
Part 2: Simulation of a stochastic Poisson process (non-homogenous)
Background: Pulse-pileup in the Gamma-ray Burst Monitor

Pulse-height distortion

Pulse-height measurement to determine photon energy
PPU distortions (continued)

\[
\lambda = 1000 \times 10^3 \text{ cps} \\
\lambda \tau \approx 2.6
\]
0th order peak

\[ f(t) = K(c_1 \ast t^\alpha - c_2 \ast t^\beta) e^{-\gamma t} \]

Single Pulse peak “zeroth order”:

\[ (t_p^0) \equiv \text{peak time of a single pulse at } \tau = 0 \]

\[
\frac{d}{dt} f(t) \bigg|_{t_p^0} = f'(t_p^0)
\]

\[
= \frac{d}{dt} \left[ (a t^\alpha - b t^\beta) e^{-c t} \right] \bigg|_{t_p^0}
\]

\[
= (a \alpha t^{\alpha - 1} - b \beta t^{\beta - 1}) e^{-c t} - c (a t^\alpha - b t^\beta) e^{-c t} \bigg|_{t_p^0}
\]

\[= 0 \]

\[\Rightarrow a\{\alpha (t_p^0)^{\alpha - 1} - c (t_p^0)^\alpha\} + b\{c (t_p^0)^\beta - \beta (t_p^0)^{\beta - 1}\} = 0 \]

\[\Rightarrow t_p^0 \approx 0.388 \mu s \]
\( (t_p^1) \equiv \text{peak time of two pulses } s_1 = t_1 - t_0 \)

\[
\frac{d}{dt} \left[ V_0 f(t) + V_1 f(t - s_1) \right]_{t_p^1} = V_0 f'(t_p^1) + V_1 f'(t_p^1 - s_1)
\]

\[
= V_0 e^{-ct_p^1} \left\{ \left( a \alpha (t_p^1)^{\alpha - 1} - b \beta (t_p^1)^{\beta - 1} \right) - c \left( a (t_p^1)^\alpha - b (t_p^1)^\beta \right) \right\} + V_1 e^{-c(t_p^1 - s_1)} \left\{ \left( a \alpha (t_p^1 - s_1)^{\alpha - 1} - b \beta (t_p^1 - s_1)^{\beta - 1} \right) - c \left( a (t_p^1 - s_1)^\alpha - b (t_p^1 - s_1)^\beta \right) \right\}
\]

\[
= 0
\]

\Rightarrow t_p^1 \text{ varies with } s_1, V_0, V_1
Modeling the peak time with empirical fit

Empirical model (trial and error). Parameterized by $a$. Data points are generated by simulating the instrument on a grid of $(s, v_0, v_1)$.

Gauss-Newton minimization (a non-linear least-squares fit). Minimizes statistic $S(a)$:

$$S(a) = \sum_{i=1}^{N} [y_i - t_p^1(x_i ; a)]^2$$

$$r = \{y_i - t_p^1(x_i)\}$$

$$J_{ij} = \frac{\partial t_p^1(x_i)}{\partial a_j}$$

Transpose makes this a simple 2x2 system

$$J^T J \mathbf{v} = J^T r$$

$$a^{(k+1)} = a^{(k)} + \epsilon \mathbf{v}^{(k)}$$
Results (minimum and chi-sq map)

Stable global minimum regardless of initial guess. Oversampling on the right.
Results (fit within valid data volume)
In the “thinning” technique, events are generated from a reference process, which can be a constant rate. This means events are exponentially distributed in time and easy to simulate.

They are sampled using an “importance” scheme, and kept if the following is true:

\[
\frac{\lambda(t_i)}{\Lambda} \geq R[0, 1]
\]
Statistics

Binned light curve (typical analysis technique in astrophysics)

NHPP “thinned” simulation (a better lightcurve)

Variation from the average rate is the same if we assume the binned lightcurve and treat counts / bin as Poisson deviates (LEFT), and if do the NHPP simulation (RIGHT). An event list can be approximated from the lefthand case by decreasing the binsize to smaller and smaller values.