Midterm #1: Section 1

Full Name: ..... **Signature**.....

Note: You need to **SHOW** all your **WORK** in order to have full **CREDIT**.

There are 8 problems on the Midterm.

Exercise 1. *(30 points)*

- *Find an equation of the sphere that passes through the point* $P(2, -1, 2)$ *and whose center is* $C(0, -1, 1)$.

- *We are interested in the intersection of the sphere from the first part of this exercise with the coordinates system.*
  Find the intersection of the sphere from the first part with:
  - *the* x*-axis*
  - *the* zx*-plane*

Exercise 2. *(40 points)*

- *How do you tell if:*
  - *two vectors are parallel?*
  - *two vectors are perpendicular?*
  - *two plane are parallel?*

- *How do you find the distance:*
  - *from a point to a line ?*
  - *from a point to a plane ?*

- *True or False (If it is false give an example that disproves the statement.)*
  - *For any vectors $\vec{u}$ and $\vec{v}$ in V_3,* $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$
  - *For any vectors $\vec{u}$ and $\vec{v}$ in V_3,* $\vec{u} \times \vec{v} = \vec{v} \times \vec{u}$
  - *For any vectors $\vec{u}$ and $\vec{v}$ in V_3,* $(\vec{u} \times \vec{v}) \cdot \vec{v} = 0$
  - *For any vectors $\vec{u}$ and $\vec{v}$ in V_3,* $(\vec{u} + \vec{v}) \times \vec{v} = \vec{u} \times \vec{v}$
  - *The cross product of two unit vectors is a unit vector.*

Exercise 3. *(50 points)*
If $\vec{u} = 2\vec{i} + \vec{j} + \vec{k}$ *and* $\vec{v} = \vec{i} - \vec{j} + 2\,\vec{k}$

- *Find the angle between the vectors* $\vec{u}$ *and* $\vec{v}$ *(15 points)*

- *For what values of* b *are the vectors* $\vec{v}$ *and* $\vec{w} = b\,\vec{i} + b^2\,\vec{j}$ *orthogonal? (15 points)*

- *Find two unit vectors orthogonal to both* $\vec{u}$ *and* $\vec{v}$ *(20 points)*

Exercise 4. *(40 points)*

- *Find the parametric equations for the line through* $(2, 5, 3)$ *and perpendicular to the plane* $x - 2y + x - 3 = 0$

- *Recall that two planes are parallel if their normal vector are collinear. Find an equation of the plane through* $(0, 0, 5)$ *parallel to the?*
  - *the xy-plane;*
  - *the yz-plane;*
  - *the zx-plane;*

Exercise 5. *(20 points)*
Determine whether the two lines

$$L_1 : \frac{x-1}{-4} = \frac{y-3}{-4} = \frac{z-2}{2} \qquad L_2 : \frac{x-2}{1} = \frac{y-6}{-1} = \frac{z+2}{3}$$

are parallel, skew, or intersecting. If they intersect, find the point of intersection.

Exercise 6. *(20 points)*

- *The following is an equation of a surface in cartesian coordinates.*

$$x^2 + y^2 + z^2 - 4z = 0$$

  - *Write the equation in cylindrical coordinates.*

  - *Write the equation in spherical coordinates*

- *Identify the surface whose equation is given by*

$$\rho \sin(\phi) = 6$$

Exercise 7. *(Bonus 20 points)*
A plane is capable of flying at a speed of 180km/h in still air. The pilot takes off from an airfield and heads due north according to the plane compass. After 30 minutes of flight time, the pilot notices that, due to the wind, the plane has actually traveled 95 km at an angle 5^o east of north.

- *What is the wind speed?*

- *In what direction should the pilot have headed to reach the intended destination? (the angle with respect to the northern direction)*