Midterm #3: Section 1

Full Name: ..... **Signature**.....

Note: You need to **SHOW** all your **WORK** including **all steps** in order to have full **CREDIT**.

There are 4 problems and a bonus exercise on the Midterm.

Exercise 1. *(50 points)*

- *Evaluate the double integral*

$$\int\int_D y\sqrt{y^2-x^2}\,dA \quad \textit{with} \quad D=\{(x,y)|0\le x\le 1,\ ;\ 0\le y\le x\}$$

- *find the volume of the solid bounded by the two cylinders*

$$x^2+y^2=4 \quad \textit{and} \quad y^2+z^2=4$$

Exercise 2. *(40 points)*
We are intersted in evaluating the following integral

$$I \quad = \quad \int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}\, dx\, dy$$

- *Sketch the region of integration and change the order of integration of I*

- *Evaluate I after reversing the order of integration*

Exercise 3. *(30 points)*
Use double integrals to find the area inside the circle $r = 4\cos(\theta)$ *and outside the circle* $r = 2$ *in the first quadrant.*

Exercise 4. *(30 points)*
Use triple integral to find the mass, center of mass of a solid occupying the hemisphere $x^2 + y^2 + z^2 \leq 4, \quad z \geq 0$ *and with density function given by*

$$\sigma(x, y, z) = x^2 + y^2 + z^2$$

Exercise 5. *(Bonus)*
True or False? Justify your answer.

1. $$\int_0^5 \int_{-1}^4 x \sin(x-y)\, dx\, dy = \int_{-1}^4 \int_0^5 x \sin(x-y)\, dy\, dx$$
.

2. *The integral* $\int \int \int_E kr^3\, dz\, dr\, d\theta$ *represents the moment of inertia about the z-axis of a solid* E *with constant density* k.

3. *The vector field* $F(x,y) =< y, x >$ *is conservative*

4. *The line integral* $\int_C F(x,y)\, ds = 0$ *if the vector field* $F(x,y) = y\vec{i} + x\vec{j}$ *and* C *is the unit circle centered at the origin.*

5. *The gradient of* $f(x,y) = \ln(x+2y)$ *is* $< \frac{1}{x+2y}, \frac{1}{x+2y} >$