Final Exam: MATH 201 Section 1

Full Name: ....

Student ID Number...

Note: You need to **SHOW** all your **WORK** in order to have full **CREDIT**.
The use of **CALCULATOR is prohibited** during the exam.

There are 10 problems (30 points each) on the Exam and one bonus problem worths 40 points on this Final Exam. Read each question carefully

The Final is 2 and 1/2 hours long or 150 minutes

Homework and Quizzes: 150 pts	Midterms: 550 pts	Final Exam : 300 pts

Final Grade over 1000	Letter Grade

Note that you need at least 600/1000 to make a grade of C^{-}.

Exercise 1. *(30 points)*
If $\vec{u} = \vec{i} - \vec{j} + \vec{k}$ *and* $\vec{v} = \vec{i} + \vec{j} + \vec{k}$

- *Find the cosine of the angle between the vectors* $\vec{u}$ *and* $\vec{v}$

- *For what values of the constant* b *are the vectors* $\vec{v}$ *and* $\vec{w} = \vec{i} + b\,\vec{j}$ *orthogonal?*

- *Find the area of the parallelepiped spanned by the vectors* $\vec{u}$ *and* $\vec{v}$

Exercise 2. *(30 points)*

- *Find the vector equation of the line through* $P(2,1,3)$ *perpendicular to the plane* $3x - 4y + z - 5 = 0$

- *Two particles* P_1 *and* P_2 *have position vector* $\vec{r}_1(t) =< 1, t+3, -t >$ *and* $\vec{r}_2(t) =< 2t-1, 1+3t^2, t^3-2 >$ *respectively at time* t*. Do the particles collide? If they collide, at what point does the collision occur?*

Exercise 3. *(30 points)*

- *The pressure, Volume and Temperature of an ideal gas are related by* $PV = 8.31T$ *or that as a function of Temperature* T *and Volume* V*, the pressure* P *is given by* $P(T,V) = \frac{8.31T}{V}$ *where* P *is measured in kilopascals,* V *in liters and* T *in kelvins. Use* ***differentials*** *to find the approximate change in the pressure if the volume increases from* $12L$ *to* $12.3L$ *and the temperature decreases from* $310K$ *to* $305K$*.*

- *let* $f(x,y) = \sqrt{20 - x^2 - 7y^2}$,
  - *Find the linear approximation to* $f(x,y)$ *at* $(2,1)$
  - *Use the linear approximation in the previous part to find an approximation of* $f(1.95, 1.08)$*.* No credit if you don't show your work for this question.

Exercise 4. *(30 points)*

- *Suppose* $(0,2)$ *is a critical point of the function* g *with continuous second derivatives. In each case what can you say about* g*? (is it a local min, local max, a saddle point or none of the above?* ***No credit if there is no justification****)*

  - $g_{xx}(0,2) = -1, \quad g_{xy}(0,2) = 6, \quad g_{yy}(0,2) = 1$

  - $g_{xx}(0,2) = -1, \quad g_{xy}(0,2) = 2, \quad g_{yy}(0,2) = -8$

  - $g_{xx}(0,2) = 4, \quad g_{xy}(0,2) = 6, \quad g_{yy}(0,2) = 9$

- *Find the local maximum , local minimum or saddle points if they exist, of the function*
$$f(x,y) = 2x^2 + 2xy - y^2 - x - 5y + 1$$

Exercise 5. *(30 points)*

- *Evaluate* $\int\int_D x^3y^2\,dA$ *if* $D = \{(x,y)/0 \leq x \leq 2, \quad -x \leq y \leq x\}$

- *Use double integral to find the area of the region enclosed by the curve* $r = 2 + \sin\theta$

Exercise 6. *(30 points)*
Suppose that over a certain region in space the electrical potential V is given by $V(x,y,z) = 5x^2 - 3xy + xyz$.

1. *Find the rate of change of the potential (or the directional derivative) at $P(1,1,2)$ in the direction of the vector $\vec{v} = \vec{i} + \vec{j} - \vec{k}$*

2. *In which direction does V changes most rapidly at P?*

3. *What is the maximum rate of change of V at P?*

Exercise 7. *(30 points)*
Find the volume of the solid bounded by $z = \sqrt{y^3 + 1}$, $z = 0$, $y = 0$, $x = 1$ *and* $x = y^2$ *in the first octant.*

Exercise 8. *(30 points)*
Find the mass, center of mass of a solid with density function

$$\sigma(x, y, z) \quad = \quad \frac{e^{\sqrt{x^2+y^2+z^2}}}{x^2 + y^2 + z^2}$$

inclosed by the sphere $x^2 + y^2 + z^2 = 4$ *in the first octant.*

Exercise 9. *(30 points)*

1. *Evaluate the integral* $J = \int_C x^2 y\, ds$ *where* C *is the upper half circle joining* $(1,0)$ *to* $(-1,0)$

2. *Show that the vector field* $\vec{F}(x,y) = < 4x^3 + 2xy\ ,\ x^2 + 5y^4 >$ *is coservative. And then find a potential function* f *such that* $\vec{F} = \vec{\nabla} f$

3. *Evaluate integral* $K = \int_C (4x^3 + 2xy)dx + (x^2 + 5y^4)dy$ *if* C *is any path from* $(1,1)$ *to* $(1,2)$.

Exercise 10. *(30 points)*

1. *Use Green's Theorem to evaluate the line integral* $L = \int_C \vec{F} \cdot d\vec{r}$ *where* $\vec{F}(x,y) = < x^2y\,,\, -xy^2 >$ *and* C *is the circle* $x^2+y^2 = 4$ *with counterclockwise orientation.*

2. *Use the Divergence Theorem to calculate the surface integral* $\int\int_S \vec{F} \cdot d\vec{S}$ *(This is, calculate the Flux of* $\vec{F}$ *across* S*.) If* $\vec{F}(x,y,z) = < x^3, y^3, z^3 >$ *where* S *is the surface of the solid bounded by the cylinder* $x^2 + y^2 = 1$ *and the planes* $z = 0$ *and* $z = 2$*.*

Exercise 11. *(40 points) Bonus*

1. *True or False ? (Justify your answer)*

   (a) *If f has continuous partial derivatives and C is any circle, then* $\int_C \vec{\nabla} f \cdot d\vec{r} = 0$.

   (b) *If $\vec{F}$ is a vector field, then div$\vec{F}$ is a scalar function.*

   (c) *If $\vec{F}$ is a vector field, then curl$\vec{F}$ is a vector field*

   (d) *If f has continuous partial derivatives of all orders, then* $\mathrm{curl}(\mathrm{div}\vec{\nabla} f)$ *is defined.*

   (e) *If* $\vec{F} = P\vec{i} + Q\vec{j} + R\vec{k}$ *and* $\mathrm{curl}(\vec{F}) = 0$, *then $\vec{F}$ is conservative*

   (f) *The integral* $\int_0^{2\pi} \int_0^2 \int_0^2 r dz\, dr\, d\theta$ *represents the volume enclosed by the cone* $z = \sqrt{x^2 + y^2}$ *and the plane* $z = 2$

   (g) *If* $\vec{F} = \vec{\nabla} f$ *then* $\int_C \vec{F} \cdot d\vec{r} = f(\vec{r}(a)) - f(\vec{r}(b))$ *for any curve C given by $\vec{r}(t)$ with* $a \leq t \leq b$

2. *If f has a local minimum at (a, b) and f is differentiable at (a, b) then* $\vec{\nabla} f(a, b) = 0$

3. *The gradient of the vector function* $\vec{F}(t) =< \sin x(t), e^{\sin y(t)}, \ln(\sin z(t)) >$ *is* $< x'(t) \cos x(t), y'(t) \cos y(t) e^{\sin y(t)}, z'(t) \cot z(t) >$

4. *For any two vectors $\vec{u}$ and $\vec{v}$ we have* $(\vec{u} \times \vec{v}) \cdot \vec{u} = \vec{0}$