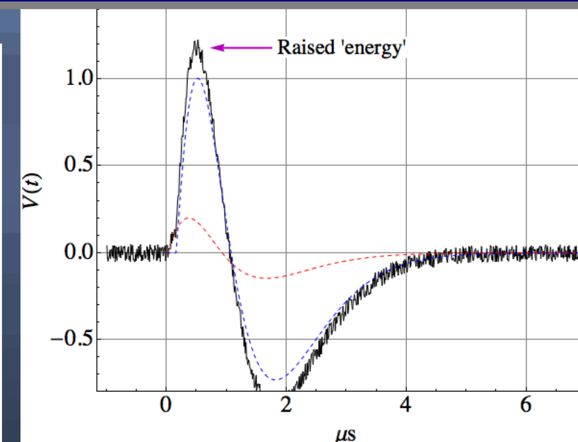
- Part 2: Simulation of a stochastic Poisson process (non-homogenous)

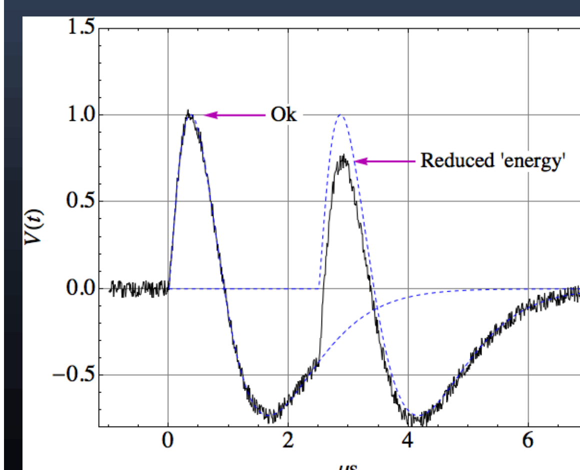
Background: Pulse-pileup in the *Gamma-ray Burst Monitor*

Pulse-height measurement to determine photon energy

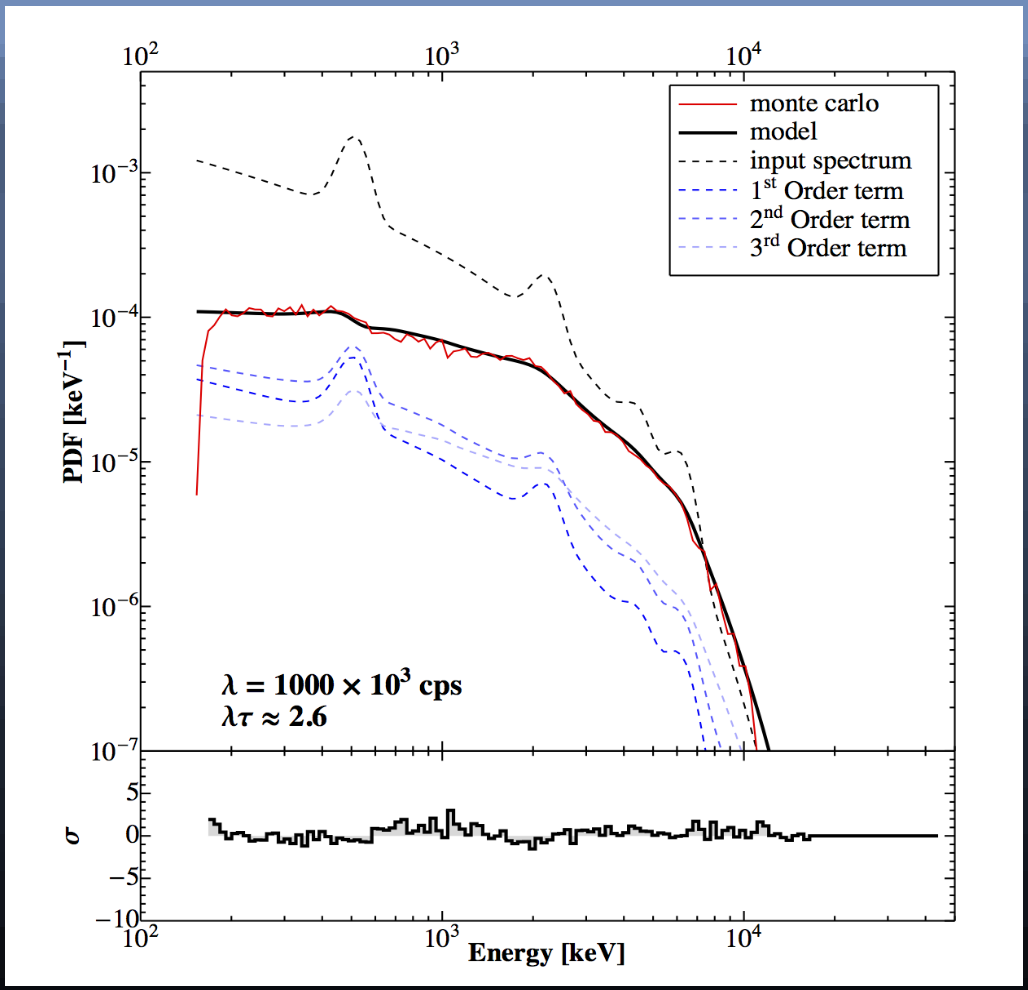


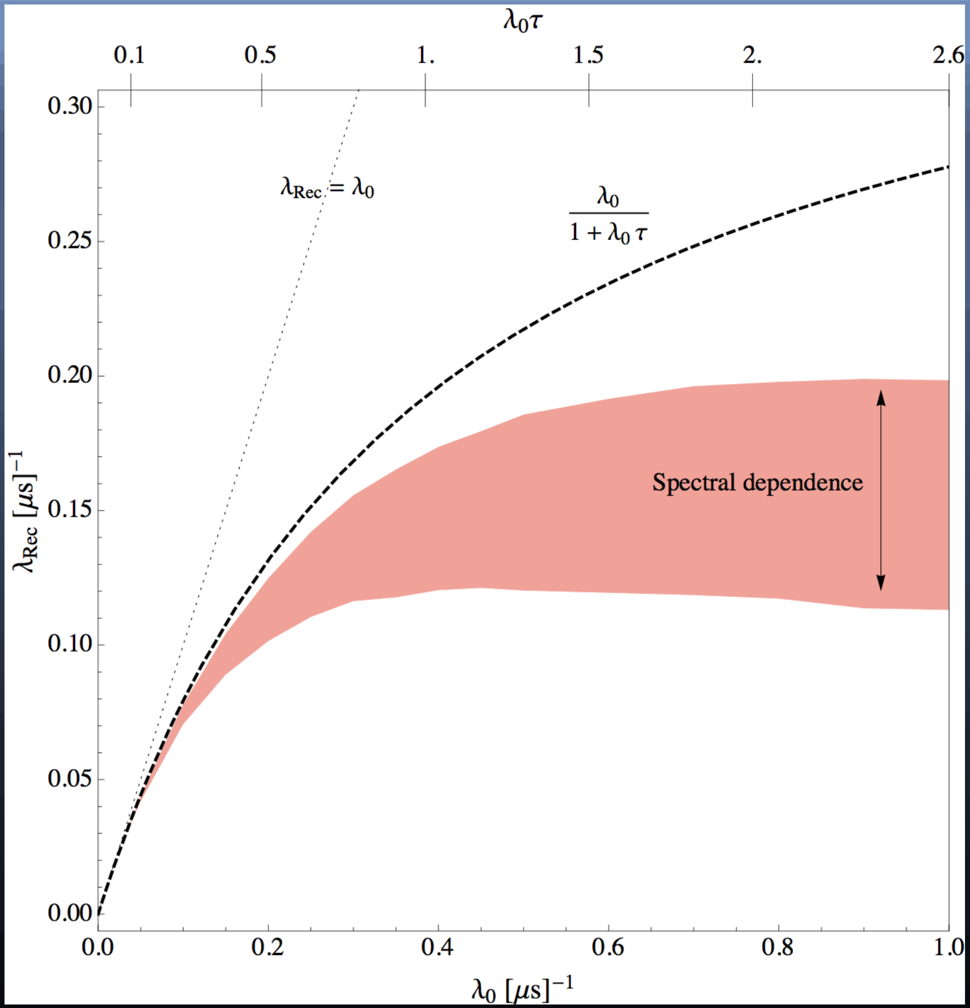
Pulse-height distortion





PPU distortions (continued)





0^{th} order peak

$$f(t) = K(c_1 * t^\alpha - c_2 * t^\beta)e^{-\gamma t}$$

Single Pulse peak "zeroth order":

$$(t_p^0) \equiv \text{ peak time of a single pulse at } \tau = 0$$

$$\begin{aligned}
\frac{d}{dt} f(t)\Big|_{t_p^0} &= f'(t_p^0) \\
&= \frac{d}{dt}\left[\left(at^\alpha - bt^\beta\right)e^{-ct}\right]\Big|_{t_p^0} \\
&= \left(a\alpha t^{\alpha-1} - b\beta t^{\beta-1}\right)e^{-ct} - c\left(at^\alpha - bt^\beta\right)e^{-ct}\Big|_{t_p^0} \\
&= 0
\end{aligned}$$

$$\Rightarrow a\{\alpha\ (t_p^0)^{\alpha-1} - c\ (t_p^0)^\alpha\} + b\{c\ (t_p^0)^\beta - \beta\ (t_p^0)^{\beta-1}\} = 0$$

$$\Rightarrow t_p^0 \cong 0.388\mu s$$

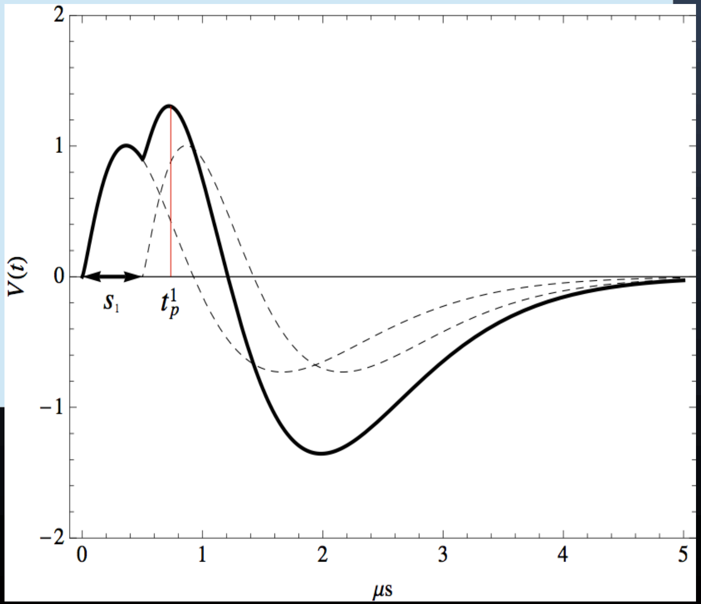
1st order peak-time

$$(t_p^1) \equiv \text{peak time of two pulses } s_1 = t_1 - t_0$$

$$\begin{aligned}
\frac{d}{dt}\Big[V_0 f(t) + V_1 f(t-s_1)\Big]_{t_p^1} &= V_0 f'(t_p^1) + V_1 f'(t_p^1 - s_1) \\
&= V_0 e^{-ct_p^1}\Big\{ \left(a\alpha(t_p^1)^{\alpha-1} - b\beta(t_p^1)^{\beta-1}\right) - c\left(a(t_p^1)^{\alpha} - b(t_p^1)^{\beta}\right) \Big\} + \\
&\quad V_1 e^{-c(t_p^1 - s_1)}\Big\{ \left(a\alpha(t_p^1-s_1)^{\alpha-1} - b\beta(t_p^1-s_1)^{\beta-1}\right) - \\
&\quad c\left(a(t_p^1-s_1)^{\alpha} - b(t_p^1-s_1)^{\beta}\right) \Big\} \\
&= 0
\end{aligned}$$

$$\Rightarrow t_p^1 \text{ varies with } s_1, V_0, V_1$$

Summed peak is a function of separation and energies (voltage). Too complicated to invert in closed form



Modeling the peak time with empirical fit

$$t_p^1(s_1, v_0, v_1; \mathbf{a}) = \frac{v_0 * t_p^0 + v_1 * (t_p^0 + s_1)}{v_0 + v_1} + e^{-a_1\left(\frac{v_0 - v_1}{v_0 + v_1}\right)^2}(e^{a_2 s_1^2} - 1)$$

Empirical model (trial and error). Parameterized by **a.** Data points are generated by simulating the instrument on a grid of (s, v0, v1).

Gauss-Newton minimization (a non-linear least-squares fit). Minimizes statistic *S*(**a**):

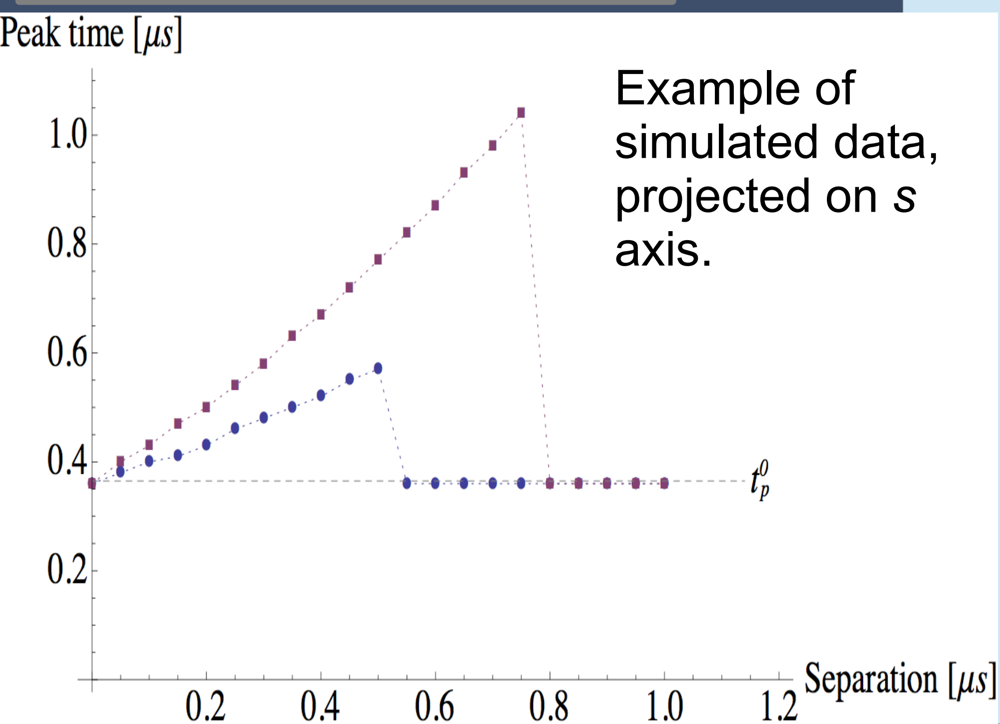
$$S(\mathbf{a}) = \sum_{i=1}^{N} [y_i - t_p^1(\mathbf{x}_i \ ; \ \mathbf{a})]^2$$

$$\mathbf{r} = \{y_i - t_p^1(\mathbf{x}_i)\}$$

$$\mathrm{J}_{ij} = \frac{\partial t_p^1(\mathbf{x}_i)}{\partial a_j}$$ Jacobian

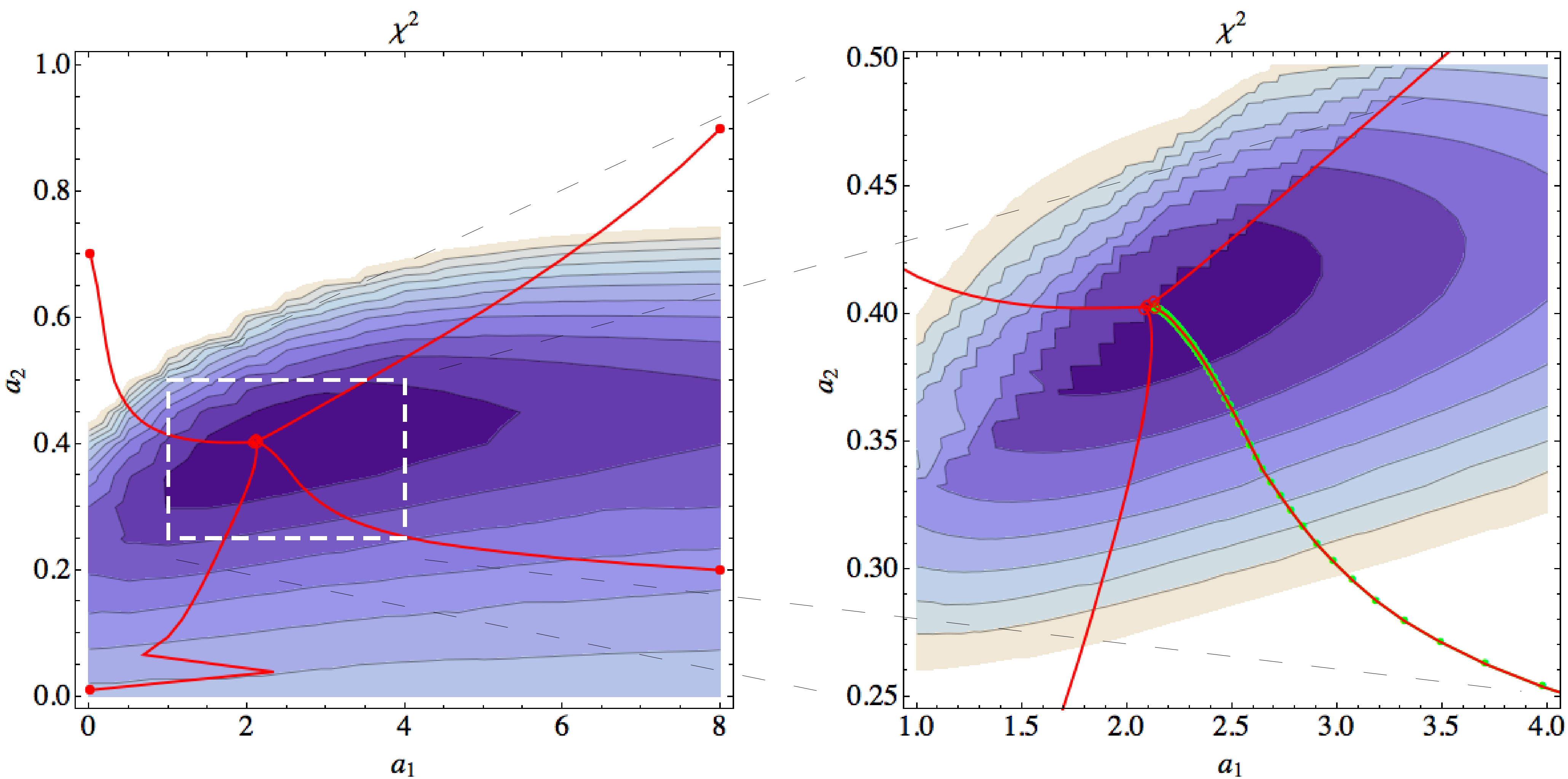
$$(\mathbf{J}^\mathsf{T}\mathbf{J})\mathbf{v} = \mathbf{J}^\mathsf{T}\mathbf{r}$$ Transpose makes this a simple 2x2 system

$$\mathbf{a}^{(k+1)} = \mathbf{a}^{(k)} + \epsilon \mathbf{v}^{(k)}$$



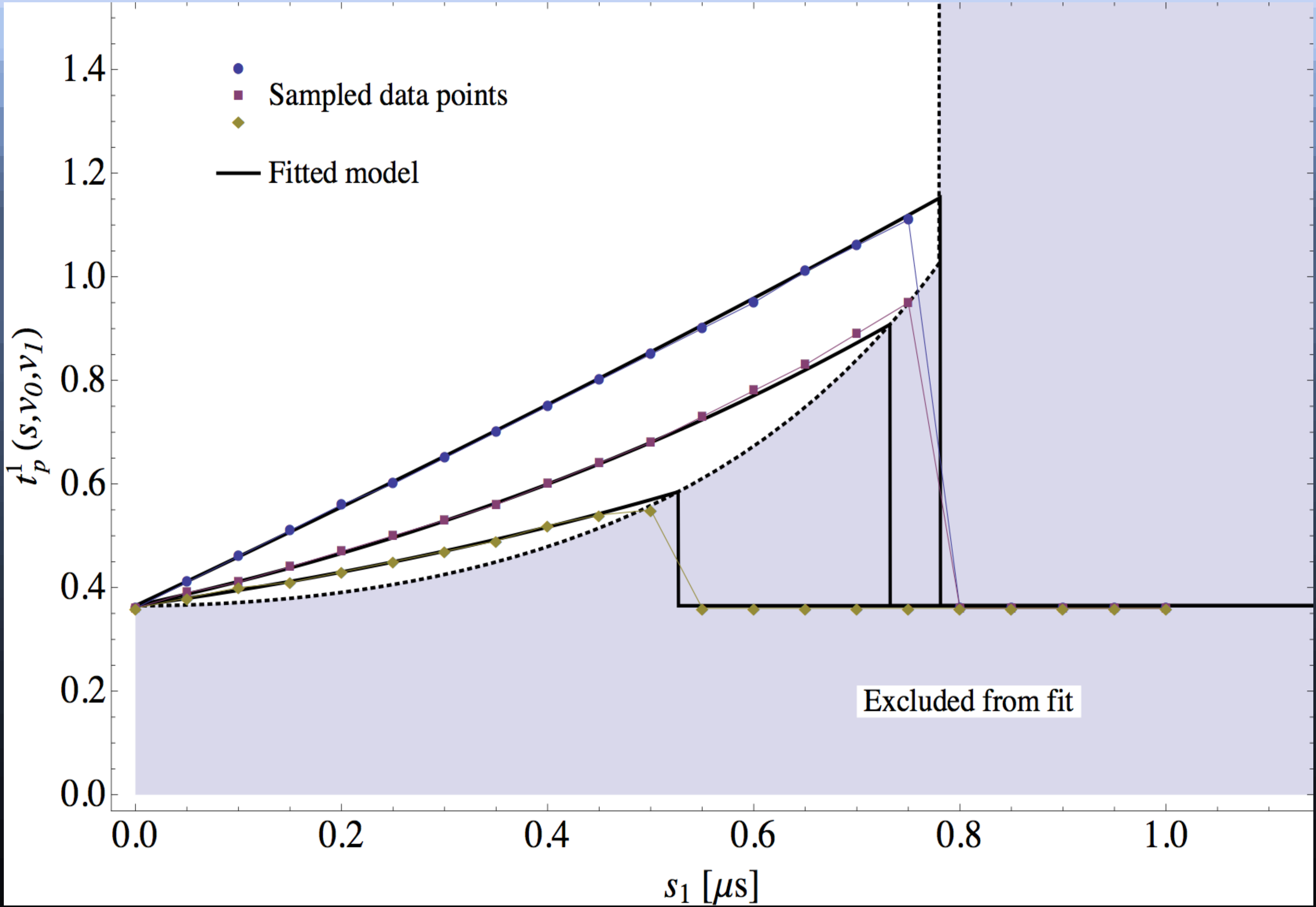
Example of simulated data, projected on *s* axis.

Results (minimum and chi-sq map)



Stable global minimum regardless of initial guess. Oversampling on the right.

Results (fit within valid data volume)



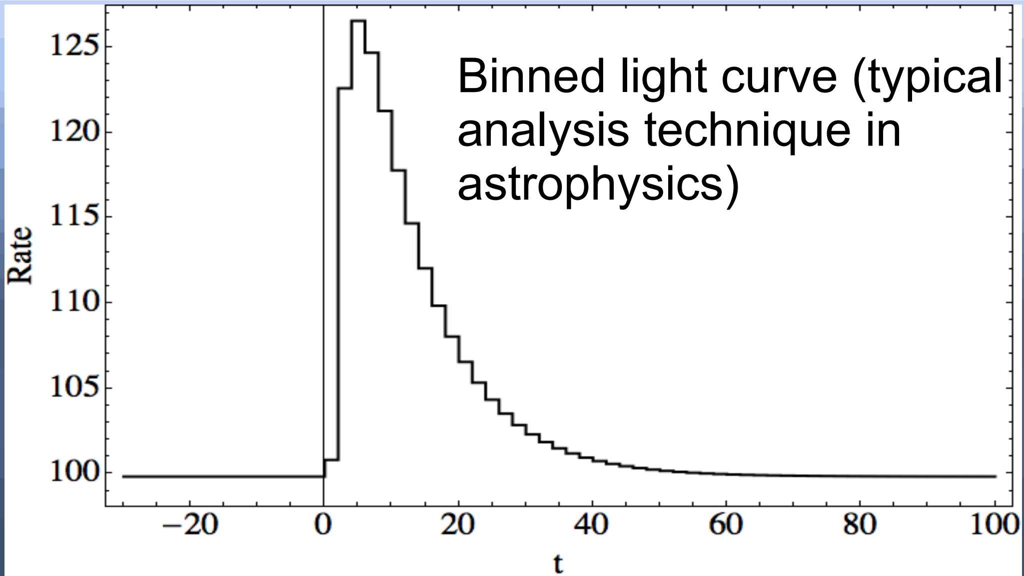
Part 2: Variable Poisson process by thinning

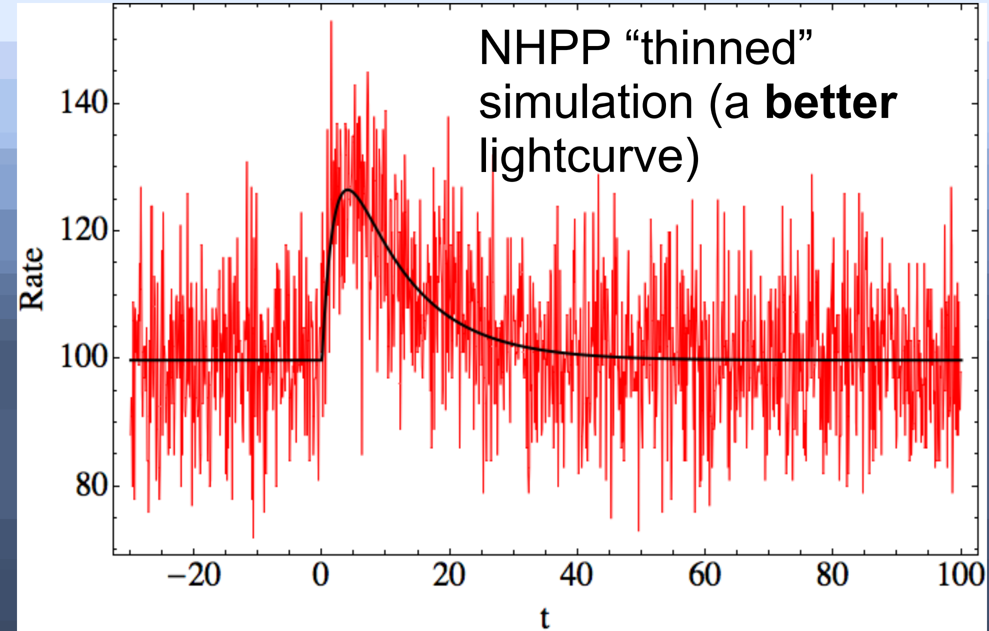
In the "thinning" technique, events are generate from a reference process, which can be a constant rate. This means events are exponentially distributed in time and easy to simulate.

They are sampled using an "importance" scheme, and kept if the following is true:

$$\frac{\lambda(t_i)}{\Lambda} \geq \mathrm{R}[0,1]$$

Statistics





Variation from the average rate is the same if we assume the binned lightcurve and treat counts / bin as Poisson deviates (LEFT), and if do the NHPP simulation (RIGHT). An event list can be approximated from the lefthand case by decreasing the binsize to smaller and smaller values