3

Programming with MATLAB

CHAPTER OBJECTIVES

The primary objective of this chapter is to learn how to write M-file programs to implement numerical methods. Specific objectives and topics covered are

- Learning how to create well-documented M-files in the edit window and invoke them from the command window.
- Understanding how script and function files differ.
- Understanding how to incorporate help comments in functions.
- Knowing how to set up M-files so that they interactively prompt users for information and display results in the command window.
- Understanding the role of subfunctions and how they are accessed.
- Knowing how to create and retrieve data files.
- Learning how to write clear and well-documented M-files by employing structured programming constructs to implement logic and repetition.
- Recognizing the difference between `if...elseif` and `switch` constructs.
- Recognizing the difference between `for...end` and `while` structures.
- Knowing how to animate MATLAB plots.
- Understanding what is meant by vectorization and why it is beneficial.
- Understanding how anonymous functions can be employed to pass functions to function function M-files.

YOU'VE GOT A PROBLEM

In Chap. 1, we used a force balance to develop a mathematical model to predict the fall velocity of a bungee jumper. This model took the form of the following differential equation:

$$\frac{dv}{dt} = g - \frac{c_d}{m} v|v|$$

We also learned that a numerical solution of this equation could be obtained with Euler's method:

$$v_{i+1} = v_i + \frac{dv_i}{dt}\Delta t$$

This equation can be implemented repeatedly to compute velocity as a function of time. However, to obtain good accuracy, many small steps must be taken. This would be extremely laborious and time consuming to implement by hand. However, with the aid of MATLAB, such calculations can be performed easily.

So our problem now is to figure out how to do this. This chapter will introduce you to how MATLAB M-files can be used to obtain such solutions.

3.1 M-FILES

The most common way to operate MATLAB is by entering commands one at a time in the command window. M-files provide an alternative way of performing operations that greatly expand MATLAB's problem-solving capabilities. An *M-file* consists of a series of statements that can be run all at once. Note that the nomenclature "M-file" comes from the fact that such files are stored with a `.m` extension. M-files come in two flavors: script files and function files.

3.1.1 Script Files

A script file is merely a series of MATLAB commands that are saved on a file. They are useful for retaining a series of commands that you want to execute on more than one occasion. The script can be executed by typing the file name in the command window or by invoking the menu selections in the edit window: **Debug, Run.**

EXAMPLE 3.1 Script File

Problem Statement. Develop a script file to compute the velocity of the free-falling bungee jumper for the case where the initial velocity is zero.

Solution. Open the editor with the menu selection: **File, New, M-file.** Type in the following statements to compute the velocity of the free-falling bungee jumper at a specific time [recall Eq. (1.9)]:

```
g = 9.81; m = 68.1; t = 12; cd = 0.25;
v = sqrt(g * m / cd) * tanh(sqrt(g * cd / m) * t)
```

Save the file as `scriptdemo.m`. Return to the command window and type

```
>>scriptdemo
```

The result will be displayed as

```
v =
   50.6175
```

Thus, the script executes just as if you had typed each of its lines in the command window.

As a final step, determine the value of `g` by typing

```
>> g

g =
    9.8100
```

So you can see that even though `g` was defined within the script, it retains its value back in the command workspace. As we will see in the following section, this is an important distinction between scripts and functions.

3.1.2 Function Files

Function files are M-files that start with the word `function`. In contrast to script files, they can accept input arguments and return outputs. Hence they are analogous to user-defined functions in programming languages such as Fortran, Visual Basic or *C*.

The syntax for the function file can be represented generally as

```
function outvar = funcname(arglist)
% helpcomments
statements
outvar = value;
```

where *outvar* = the name of the output variable, *funcname* = the function's name, *arglist* = the function's argument list (i.e., comma-delimited values that are passed into the function), *helpcomments* = text that provides the user with information regarding the function (these can be invoked by typing `Help` *funcname* in the command window), and *statements* = MATLAB statements that compute the *value* that is assigned to *outvar*.

Beyond its role in describing the function, the first line of the *helpcomments*, called the *H1 line,* is the line that is searched by the `lookfor` command (recall Sec. 2.6). Thus, you should include key descriptive words related to the file on this line.

The M-file should be saved as *funcname*`.m`. The function can then be run by typing *funcname* in the command window as illustrated in the following example. Note that even though MATLAB is case-sensitive, your computer's operating system may not be. Whereas MATLAB would treat function names like `freefall` and `FreeFall` as two different variables, your operating system might not.

EXAMPLE 3.2 Function File

Problem Statement. As in Example 3.1, compute the velocity of the free-falling bungee jumper but now use a function file for the task.

Solution. Type the following statements in the file editor:

```
function v = freefall(t, m, cd)
% freefall: bungee velocity with second-order drag
% v=freefall(t,m,cd) computes the free-fall velocity
%                    of an object with second-order drag
% input:
```

```
%   t = time (s)
%   m = mass (kg)
%   cd = second-order drag coefficient (kg/m)
% output:
%   v = downward velocity (m/s)

g = 9.81;     % acceleration of gravity
v = sqrt(g * m / cd)*tanh(sqrt(g * cd / m) * t);
```

Save the file as `freefall.m`. To invoke the function, return to the command window and type in

```
>> freefall(12,68.1,0.25)
```

The result will be displayed as

```
ans =
   50.6175
```

One advantage of a function M-file is that it can be invoked repeatedly for different argument values. Suppose that you wanted to compute the velocity of a 100-kg jumper after 8 s:

```
>> freefall(8,100,0.25)

ans =
   53.1878
```

To invoke the help comments type

```
>> help freefall
```

which results in the comments being displayed

```
freefall: bungee velocity with second-order drag
  v=freefall(t,m,cd) computes the free-fall velocity
                     of an object with second-order drag
input:
  t = time (s)
  m = mass (kg)
  cd = second-order drag coefficient (kg/m)
output:
  v = downward velocity (m/s)
```

If at a later date, you forgot the name of this function, but remembered that it involved bungee jumping, you could enter

```
>> lookfor bungee
```

and the following information would be displayed

```
freefall.m - bungee velocity with second-order drag
```

Note that, at the end of the previous example, if we had typed

```
>> g
```

the following message would have been displayed

```
??? Undefined function or variable 'g'.
```

So even though `g` had a value of 9.81 within the M-file, it would not have a value in the command workspace. As noted previously at the end of Example 3.1, this is an important distinction between functions and scripts. The variables within a function are said to be *local* and are erased after the function is executed. In contrast, the variables in a script retain their existence after the script is executed.

Function M-files can return more than one result. In such cases, the variables containing the results are comma-delimited and enclosed in brackets. For example, the following function, `stats.m`, computes the mean and the standard deviation of a vector:

```
function [mean, stdev] = stats(x)
n = length(x);
mean = sum(x)/n;
stdev = sqrt(sum((x-mean).^2/(n-1)));
```

Here is an example of how it can be applied:

```
>> y = [8 5 10 12 6 7.5 4];
>> [m,s] = stats(y)

m =
   7.5000

s =
   2.8137
```

Although we will also make use of script M-files, function M-files will be our primary programming tool for the remainder of this book. Hence, we will often refer to function M-files as simply M-files.

3.1.3 Subfunctions

Functions can call other functions. Although such functions can exist as separate M-files, they may also be contained in a single M-file. For example, the M-file in Example 3.2 (without comments) could have been split into two functions and saved as a single M-file[1]:

```
function v = freefallsubfunc(t, m, cd)
v = vel(t, m, cd);
end

function v = vel(t, m, cd)
g = 9.81;
v = sqrt(g * m / cd)*tanh(sqrt(g * cd / m) * t);
end
```

[1] Note that although `end` statements are not used to terminate single-function M-files, they are included when subfunctions are involved to demarcate the boundaries between the main function and the subfunctions.

This M-file would be saved as `freefallsubfunc.m`. In such cases, the first function is called the *main* or *primary function*. It is the only function that is accessible to the command window and other functions and scripts. All the other functions (in this case, `vel`) are referred to as *subfunctions*.

A subfunction is only accessible to the main function and other subfunctions within the M-file in which it resides. If we run `freefallsubfunc` from the command window, the result is identical to Example 3.2:

```
>> freefallsubfunc(12,68.1,0.25)

ans =
   50.6175
```

However, if we attempt to run the subfunction `vel`, an error message occurs:

```
>> vel(12,68.1,.25)
??? Undefined function or method 'vel' for input arguments
of type 'double'.
```

3.2 INPUT-OUTPUT

As in Section 3.1, information is passed into the function via the argument list and is output via the function's name. Two other functions provide ways to enter and display information directly using the command window.

The `input` Function. This function allows you to prompt the user for values directly from the command window. Its syntax is

```
n = input('promptstring')
```

The function displays the *promptstring*, waits for keyboard input, and then returns the value from the keyboard. For example,

```
m = input('Mass (kg): ')
```

When this line is executed, the user is prompted with the message

```
Mass (kg):
```

If the user enters a value, it would then be assigned to the variable `m`.

The `input` function can also return user input as a string. To do this, an `'s'` is appended to the function's argument list. For example,

```
name = input('Enter your name: ','s')
```

The `disp` Function. This function provides a handy way to display a value. Its syntax is

```
disp(value)
```

where *value* = the value you would like to display. It can be a numeric constant or variable, or a string message enclosed in hyphens. Its application is illustrated in the following example.

EXAMPLE 3.3 An Interactive M-File Function

Problem Statement. As in Example 3.2, compute the velocity of the free-falling bungee jumper, but now use the `input` and `disp` functions for input/output.

Solution. Type the following statements in the file editor:

```
function freefalli
% freefalli: interactive bungee velocity
%   freefalli interactive computation of the
%             free-fall velocity of an object
%             with second-order drag.
g = 9.81;     % acceleration of gravity
m = input('Mass (kg): ');
cd = input('Drag coefficient (kg/m): ');
t = input('Time (s): ');
disp(' ')
disp('Velocity (m/s):')
disp(sqrt(g * m / cd)*tanh(sqrt(g * cd / m) * t))
```

Save the file as `freefalli.m`. To invoke the function, return to the command window and type

```
>> freefalli
Mass (kg): 68.1
Drag coefficient (kg/m): 0.25
Time (s): 12

Velocity (m/s):
   50.6175
```

The `fprintf` Function. This function provides additional control over the display of information. A simple representation of its syntax is

```
fprintf('format', x, ...)
```

where *`format`* is a string specifying how you want the value of the variable *`x`* to be displayed. The operation of this function is best illustrated by examples.

A simple example would be to display a value along with a message. For instance, suppose that the variable `velocity` has a value of 50.6175. To display the value using eight digits with four digits to the right of the decimal point along with a message, the statement along with the resulting output would be

```
>> fprintf('The velocity is %8.4f m/s\n', velocity)
The velocity is 50.6175 m/s
```

This example should make it clear how the format string works. MATLAB starts at the left end of the string and displays the labels until it detects one of the symbols: `%` or `\`. In our example, it first encounters a `%` and recognizes that the following text is a format code. As in Table 3.1, the *format codes* allow you to specify whether numeric values are

TABLE 3.1 Commonly used format and control codes employed with the `fprintf` function.

Format Code	Description
`%d`	Integer format
`%e`	Scientific format with lowercase e
`%E`	Scientific format with uppercase E
`%f`	Decimal format
`%g`	The more compact of `%e` or `%f`
Control Code	**Description**
`\n`	Start new line
`\t`	Tab

displayed in integer, decimal, or scientific format. After displaying the value of `velocity`, MATLAB continues displaying the character information (in our case the units: `m/s`) until it detects the symbol `\`. This tells MATLAB that the following text is a control code. As in Table 3.1, the *control codes* provide a means to perform actions such as skipping to the next line. If we had omitted the code `\n` in the previous example, the command prompt would appear at the end of the label `m/s` rather than on the next line as would typically be desired.

The `fprintf` function can also be used to display several values per line with different formats. For example,

```
>> fprintf('%5d %10.3f %8.5e\n',100,2*pi,pi);
  100        6.283 3.14159e+000
```

It can also be used to display vectors and matrices. Here is an M-file that enters two sets of values as vectors. These vectors are then combined into a matrix, which is then displayed as a table with headings:

```
function fprintfdemo
x = [1 2 3 4 5];
y = [20.4 12.6 17.8 88.7 120.4];
z = [x;y];
fprintf('     x       y\n');
fprintf('%5d %10.3f\n',z);
```

The result of running this M-file is

```
>> fprintfdemo
    x       y
    1    20.400
    2    12.600
    3    17.800
    4    88.700
    5   120.400
```

3.2.1 Creating and Accessing Files

MATLAB has the capability to both read and write data files. The simplest approach involves a special type of binary file, called a *MAT-file,* which is expressly designed for implementation within MATLAB. Such files are created and accessed with the `save` and `load` commands.

The `save` command can be used to generate a MAT-file holding either the entire workspace or a few selected variables. A simple representation of its syntax is

```
save filename var1 var2 ... varn
```

This command creates a MAT-file named *filename*`.mat` that holds the variables *var1* through *varn*. If the variables are omitted, all the workspace variables are saved. The `load` command can subsequently be used to retrieve the file:

```
load filename var1 var2 ... varn
```

which retrieves the variables *var1* through *varn* from *filename*`.mat`. As was the case with `save`, if the variables are omitted, all the variables are retrieved.

For example, suppose that you use Eq. (1.9) to generate velocities for a set of drag coefficients:

```
>> g=9.81;m=80;t=5;
>> cd=[.25 .267 .245 .28 .273]';
>> v=sqrt(g*m ./cd).*tanh(sqrt(g*cd/m)*t);
```

You can then create a file holding the values of the drag coefficients and the velocities with

```
>> save veldrag v cd
```

To illustrate how the values can be retrieved at a later time, remove all variables from the workspace with the `clear` command,

```
>> clear
```

At this point, if you tried to display the velocities you would get the result:

```
>> v
??? Undefined function or variable 'v'.
```

However, you can recover them by entering

```
>> load veldrag
```

Now, the velocities are available as can be verified by typing

```
>> who

Your variables are:
cd  v
```

Although MAT-files are quite useful when working exclusively within the MATLAB environment, a somewhat different approach is required when interfacing MATLAB with other programs. In such cases, a simple approach is to create text files written in ASCII format.

ASCII files can be generated in MATLAB by appending `-ascii` to the `save` command. In contrast to MAT-files where you might want to save the entire workspace, you would typically save a single rectangular matrix of values. For example,

```
>> A=[5 7 9 2;3 6 3 9];
>> save simpmatrix.txt -ascii
```

In this case, the `save` command stores the values in A in 8-digit ASCII form. If you want to store the numbers in double precision, just append `-ascii -double`. In either case, the file can be accessed by other programs such as spreadsheets or word processors. For example, if you open this file with a text editor, you will see

```
5.0000000e+000  7.0000000e+000  9.0000000e+000  2.0000000e+000
3.0000000e+000  6.0000000e+000  3.0000000e+000  9.0000000e+000
```

Alternatively, you can read the values back into MATLAB with the `load` command,

```
>> load simpmatrix.txt
```

Because `simpmatrix.txt` is not a MAT-file, MATLAB creates a double precision array named after the *`filename`*:

```
>> simpmatrix
simpmatrix =
     5     7     9     2
     3     6     3     9
```

Alternatively, you could use the `load` command as a function and assign its values to a variable as in

```
>> A = load('simpmatrix.txt')
```

The foregoing material covers but a small portion of MATLAB's file management capabilities. For example, a handy import wizard can be invoked with the menu selections: **File, Import Data.** As an exercise, you can demonstrate the import wizards convenience by using it to open `simpmatrix.txt`. In addition, you can always consult `help` to learn more about this and other features.

3.3 STRUCTURED PROGRAMMING

The simplest of all M-files perform instructions sequentially. That is, the program statements are executed line by line starting at the top of the function and moving down to the end. Because a strict sequence is highly limiting, all computer languages include statements allowing programs to take nonsequential paths. These can be classified as

- *Decisions* (or Selection). The branching of flow based on a decision.
- *Loops* (or Repetition). The looping of flow to allow statements to be repeated.

3.3.1 Decisions

The `if` Structure. This structure allows you to execute a set of statements if a logical condition is true. Its general syntax is

```
if condition
  statements
end
```

where *condition* is a logical expression that is either true or false. For example, here is a simple M-file to evaluate whether a grade is passing:

```
function grader(grade)
% grader(grade):
%   determines whether grade is passing
% input:
%   grade = numerical value of grade (0-100)
% output:
%   displayed message
if grade >= 60
  disp('passing grade')
end
```

The following illustrates the result

```
>> grader(95.6)

passing grade
```

For cases where only one statement is executed, it is often convenient to implement the `if` structure as a single line,

```
if grade > 60, disp('passing grade'), end
```

This structure is called a *single-line if*. For cases where more than one statement is implemented, the multiline if structure is usually preferable because it is easier to read.

Error Function. A nice example of the utility of a single-line if is to employ it for rudimentary error trapping. This involves using the `error` function which has the syntax,

```
error(msg)
```

When this function is encountered, it displays the text message *msg*, indicates where the error occurred, and causes the M-file to terminate and return to the command window.

An example of its use would be where we might want to terminate an M-file to avoid a division by zero. The following M-file illustrates how this could be done:

```
function f = errortest(x)
if x == 0, error('zero value encountered'), end
f = 1/x;
```

If a nonzero argument is used, the division would be implemented successfully as in

```
>> errortest(10)

ans =
    0.1000
```

However, for a zero argument, the function would terminate prior to the division and the error message would be displayed in red typeface:

```
>> errortest(0)

??? Error using ==> errortest at 2
zero value encountered
```

TABLE 3.2 Summary of relational operators in MATLAB.

Example	Operator	Relationship
`x == 0`	`==`	Equal
`unit ~= 'm'`	`~=`	Not equal
`a < 0`	`<`	Less than
`s > t`	`>`	Greater than
`3.9 <= a/3`	`<=`	Less than or equal to
`r >= 0`	`>=`	Greater than or equal to

Logical Conditions. The simplest form of the *condition* is a single relational expression that compares two values as in

$$value_1 \ relation \ value_2$$

where the *values* can be constants, variables, or expressions and the *relation* is one of the relational operators listed in Table 3.2.

MATLAB also allows testing of more than one logical condition by employing logical operators. We will emphasize the following:

- `~` *(Not)*. Used to perform logical negation on an expression.

  ~expression

  If the *expression* is true, the result is false. Conversely, if the *expression* is false, the result is true.
- `&` *(And)*. Used to perform a logical conjunction on two expressions.

  $expression_1$ `&` $expression_2$

  If both *expressions* evaluate to true, the result is true. If either or both *expressions* evaluates to false, the result is false.
- `|` *(Or)*. Used to perform a logical disjunction on two expressions.

  $expression_1$ `|` $expression_2$

  If either or both *expressions* evaluate to true, the result is true.

Table 3.3 summarizes all possible outcomes for each of these operators. Just as for arithmetic operations, there is a priority order for evaluating logical operations. These

TABLE 3.3 A truth table summarizing the possible outcomes for logical operators employed in MATLAB. The order of priority of the operators is shown at the top of the table.

		Highest →		Lowest
x	y	~x	x & y	x | y
T	T	F	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	F

are from highest to lowest: ~, & and |. In choosing between operators of equal priority, MATLAB evaluates them from left to right. Finally, as with arithmetic operators, parentheses can be used to override the priority order.

Let's investigate how the computer employs the priorities to evaluate a logical expression. If `a = -1`, `b = 2`, `x = 1`, and `y = 'b'`, evaluate whether the following is true or false:

```
a * b > 0 & b == 2 & x > 7 | ~(y > 'd')
```

To make it easier to evaluate, substitute the values for the variables:

```
-1 * 2 > 0 & 2 == 2 & 1 > 7 | ~('b' > 'd')
```

The first thing that MATLAB does is to evaluate any mathematical expressions. In this example, there is only one: `-1 * 2`,

```
-2 > 0 & 2 == 2 & 1 > 7 | ~('b' > 'd')
```

Next, evaluate all the relational expressions

```
-2 > 0 & 2 == 2 & 1 > 7 | ~('b' > 'd')
   F   &   T   &   F   | ~     F
```

At this point, the logical operators are evaluated in priority order. Since the ~ has highest priority, the last expression (`~F`) is evaluated first to give

```
F & T & F | T
```

The & operator is evaluated next. Since there are two, the left-to-right rule is applied and the first expression (`F & T`) is evaluated:

```
F & F | T
```

The & again has highest priority

```
F | T
```

Finally, the | is evaluated as true. The entire process is depicted in Fig. 3.1.

The `if...else` Structure. This structure allows you to execute a set of statements if a logical condition is true and to execute a second set if the condition is false. Its general syntax is

```
if condition
  statements1
else
  statements2
end
```

The `if...elseif` Structure. It often happens that the false option of an `if...else` structure is another decision. This type of structure often occurs when we have more than two options for a particular problem setting. For such cases, a special form of decision structure, the `if...elseif` has been developed. It has the general syntax

```
if condition1
  statements1
elseif condition2
  statements2
```

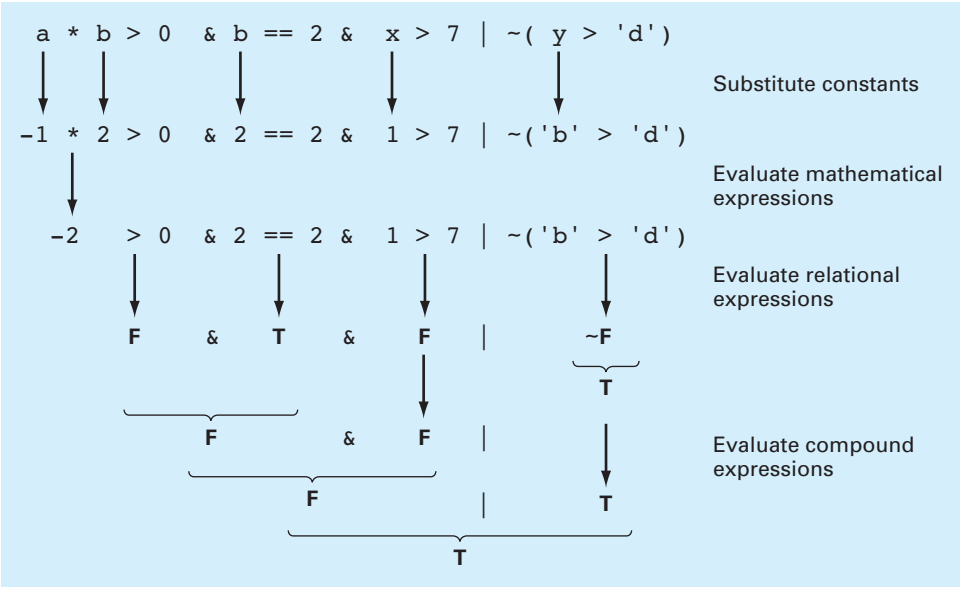


FIGURE 3.1
A step-by-step evaluation of a complex decision.

```
elseif condition3
  statements3
    .
    .
    .
else
  statementselse
end
```

EXAMPLE 3.4 `if` Structures

Problem Statement. For a scalar, the built-in MATLAB `sign` function returns the sign of its argument (−1, 0, 1). Here's a MATLAB session that illustrates how it works:

```
>> sign(25.6)
ans =
     1
>> sign(-0.776)
ans =
    -1
>> sign(0)
ans =
     0
```

Develop an M-file to perform the same function.

Solution. First, an `if` structure can be used to return `1` if the argument is positive:

```
function sgn = mysign(x)
% mysign(x) returns 1 if x is greater than zero.
if x > 0
  sgn = 1;
end
```

This function can be run as

```
>> mysign(25.6)

ans =
     1
```

Although the function handles positive numbers correctly, if it is run with a negative or zero argument, nothing is displayed. To partially remedy this shortcoming, an `if...else` structure can be used to display `-1` if the condition is false:

```
function sgn = mysign(x)
% mysign(x) returns 1 if x is greater than zero.
%                   -1 if x is less than or equal to zero.
if x > 0
  sgn = 1;
else
  sgn = -1;
end
```

This function can be run as

```
>> mysign(-0.776)

ans =
    -1
```

Although the positive and negative cases are now handled properly, `-1` is erroneously returned if a zero argument is used. An `if...elseif` structure can be used to incorporate this final case:

```
function sgn = mysign(x)
% mysign(x) returns 1 if x is greater than zero.
%                   -1 if x is less than zero.
%                   0 if x is equal to zero.
if x > 0
  sgn = 1;
elseif x < 0
  sgn = -1;
else
  sgn = 0;
end
```

The function now handles all possible cases. For example,

```
>> mysign(0)

ans =
     0
```

The `switch` Structure. The `switch` structure is similar in spirit to the `if...elseif` structure. However, rather than testing individual conditions, the branching is based on the value of a single test expression. Depending on its value, different blocks of code are implemented. In addition, an optional block is implemented if the expression takes on none of the prescribed values. It has the general syntax

```
switch testexpression
  case value1
    statements1
  case value2
    statements2
   .
   .
   .
  otherwise
    statementsotherwise
end
```

As an example, here is function that displays a message depending on the value of the string variable, `grade`.

```
grade = 'B';
switch grade
  case 'A'
    disp('Excellent')
  case 'B'
    disp('Good')
  case 'C'
    disp('Mediocre')
  case 'D'
    disp('Whoops')
  case 'F'
    disp('Would like fries with your order?')
  otherwise
    disp('Huh!')
end
```

When this code was executed, the message "Good" would be displayed.

Variable Argument List. MATLAB allows a variable number of arguments to be passed to a function. This feature can come in handy for incorporating default values into your functions. A *default value* is a number that is automatically assigned in the event that the user does not pass it to a function.

As an example, recall that earlier in this chapter, we developed a function `freefall`, which had three arguments:

```
v = freefall(t,m,cd)
```

Although a user would obviously need to specify the time and mass, they might not have a good idea of an appropriate drag coefficient. Therefore, it would be nice to have the program supply a value if they omitted it from the argument list.

MATLAB has a function called `nargin` that provides the number of input arguments supplied to a function by a user. It can be used in conjunction with decision structures like

the `if` or `switch` constructs to incorporate default values as well as error messages into your functions. The following code illustrates how this can be done for `freefall`:

```
function v = freefall2(t, m, cd)
% freefall2: bungee velocity with second-order drag
%   v=freefall2(t,m,cd) computes the free-fall velocity
%                       of an object with second-order drag.
% input:
%   t = time (s)
%   m = mass (kg)
%   cd = drag coefficient (default = 0.27 kg/m)
% output:
%   v = downward velocity (m/s)
switch nargin
  case 0
    error('Must enter time and mass')
  case 1
    error('Must enter mass')
  case 2
    cd = 0.27;
end
g = 9.81;     % acceleration of gravity
v = sqrt(g * m / cd)*tanh(sqrt(g * cd / m) * t);
```

Notice how we have used a `switch` structure to either display error messages or set the default, depending on the number of arguments passed by the user. Here is a command window session showing the results:

```
>> freefall2(12,68.1,0.25)

ans =
   50.6175

>> freefall2(12,68.1)

ans =
   48.8747

>> freefall2(12)

??? Error using ==> freefall2 at 15
Must enter mass

>> freefall2()

??? Error using ==> freefall2 at 13
Must enter time and mass
```

Note that `nargin` behaves a little differently when it is invoked in the command window. In the command window, it must include a string argument specifying the function and it returns the number of arguments in the function. For example,

```
>> nargin('freefall2')

ans =
     3
```

3.3.2 Loops

As the name implies, loops perform operations repetitively. There are two types of loops, depending on how the repetitions are terminated. A *for loop* ends after a specified number of repetitions. A *while loop* ends on the basis of a logical condition.

The `for...end` Structure. A `for` loop repeats statements a specific number of times. Its general syntax is

```
for index = start:step:finish
  statements
end
```

The `for` loop operates as follows. The *index* is a variable that is set at an initial value, *start*. The program then compares the *index* with a desired final value, *finish*. If the *index* is less than or equal to the *finish*, the program executes the *statements*. When it reaches the `end` line that marks the end of the loop, the *index* variable is increased by the *step* and the program loops back up to the `for` statement. The process continues until the *index* becomes greater than the *finish* value. At this point, the loop terminates as the program skips down to the line immediately following the `end` statement.

Note that if an increment of `1` is desired (as is often the case), the *step* can be dropped. For example,

```
for i = 1:5
  disp(i)
end
```

When this executes, MATLAB would display in succession, `1, 2, 3, 4, 5`. In other words, the default *step* is `1`.

The size of the *step* can be changed from the default of `1` to any other numeric value. It does not have to be an integer, nor does it have to be positive. For example, step sizes of `0.2`, `-1`, or `-5`, are all acceptable.

If a negative *step* is used, the loop will "countdown" in reverse. For such cases, the loop's logic is reversed. Thus, the *finish* is less than the *start* and the loop terminates when the `index` is less than the *finish*. For example,

```
for j = 10:-1:1
  disp(j)
end
```

When this executes, MATLAB would display the classic "countdown" sequence: `10, 9, 8, 7, 6, 5, 4, 3, 2, 1`.

EXAMPLE 3.5 Using a `for` Loop to Compute the Factorial

Problem Statement. Develop an M-file to compute the factorial.[2]

$$0! = 1$$
$$1! = 1$$
$$2! = 1 \times 2 = 2$$

[2] Note that MATLAB has a built-in function `factorial` that performs this computation.

$3! = 1 \times 2 \times 3 = 6$
$4! = 1 \times 2 \times 3 \times 4 = 24$
$5! = 1 \times 2 \times 3 \times 4 \times 5 = 120$
$\vdots$

Solution. A simple function to implement this calculation can be developed as

```
function fout = factor(n)
% factor(n):
%   Computes the product of all the integers from 1 to n.
x = 1;
for i = 1:n
  x = x * i;
end
fout = x;
end
```

which can be run as

```
>> factor(5)

ans =
   120
```

This loop will execute 5 times (from 1 to 5). At the end of the process, `x` will hold a value of 5! (meaning 5 factorial or $1 \times 2 \times 3 \times 4 \times 5 = 120$).

Notice what happens if $n = 0$. For this case, the `for` loop would not execute, and we would get the desired result, $0! = 1$.

Vectorization. The `for` loop is easy to implement and understand. However, for MATLAB, it is not necessarily the most efficient means to repeat statements a specific number of times. Because of MATLAB's ability to operate directly on arrays, *vectorization* provides a much more efficient option. For example, the following `for` loop structure:

```
i = 0;
for t = 0:0.02:50
  i = i + 1;
  y(i) = cos(t);
end
```

can be represented in vectorized form as

```
t = 0:0.02:50;
y = cos(t);
```

It should be noted that for more complex code, it may not be obvious how to vectorize the code. That said, wherever possible, vectorization is recommended.

Preallocation of Memory. MATLAB automatically increases the size of arrays every time you add a new element. This can become time consuming when you perform actions such as adding new values one at a time within a loop. For example, here is some code that

sets value of elements of `y` depending on whether or not values of `t` are greater than one:

```
t = 0:.01:5;
for i = 1:length(t)
  if t(i)>1
    y(i) = 1/t(i);
  else
    y(i) = 1;
  end
end
```

For this case, MATLAB must resize `y` every time a new value is determined. The following code preallocates the proper amount of memory by using a vectorized statement to assign ones to `y` prior to entering the loop.

```
t = 0:.01:5;
y = ones(size(t));
for i = 1:length(t)
  if t(i)>1
    y(i) = 1/t(i);
  end
end
```

Thus, the array is only sized once. In addition, preallocation helps reduce memory fragmentation, which also enhances efficiency.

The `while` Structure. A `while` loop repeats as long as a logical condition is true. Its general syntax is

```
while condition
  statements
end
```

The *`statements`* between the `while` and the `end` are repeated as long as the *`condition`* is true. A simple example is

```
x = 8
while x > 0
  x = x - 3;
  disp(x)
end
```

When this code is run, the result is

```
x =
     8
     5
     2
    -1
```

The `while...break` Structure. Although the `while` structure is extremely useful, the fact that it always exits at the beginning of the structure on a false result is somewhat constraining. For this reason, languages such as Fortran 90 and Visual Basic have special structures that allow loop termination on a true condition anywhere in the loop. Although such structures are currently not available in MATLAB, their functionality can be mimicked

by a special version of the `while` loop. The syntax of this version, called a *`while...break` structure,* can be written as

```
while (1)
  statements
  if condition, break, end
  statements
end
```

where `break` terminates execution of the loop. Thus, a single line `if` is used to exit the loop if the condition tests true. Note that as shown, the `break` can be placed in the middle of the loop (i.e., with statements before and after it). Such a structure is called a *midtest loop.*

If the problem required it, we could place the `break` at the very beginning to create a *pretest loop.* An example is

```
while (1)
  If x < 0, break, end
  x = x - 5;
end
```

Notice how `5` is subtracted from `x` on each iteration. This represents a mechanism so that the loop eventually terminates. Every decision loop must have such a mechanism. Otherwise it would become a so-called *infinite loop* that would never stop.

Alternatively, we could also place the `if...break` statement at the very end and create a *posttest loop,*

```
while (1)
  x = x - 5;
  if x < 0, break, end
end
```

It should be clear that, in fact, all three structures are really the same. That is, depending on where we put the exit (beginning, middle, or end) dictates whether we have a pre-, mid- or posttest. It is this simplicity that led the computer scientists who developed Fortran 90 and Visual Basic to favor this structure over other forms of the decision loop such as the conventional `while` structure.

The `pause` Command. There are often times when you might want a program to temporarily halt. The command `pause` causes a procedure to stop and wait until any key is hit. A nice example involves creating a sequence of plots that a user might want to leisurely peruse before moving on to the next. The following code employs a `for` loop to create a sequence of interesting plots that can be viewed in this manner:

```
for n = 3:10
  mesh(magic(n))
  pause
end
```

The `pause` can also be formulated as `pause(n)`, in which case the procedure will halt for `n` seconds. This feature can be demonstrated by implementing it in conjunction with several other useful MATLAB functions. The `beep` command causes the computer to emit

a beep sound. Two other functions, `tic` and `toc`, work together to measure elapsed time. The `tic` command saves the current time that `toc` later employs to display the elapsed time. The following code then confirms that `pause(n)` works as advertised complete with sound effects:

```
tic
beep
pause(5)
beep
toc
```

When this code is run, the computer will beep. Five seconds later it will beep again and display the following message:

```
Elapsed time is 5.006306 seconds.
```

By the way, if you ever have the urge to use the command `pause(inf)`, MATLAB will go into an infinite loop. In such cases, you can return to the command prompt by typing **Ctrl+c** or **Ctrl+Break.**

Although the foregoing examples might seem a tad frivolous, the commands can be quite useful. For instance, `tic` and `toc` can be employed to identify the parts of an algorithm that consume the most execution time. Further, the **Ctrl+c** or **Ctrl+Break** key combinations come in real handy in the event that you inadvertently create an infinite loop in one of your M-files.

3.3.3 Animation

There are two simple ways to animate a plot in MATLAB. First, if the computations are sufficiently quick, the standard `plot` function can be employed in a way that the animation can appear smooth. Here is a code fragment that indicates how a `for` loop and standard plotting functions can be employed to animate a plot,

```
% create animation with standard plot functions
for j=1:n
  plot commands
end
```

Thus, because we do not include `hold on`, the plot will refresh on each loop iteration. Through judicious use of axis commands, this can result in a smoothly changing image.

Second, there are special functions, `getframe` and `movie`, that allow you to capture a sequence of plots and then play them back. As the name implies, the `getframe` function captures a snapshot (*pixmap*) of the current axes or figure. It is usually used in a `for` loop to assemble an array of movie frames for later playback with the `movie` function, which has the following syntax:

```
movie(m,n,fps)
```

where *m* = the vector or matrix holding the sequence of frames constituting the movie, *n* = an optional variable specifying how many times the movie is to be repeated (if it is omitted, the movie plays once), and *fps* = an optional variable that specifies the movie's *frame rate* (if it is omitted, the default is 12 frames per second). Here is a code

fragment that indicates how a `for` loop along with the two functions can be employed to create a movie,

```
% create animation with standard plot functions
for j=1:n
  plot commands
  M(j) = getframe;
end
movie(M)
```

Each time the loop executes, the *plot commands* create an updated version of a plot, which is then stored in the vector `M`. After the loop terminates, the `n` images are then played back by `movie`.

EXAMPLE 3.6 Animation of Projectile Motion

Problem Statement. In the absence of air resistance, the Cartesian coordinates of a projectile launched with an initial velocity (v_0) and angle (θ_0) can be computed with

$$x = v_0 \cos(\theta_0)t$$
$$y = v_0 \sin(\theta_0)t - 0.5gt^2$$

where $g = 9.81$ m/s^2. Develop a script to generate an animated plot of the projectile's trajectory given that $v_0 = 5$ m/s and $\theta_0 = 45°$.

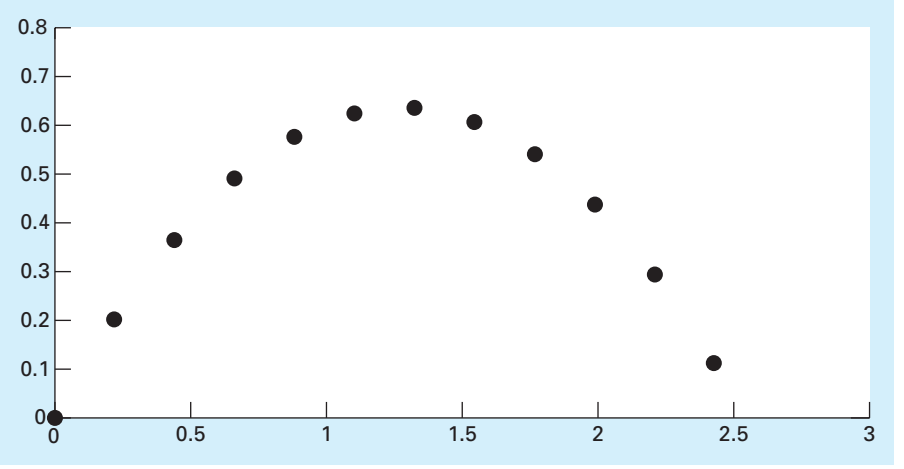
Solution. A script to generate the animation can be written as

```
clc,clf,clear
g=9.81; theta0=45*pi/180; v0=5;
t(1)=0;x=0;y=0;
plot(x,y,'o','MarkerFaceColor','b','MarkerSize',8)
axis([0 3 0 0.8])
M(1)=getframe;
dt=1/128;
for j = 2:1000
  t(j)=t(j-1)+dt;
  x=v0*cos(theta0)*t(j);
  y=v0*sin(theta0)*t(j)-0.5*g*t(j)^2;
  plot(x,y,'o','MarkerFaceColor','b','MarkerSize',8)
  axis([0 3 0 0.8])
  M(j)=getframe;
  if y<=0, break, end
end
pause
movie(M,1)
```

Several features of this script bear mention. First, notice that we have fixed the ranges for the x and y axes. If this is not done, the axes will rescale and cause the animation to jump around. Second, we terminate the `for` loop when the projectile's height y falls below zero.

When the script is executed, two animations will be displayed (we've placed a `pause` between them). The first corresponds to the sequential generation of the frames within the loop, and the second corresponds to the actual movie. Although we cannot show the results here, the trajectory for both cases will look like Fig. 3.2. You should enter and run the foregoing script in MATLAB to see the actual animation.

FIGURE 3.2
Plot of a projectile's trajectory.



3.4 NESTING AND INDENTATION

We need to understand that structures can be "nested" within each other. *Nesting* refers to placing structures within other structures. The following example illustrates the concept.

EXAMPLE 3.7 Nesting Structures

Problem Statement. The roots of a quadratic equation

$$f(x) = ax^2 + bx + c$$

can be determined with the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Develop a function to implement this formula given values of the coeffcients.

Solution. *Top-down design* provides a nice approach for designing an algorithm to compute the roots. This involves developing the general structure without details and then refining the algorithm. To start, we first recognize that depending on whether the parameter a is zero, we will either have "special" cases (e.g., single roots or trivial values) or conventional cases using the quadratic formula. This "big-picture" version can be programmed as

```
function quadroots(a, b, c)
% quadroots: roots of quadratic equation
%   quadroots(a,b,c): real and complex roots
%                     of quadratic equation
% input:
%   a = second-order coefficient
```

```
%   b = first-order coefficient
%   c = zero-order coefficient
% output:
%   r1 = real part of first root
%   i1 = imaginary part of first root
%   r2 = real part of second root
%   i2 = imaginary part of second root
if a == 0
  %special cases
else
  %quadratic formula
end
```

Next, we develop refined code to handle the "special" cases:

```
%special cases
if b ~= 0
  %single root
  r1 = -c / b
else
  %trivial solution
  disp('Trivial solution. Try again')
end
```

And we can develop refined code to handle the quadratic formula cases:

```
%quadratic formula
d = b ^ 2 - 4 * a * c;
if d >= 0
  %real roots
  r1 = (-b + sqrt(d)) / (2 * a)
  r2 = (-b - sqrt(d)) / (2 * a)
else
  %complex roots
  r1 = -b / (2 * a)
  i1 = sqrt(abs(d)) / (2 * a)
  r2 = r1
  i2 = -i1
end
```

We can then merely substitute these blocks back into the simple "big-picture" framework to give the final result:

```
function quadroots(a, b, c)
% quadroots: roots of quadratic equation
%   quadroots(a,b,c): real and complex roots
%                     of quadratic equation
% input:
%   a = second-order coefficient
%   b = first-order coefficient
%   c = zero-order coefficient
% output:
%   r1 = real part of first root
%   i1 = imaginary part of first root
```

```
%   r2 = real part of second root
%   i2 = imaginary part of second root
if a == 0
  %special cases
  if b ~= 0
    %single root
    r1 = -c / b
  else
    %trivial solution
    disp('Trivial solution. Try again')
  end
else
  %quadratic formula
  d = b ^ 2 - 4 * a * c;    %discriminant
  if d >= 0
    %real roots
    r1 = (-b + sqrt(d)) / (2 * a)
    r2 = (-b - sqrt(d)) / (2 * a)
  else
    %complex roots
    r1 = -b / (2 * a)
    i1 = sqrt(abs(d)) / (2 * a)
    r2 = r1
    i2 = -i1
  end
end
```

As highlighted by the shading, notice how indentation helps to make the underlying logical structure clear. Also notice how "modular" the structures are. Here is a command window session illustrating how the function performs:

```
>> quadroots(1,1,1)

r1 =
   -0.5000
i1 =
    0.8660
r2 =
   -0.5000
i2 =
   -0.8660

>> quadroots(1,5,1)

r1 =
   -0.2087
r2 =
   -4.7913

>> quadroots(0,5,1)

r1 =
   -0.2000
```

```
>> quadroots(0,0,0)
Trivial solution. Try again
```

3.5 PASSING FUNCTIONS TO M-FILES

Much of the remainder of the book involves developing functions to numerically evaluate other functions. Although a customized function could be developed for every new equation we analyzed, a better alternative is to design a generic function and pass the particular equation we wish to analyze as an argument. In the parlance of MATLAB, these functions are given a special name: *function functions*. Before describing how they work, we will first introduce anonymous functions, which provide a handy means to define simple user-defined functions without developing a full-blown M-file.

3.5.1 Anonymous Functions

Anonymous functions allow you to create a simple function without creating an M-file. They can be defined within the command window with the following syntax:

```
fhandle = @(arglist) expression
```

where `fhandle` = the function handle you can use to invoke the function, `arglist` = a comma separated list of input arguments to be passed to the function, and `expression` = any single valid MATLAB expression. For example,

```
>> f1=@(x,y) x^2 + y^2;
```

Once these functions are defined in the command window, they can be used just as other functions:

```
>> f1(3,4)

ans =
    25
```

Aside from the variables in its argument list, an anonymous function can include variables that exist in the workspace where it is created. For example, we could create an anonymous function $f(x) = 4x^2$ as

```
>> a = 4;
>> b = 2;
>> f2=@(x) a*x^b;
>> f2(3)

ans = 36
```

Note that if subsequently we enter new values for `a` and `b`, the anonymous function does not change:

```
>> a = 3;
>> f2(3)

ans = 36
```

Thus, the function handle holds a snapshot of the function at the time it was created. If we want the variables to take on values, we must recreate the function. For example, having changed `a` to 3,

```
>> f2=@(x) a*x^b;
```

with the result

```
>> f2(3)

ans =
    27
```

It should be noted that prior to MATLAB 7, `inline` functions performed the same role as anonymous functions. For example, the anonymous function developed above, `f1`, could be written as

```
>> f1=inline('x^2 + y^2','x','y');
```

Although they are being phased out in favor of anonymous function, some readers might be using earlier versions, and so we thought it would be helpful to mention them. MATLAB help can be consulted to learn more about their use and limitations.

3.5.2 Function Functions

Function functions are functions that operate on other functions which are passed to it as input arguments. The function that is passed to the function function is referred to as the *passed function*. A simple example is the built-in function `fplot`, which plots the graphs of functions. A simple representation of its syntax is

```
fplot(func,lims)
```

where *func* is the function being plotted between the x-axis limits specified by *lims* = [*xmin xmax*]. For this case, *func* is the passed function. This function is "smart" in that it automatically analyzes the function and decides how many values to use so that the plot will exhibit all the function's features.

Here is an example of how `fplot` can be used to plot the velocity of the free-falling bungee jumper. The function can be created with an anonymous function:

```
>> vel=@(t) ...
sqrt(9.81*68.1/0.25)*tanh(sqrt(9.81*0.25/68.1)*t);
```

We can then generate a plot from $t = 0$ to 12 as

```
>> fplot(vel,[0 12])
```

The result is displayed in Fig. 3.3.

Note that in the remainder of this book, we will have many occasions to use MATLAB's built-in function functions. As in the following example, we will also be developing our own.

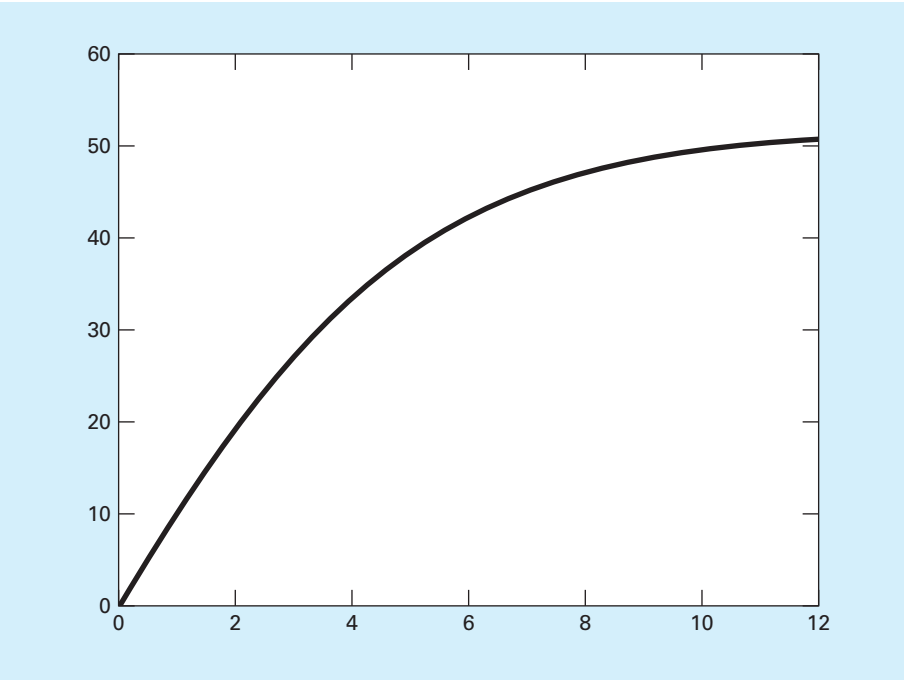


FIGURE 3.3
A plot of velocity versus time generated with the `fplot` function.

EXAMPLE 3.8 Building and Implementing a Function Function

Problem Statement. Develop an M-file function function to determine the average value of a function over a range. Illustrate its use for the bungee jumper velocity over the range from $t = 0$ to 12 s:

$$v(t) = \sqrt{\frac{gm}{c_d}}\tanh\left(\sqrt{\frac{gc_d}{m}}t\right)$$

where $g = 9.81$, $m = 68.1$, and $c_d = 0.25$.

Solution. The average value of the function can be computed with standard MATLAB commands as

```
>> t=linspace(0,12);
>> v=sqrt(9.81*68.1/0.25)*tanh(sqrt(9.81*0.25/68.1)*t);
>> mean(v)

ans =
   36.0870
```

Inspection of a plot of the function (Fig. 3.3) shows that this result is a reasonable estimate of the curve's average height.

We can write an M-file to perform the same computation:

```
function favg = funcavg(a,b,n)
% funcavg: average function height
%   favg=funcavg(a,b,n): computes average value
%                        of function over a range
% input:
%   a = lower bound of range
%   b = upper bound of range
%   n = number of intervals
% output:
%   favg = average value of function
x = linspace(a,b,n);
y = func(x);
favg = mean(y);
end

function f = func(t)
f=sqrt(9.81*68.1/0.25)*tanh(sqrt(9.81*0.25/68.1)*t);
end
```

The main function first uses `linspace` to generate equally spaced x values across the range. These values are then passed to a subfunction `func` in order to generate the corresponding y values. Finally, the average value is computed. The function can be run from the command window as

```
>> funcavg (0,12,60)

ans =
   36.0127
```

Now let's rewrite the M-file so that rather than being specific to `func`, it evaluates a nonspecific function name `f` that is passed in as an argument:

```
function favg = funcavg (f,a,b,n)
% funcavg: average function height
%   favg=funcavg(f,a,b,n): computes average value
%                          of function over a range
% input:
%   f = function to be evaluated
%   a = lower bound of range
%   b = upper bound of range
%   n = number of intervals
% output:
%   favg = average value of function
x = linspace(a,b,n);
y = f(x);
favg = mean(y);
```

Because we have removed the subfunction `func`, this version is truly generic. It can be run from the command window as

```
>> vel=@(t) ...
sqrt(9.81*68.1/0.25)*tanh(sqrt(9.81*0.25/68.1)*t);
>> funcavg(vel,0,12,60)
```

```
ans =
   36.0127
```

To demonstrate its generic nature, `funcavg` can easily be applied to another case by merely passing it a different function. For example, it could be used to determine the average value of the built-in `sin` function between 0 and 2π as

```
>> funcavg(@sin,0,2*pi,180)

ans =
 -6.3001e-017
```

Does this result make sense?

We can see that `funcavg` is now designed to evaluate any valid MATLAB expression. We will do this on numerous occasions throughout the remainder of this text in a number of contexts ranging from nonlinear equation solving to the solution of differential equations.

3.5.3 Passing Parameters

Recall from Chap. 1 that the terms in mathematical models can be divided into dependent and independent variables, parameters, and forcing functions. For the bungee jumper model, the velocity (v) is the dependent variable, time (t) is the independent variable, the mass (m) and drag coefficient (c_d) are parameters, and the gravitational constant (g) is the forcing function. It is commonplace to investigate the behavior of such models by performing a *sensitivity analysis*. This involves observing how the dependent variable changes as the parameters and forcing functions are varied.

In Example 3.8, we developed a function function, `funcavg`, and used it to determine the average value of the bungee jumper velocity for the case where the parameters were set at $m = 68.1$ and $c_d = 0.25$. Suppose that we wanted to analyze the same function, but with different parameters. Of course, we could retype the function with new values for each case, but it would be preferable to just change the parameters.

As we learned in Sec. 3.5.1, it is possible to incorporate parameters into anonymous functions. For example, rather than "wiring" the numeric values, we could have done the following:

```
>> m=68.1;cd=0.25;
>> vel=@(t) sqrt(9.81*m/cd)*tanh(sqrt(9.81*cd/m)*t);
>> funcavg(vel,0,12,60)

ans =
   36.0127
```

However, if we want the parameters to take on new values, we must recreate the anonymous function.

MATLAB offers a better alternative by adding the term `varargin` as the function function's last input argument. In addition, every time the passed function is invoked within the function function, the term `varargin{:}` should be added to the end of its

argument list (note the curly brackets). Here is how both modifications can be implemented for `funcavg` (omitting comments for conciseness):

```
function favg = funcavg(f,a,b,n,varargin)
x = linspace(a,b,n);
y = f(x,varargin{:});
favg = mean(y);
```

When the passed function is defined, the actual parameters should be added at the end of the argument list. If we used an anonymous function, this can be done as in

```
>> vel=@(t,m,cd) sqrt(9.81*m/cd)*tanh(sqrt(9.81*cd/m)*t);
```

When all these changes have been made, analyzing different parameters becomes easy. To implement the case where $m = 68.1$ and $c_d = 0.25$, we could enter

```
>> funcavg(vel,0,12,60,68.1,0.25)

ans =
   36.0127
```

An alternative case, say $m = 100$ and $c_d = 0.28$, could be rapidly generated by merely changing the arguments:

```
>> funcavg(vel,0,12,60,100,0.28)

ans =
   38.9345
```

3.6 CASE STUDY BUNGEE JUMPER VELOCITY

Background. In this section, we will use MATLAB to solve the free-falling bungee jumper problem we posed at the beginning of this chapter. This involves obtaining a solution of

$$\frac{dv}{dt} = g - \frac{c_d}{m} v|v|$$

Recall that, given an initial condition for time and velocity, the problem involved iteratively solving the formula,

$$v_{i+1} = v_i + \frac{dv_i}{dt}\Delta t$$

Now also remember that to attain good accuracy, we would employ small steps. Therefore, we would probably want to apply the formula repeatedly to step out from our initial time to attain the value at the final time. Consequently, an algorithm to solve the problem would be based on a loop.

Solution. Suppose that we started the computation at $t = 0$ and wanted to predict velocity at $t = 12$ s using a time step of $\Delta t = 0.5$ s. We would therefore need to apply the iterative equation 24 times—that is,

$$n = \frac{12}{0.5} = 24$$

3.6 CASE STUDY continued

where n = the number of iterations of the loop. Because this result is exact (i.e., the ratio is an integer), we can use a `for` loop as the basis for the algorithm. Here's an M-file to do this including a subfunction defining the differential equation:

```
function vend = velocity1(dt, ti, tf, vi)
% velocity1: Euler solution for bungee velocity
%   vend = velocity1(dt, ti, tf, vi)
%          Euler method solution of bungee
%          jumper velocity
% input:
%   dt = time step (s)
%   ti = initial time (s)
%   tf = final time (s)
%   vi = initial value of dependent variable (m/s)
% output:
%   vend = velocity at tf (m/s)
t = ti;
v = vi;
n = (tf - ti) / dt;
for i = 1:n
  dvdt = deriv(v);
  v = v + dvdt * dt;
  t = t + dt;
end
vend = v;
end

function dv = deriv(v)
dv = 9.81 - (0.25 / 68.1) * v*abs(v);
end
```

This function can be invoked from the command window with the result:

```
>> velocity1(0.5,0,12,0)

ans =
   50.9259
```

Note that the true value obtained from the analytical solution is 50.6175 (Example 3.1). We can then try a much smaller value of `dt` to obtain a more accurate numerical result:

```
>> velocity1(0.001,0,12,0)

ans =
   50.6181
```

Although this function is certainly simple to program, it is not foolproof. In particular, it will not work if the computation interval is not evenly divisible by the time step. To cover such cases, a `while . . . break` loop can be substituted in place of the shaded area (note that we have omitted the comments for conciseness):

3.6 CASE STUDY continued

```
function vend = velocity2(dt, ti, tf, vi)
t = ti;
v = vi;
h = dt;
while(1)
  if t + dt > tf, h = tf - t; end
  dvdt = deriv(v);
  v = v + dvdt * h;
  t = t + h;
  if t >= tf, break, end
end
vend = v;
end

function dv = deriv(v)
dv = 9.81 - (0.25 / 68.1) * v*abs(v);
end
```

As soon as we enter the `while` loop, we use a single line `if` structure to test whether adding `t + dt` will take us beyond the end of the interval. If not (which would usually be the case at first), we do nothing. If so, we would shorten up the interval—that is, we set the variable step `h` to the interval remaining: `tf - t`. By doing this, we guarantee that the last step falls exactly on `tf`. After we implement this final step, the loop will terminate because the condition `t >= tf` will test true.

Notice that before entering the loop, we assign the value of the time step `dt` to another variable `h`. We create this *dummy variable* so that our routine does not change the given value of `dt` if and when we shorten the time step. We do this in anticipation that we might need to use the original value of `dt` somewhere else in the event that this code were integrated within a larger program.

If we run this new version, the result will be the same as for the version based on the `for` loop structure:

```
>> velocity2(0.5,0,12,0)

ans =
   50.9259
```

Further, we can use a `dt` that is not evenly divisible into `tf - ti`:

```
>> velocity2(0.35,0,12,0)

ans =
   50.8348
```

We should note that the algorithm is still not foolproof. For example, the user could have mistakenly entered a step size greater than the calculation interval (e.g., `tf - ti = 5` and `dt = 20`). Thus, you might want to include error traps in your code to catch such errors and then allow the user to correct the mistake.

3.6 CASE STUDY continued

As a final note, we should recognize that the foregoing code is not generic. That is, we have designed it to solve the specific problem of the velocity of the bungee jumper. A more generic version can be developed as

```
function yend = odesimp(dydt, dt, ti, tf, yi)
t = ti; y = yi; h = dt;
while (1)
  if t + dt > tf, h = tf - t; end
  y = y + dydt(y) * h;
  t = t + h;
  if t >= tf, break, end
end
yend = y;
```

Notice how we have stripped out the parts of the algorithm that were specific to the bungee example (including the subfunction defining the differential equation) while keeping the essential features of the solution technique. We can then use this routine to solve the bungee jumper example, by specifying the differential equation with an anonymous function and passing its function handle to `odesimp` to generate the solution

```
>> dvdt=@(v) 9.81-(0.25/68.1)*v*abs(v);
>> odesimp(dvdt,0.5,0,12,0)

ans =
   50.9259
```

We could then analyze a different function without having to go in and modify the M-file. For example, if $y = 10$ at $t = 0$, the differential equation $dy/dt = -0.1y$ has the analytical solution $y = 10e^{-0.1t}$. Therefore, the solution at $t = 5$ would be $y(5) = 10e^{-0.1(5)} = 6.0653$. We can use `odesimp` to obtain the same result numerically as in

```
>> odesimp(@(y) -0.1*y,0.005,0,5,10)

ans =
    6.0645
```

PROBLEMS

3.1 Figure P3.1 shows a cylindrical tank with a conical base. If the liquid level is quite low, in the conical part, the volume is simply the conical volume of liquid. If the liquid level is midrange in the cylindrical part, the total volume of liquid includes the filled conical part and the partially filled cylindrical part.

Use decisional structures to write an M-file to compute the tank's volume as a function of given values of R and d. Design the function so that it returns the volume for all cases

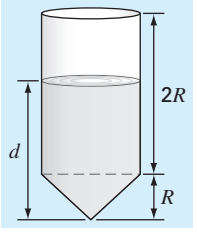


FIGURE P3.1

where the depth is less than $3R$. Return an error message ("Overtop") if you overtop the tank—that is, $d > 3R$. Test it with the following data:

R	0.9	1.5	1.3	1.3
d	1	1.25	3.8	4.0

Note that the tank's radius is R.

3.2 An amount of money P is invested in an account where interest is compounded at the end of the period. The future worth F yielded at an interest rate i after n periods may be determined from the following formula:

$$F = P(1 + i)^n$$

Write an M-file that will calculate the future worth of an investment for each year from 1 through n. The input to the function should include the initial investment P, the interest rate i (as a decimal), and the number of years n for which the future worth is to be calculated. The output should consist of a table with headings and columns for n and F. Run the program for $P = \$100{,}000$, $i = 0.05$, and $n = 10$ years.

3.3 Economic formulas are available to compute annual payments for loans. Suppose that you borrow an amount of money P and agree to repay it in n annual payments at an interest rate of i. The formula to compute the annual payment A is

$$A = P\frac{i(1 + i)^n}{(1 + i)^n - 1}$$

Write an M-file to compute A. Test it with $P = \$100{,}000$ and an interest rate of 3.3% ($i = 0.033$). Compute results for $n =$ 1, 2, 3, 4, and 5 and display the results as a table with headings and columns for n and A.

3.4 The average daily temperature for an area can be approximated by the following function:

$$T = T_{\text{mean}} + (T_{\text{peak}} - T_{\text{mean}})\cos(\omega(t - t_{\text{peak}}))$$

where T_{mean} = the average annual temperature, T_{peak} = the peak temperature, ω = the frequency of the annual variation ($= 2\pi/365$), and t_{peak} = day of the peak temperature ($\cong$205 d). Parameters for some U.S. towns are listed here:

City	T_{mean} (°C)	T_{peak} (°C)
Miami, FL	22.1	28.3
Yuma, AZ	23.1	33.6
Bismarck, ND	5.2	22.1
Seattle, WA	10.6	17.6
Boston, MA	10.7	22.9

Develop an M-file that computes the average temperature between two days of the year for a particular city. Test it for (**a**) January–February in Yuma, AZ ($t = 0$ to 59) and (**b**) July–August temperature in Seattle, WA ($t = 180$ to 242).

3.5 The cosine function can be evaluated by the following infinite series:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

Create an M-file to implement this formula so that it computes and displays the values of $\sin x$ as each term in the series is added. In other words, compute and display in sequence the values for

$$\sin x = x$$

$$\sin x = x - \frac{x^3}{3!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

$$\vdots$$

up to the order term of your choosing. For each of the preceding, compute and display the percent relative error as

$$\%\text{error} = \frac{\text{true} - \text{series approximation}}{\text{true}} \times 100\%$$

As a test case, employ the program to compute sin(0.9) for up to and including eight terms—that is, up to the term $x^{15}/15!$.

3.6 Two distances are required to specify the location of a point relative to an origin in two-dimensional space (Fig. P3.6):

- The horizontal and vertical distances (x, y) in Cartesian coordinates.
- The radius and angle (r, θ) in polar coordinates.

It is relatively straightforward to compute Cartesian coordinates (x, y) on the basis of polar coordinates (r, θ). The reverse process is not so simple. The radius can be computed by the following formula:

$$r = \sqrt{x^2 + y^2}$$

If the coordinates lie within the first and fourth coordinates (i.e., $x > 0$), then a simple formula can be used to compute θ:

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

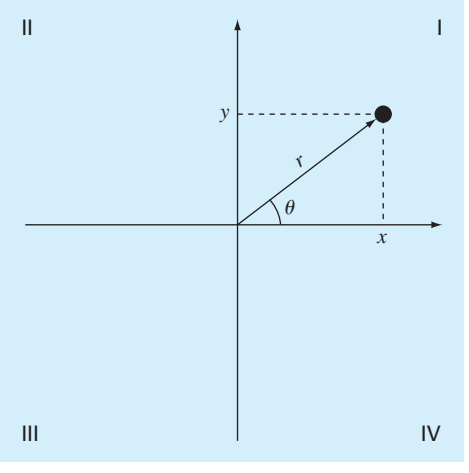


FIGURE P3.6

The difficulty arises for the other cases. The following table summarizes the possibilities:

x	y	θ
<0	>0	$\tan^{-1}(y/x) + \pi$
<0	<0	$\tan^{-1}(y/x) - \pi$
<0	=0	π
=0	>0	$\pi/2$
=0	<0	$-\pi/2$
=0	=0	0

Write a well-structured M-file using `if...elseif` structures to calculate r and θ as a function of x and y. Express the final results for θ in degrees. Test your program by evaluating the following cases:

x	y	r	θ
2	0		
2	1		
0	3		
−3	1		
−2	0		
−1	−2		
0	0		
0	−2		
2	2		

3.7 Develop an M-file to determine polar coordinates as described in Prob. 3.6. However, rather than designing the function to evaluate a single case, pass vectors of x and y. Have the function display the results as a table with columns for x, y, r, and θ. Test the program for the cases outlined in Prob. 3.6.

3.8 Develop an M-file function that is passed a numeric grade from 0 to 100 and returns a letter grade according to the scheme:

Letter	Criteria
A	90 ≤ numeric grade ≤ 100
B	80 ≤ numeric grade < 90
C	70 ≤ numeric grade < 80
D	60 ≤ numeric grade < 70
F	numeric grade < 60

The first line of the function should be

```
function grade = lettergrade(score)
```

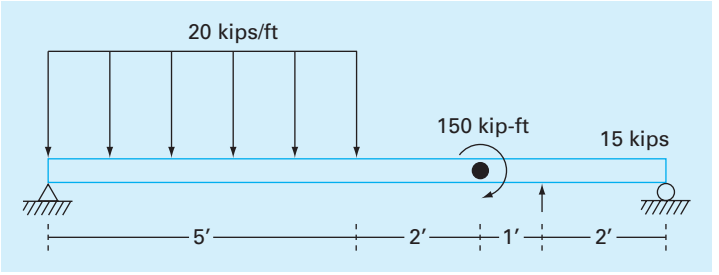


FIGURE P3.10

Design the function so that it displays an error message and terminates in the event that the user enters a value of `score` that is less than zero or greater than 100. Test your function with 89.9999, 90, 45 and 120.

3.9 Manning's equation can be used to compute the velocity of water in a rectangular open channel:

$$U = \frac{\sqrt{S}}{n}\left(\frac{BH}{B+2H}\right)^{2/3}$$

where U = velocity (m/s), S = channel slope, n = roughness coefficient, B = width (m), and H = depth (m). The following data are available for five channels:

n	*S*	*B*	*H*
0.036	0.0001	10	2
0.020	0.0002	8	1
0.015	0.0012	20	1.5
0.030	0.0007	25	3
0.022	0.0003	15	2.6

Write an M-file that computes the velocity for each of these channels. Enter these values into a matrix where each column represents a parameter and each row represents a channel. Have the M-file display the input data along with the computed velocity in tabular form where velocity is the fifth column. Include headings on the table to label the columns.

3.10 A simply supported beam is loaded as shown in Fig. P3.10. Using singularity functions, the displacement along the beam can be expressed by the equation:

$$u_y(x) = \frac{-5}{6}[\langle x-0\rangle^4 - \langle x-5\rangle^4] + \frac{15}{6}\langle x-8\rangle^3 + 75\langle x-7\rangle^2 + \frac{57}{6}x^3 - 238.25x$$

By definition, the singularity function can be expressed as follows:

$$\langle x-a\rangle^n = \begin{cases} (x-a)^n & \text{when } x > a \\ 0 & \text{when } x \le a \end{cases}$$

Develop an M-file that creates a plot of displacement (dashed line) versus distance along the beam, x. Note that $x = 0$ at the left end of the beam.

3.11 The volume V of liquid in a hollow horizontal cylinder of radius r and length L is related to the depth of the liquid h by

$$V = \left[r^2\cos^{-1}\left(\frac{r-h}{r}\right) - (r-h)\sqrt{2rh-h^2}\right]L$$

Develop an M-file to create a plot of volume versus depth. Here are the first few lines:

```
function cylinder(r, L, plot_title)
% volume of horizontal cylinder
% inputs:
% r = radius
% L = length
% plot_title = string holding plot title
```

Test your program with

```
>> cylinder(3,5,...
'Volume versus depth for horizontal...
cylindrical tank')
```

3.12 Develop a vectorized version of the following code:

```
tstart=0; tend=20; ni=8;
t(1)=tstart;
y(1)=12 + 6*cos(2*pi*t(1)/(tend-tstart));
for i=2:ni+1
  t(i)=t(i-1)+(tend-tstart)/ni;
  y(i)=10 + 5*cos(2*pi*t(i)/ ...
       (tend-tstart));
end
```

3.13 The "divide and average" method, an old-time method for approximating the square root of any positive number a, can be formulated as

$$x = \frac{x + a/x}{2}$$

Write a well-structured M-file function based on a `while...break` loop structure to implement this algorithm. Use proper indentation so that the structure is clear. At each step estimate the error in your approximation as

$$\varepsilon = \left| \frac{x_{new} - x_{old}}{x_{new}} \right|$$

Repeat the loop until ε is less than or equal to a specified value. Design your program so that it returns both the result and the error. Make sure that it can evaluate the square root of numbers that are equal to and less than zero. For the latter case, display the result as an imaginary number. For example, the square root of -4 would return $2i$. Test your program by evaluating $a = 0$, 2, 10 and -4 for $\varepsilon = 1 \times 10^{-4}$.

3.14 *Piecewise functions* are sometimes useful when the relationship between a dependent and an independent variable cannot be adequately represented by a single equation. For example, the velocity of a rocket might be described by

$$v(t) = \begin{cases} 10t^2 - 5t & 0 \le t \le 8 \\ 624 - 5t & 8 \le t \le 16 \\ 36t + 12(t-16)^2 & 16 \le t \le 26 \\ 2136e^{-0.1(t-26)} & t > 26 \\ 0 & \text{otherwise} \end{cases}$$

Develop an M-file function to compute v as a function of t. Then, develop a script that uses this function to generate a plot of v versus t for $t = -5$ to 50.

3.15 Develop an M-file function called `rounder` to round a number x to a specified number of decimal digits, n. The first line of the function should be set up as

```
function xr = rounder(x, n)
```

Test the program by rounding each of the following to 2 decimal digits: $x = 477.9587$, -477.9587, 0.125, 0.135, -0.125, and -0.135.

3.16 Develop an M-file function to determine the elapsed days in a year. The first line of the function should be set up as

```
function nd = days(mo, da, leap)
```

where `mo` = the month (1–12), `da` = the day (1–31), and `leap` = (0 for non–leap year and 1 for leap year). Test it for January 1, 1997, February 29, 2004, March 1, 2001, June 21, 2004, and December 31, 2008. Hint: A nice way to do this combines the `for` and the `switch` structures.

3.17 Develop an M-file function to determine the elapsed days in a year. The first line of the function should be set up as

```
function nd = days(mo, da, year)
```

where `mo` = the month (1–12), `da` = the day (1–31), and `year` = the year. Test it for January 1, 1997, February 29, 2004, March 1, 2001, June 21, 2004, and December 31, 2008.

3.18 Develop a function function M-file that returns the difference between the passed function's maximum and minimum value given a range of the independent variable. In addition, have the function generate a plot of the function for the range. Test it for the following cases:

(a) $f(t) = 8e^{-0.25t}\sin(t - 2)$ from $t = 0$ to 6π.
(b) $f(x) = e^{4x}\sin(1/x)$ from $x = 0.01$ to 0.2.
(c) The built-in `humps` function from $x = 0$ to 2.

3.19 Modify the function function `odesimp` developed at the end of Sec. 3.6 so that it can be passed the arguments of the passed function. Test it for the following case:

```
>> dvdt=@(v,m,cd) 9.81-(cd/m)*v^2;
>> odesimp(dvdt,0.5,0,12,-10,70,0.23)
```

3.20 A Cartesian vector can be thought of as representing magnitudes along the x-, y-, and z-axes multiplied by a unit vector (i, j, k). For such cases, the dot product of two of these vectors $\{a\}$ and $\{b\}$ corresponds to the product of their magnitudes and the cosine of the angle between their tails as in

$$\{a\} \cdot \{b\} = ab \cos\theta$$

The cross product yields another vector, $\{c\} = \{a\} \times \{b\}$, which is perpendicular to the plane defined by $\{a\}$ and $\{b\}$ such that its direction is specified by the right-hand rule. Develop an M-file function that is passed two such vectors and returns θ, $\{c\}$ and the magnitude of $\{c\}$, and generates a three-dimensional plot of the three vectors $\{a\}$, $\{b\}$, and $\{c\}$ with their origins at zero. Use dashed lines for $\{a\}$ and $\{b\}$ and a solid line for $\{c\}$. Test your function for the following cases:

(a) `a = [6 4 2]; b = [2 6 4];`
(b) `a = [3 2 -6]; b = [4 -3 1];`
(c) `a = [2 -2 1]; b = [4 2 -4];`
(d) `a = [-1 0 0]; b = [0 -1 0];`

3.21 Based on Example 3.6, develop a script to produce an animation of a bouncing ball where $v_0 = 5$ m/s and $\theta_0 = 50°$. To do this, you must be able to predict exactly when the ball hits the ground. At this point, the direction changes (the new angle will equal the negative of the angle at impact), and the

velocity will decrease in magnitude to reflect energy loss due to the collision of the ball with the ground. The change in velocity can be quantified by the *coefficient of restitution* C_R which is equal to the ratio of the velocity after to the velocity before impact. For the present case, use a value of $C_R = 0.8$.

3.22 Develop a function to produce an animation of a particle moving in a circle in Cartesian coordinates based on radial coordinates. Assume a constant radius, r, and allow the angle, θ, to increase from zero to 2 in equal increments. The function's first lines should be

```
function phasor(r, nt, nm)
% function to show the orbit of a phasor
% r = radius
% nt = number of increments for theta
% nm = number of movies
```

Test your function with

```
phasor(1, 256, 10)
```

3.23 Develop a script to produce a movie for the butterfly plot from Prob. 2.22. Use a particle located at the x-y coordinates to visualize how the plot evolves in time.