Midterm #2: Section 2

Full Name: .... **Signature**...

Note: You need to **SHOW** all your **WORK** in order to have full **CREDIT**.

There are 6 problems on the Midterm.

Exercise 1. *(40 points)*

- *At what point P do the curves*

  $\vec{r}_1(t) =< t, 2+t, 3+t^2 >$ and $\vec{r}_2(s) =< s+1, 3-2s, 4+3s^2 >$ intersect?

- *Find the cosine of their angle of intersection.*

Exercise 2. *(40 points)*
let

$$z = f(x,y) = \sqrt{5 - x^2 - y^2}$$

- *Find the domain of definition of* $z = f(x,y)$.

- *Find the linear approximation of* $z = f(x,y)$ *at* $(1,1)$

- *Use the linear approximation in the previous part to find an approximation of* $\sqrt{5-(1.1)^2-(0.9)^2}$. No credit if you don't show your work for this question.

Exercise 3. *(30 points)*

The dimensions of a right circular cone are measured as 10 m for the hight, 2 m for the radius, with a possible error of 0.1 m in each dimension. Use differentials to estimate the maximum error in calculating the volume of the cone if the volume of the cone is given by $V(h, r) = \pi r^2 h/3$.

Exercise 4. *(30 points)*
The temperature at a point (x, y, z) *is given by*

$$T(x, y, z) = 100e^{-x^2-3y^2-9z^3}$$

. *Where* T *is measured in degree* C *and* x, y, z *in meters.*

- *Find the rate of change of temperature at the point* $P(-1, -1, 1)$ *in the direction of the unit vector* $\vec{v} = \frac{1}{3}\left(\vec{i} - 2\vec{j} + 2\vec{k}\right)$.

- *The gradient vector gives the direction of fastest change. Find the maximum rate of change of* $T(x, y, z)$ *at* $P(-1, -1, 1)$

Exercise 5. *(30 points)*
Find the unit tangent $\vec{T}(t)$, the unit normal $\vec{N}(t)$ and the binormal vector $\vec{B}(t)$ if

$$\vec{r}(t) =< e^t, e^t\cos(t), e^t\sin(t) > .$$

Exercise 6. *(30 points)*
Find the local maximum , local minimum or saddle points if they exist, of the function

$$f(x, y) = 2x^2 + xy - y^2 - x + 2y + 7$$

Exercise 7. *(20 points)*
True or False? Justify your answer.

1. *If* $f(x,y,z)$ *is a function, then*

$$\lim_{(x,y,z)\to(a,b,c)} f(x,y,z) = f(a,b,c)$$

2. *the curve with vector equation* $\vec{r}(t) = \vec{i} - 3\vec{j} + 7t^2\vec{k}$ *is a line.*

3. *If* $\vec{r}(s)$ *is a differentiable vector function, then* $\frac{d}{ds}|\vec{r}(s)| = |\vec{r}\,'(s)|$

4. f *is differentiable at* (a,b) *if* $\lim_{(x,y)\to(a,b)} f_x(x,y) = f_x(a,b)$ *and* $\lim_{(x,y)\to(a,b)} f_y(x,y) = f_y(a,b)$.

5. *The gradient of* $f(x,y) = \sqrt{2xy}$ *is* $< \frac{2x}{\sqrt{2xy}}, \frac{2y}{\sqrt{2xy}} >$

6. $$\int_0^1 \left(t\vec{i} - t^3\vec{j} + 5\vec{k}\right) dt = \frac{1}{2}\vec{i} - \frac{1}{4}\vec{j} + 5\vec{k}$$

7. $$f_{xy} = \frac{\partial^2 f}{\partial x \partial y}$$