Midterm #3: Section 1

Full Name: .... **Signature**...

Note: You need to **SHOW** all your **WORK** in order to have full **CREDIT**.
The use of **CALCULATOR is prohibited** during the exam.

There are 5 problems each worth 30 points and a bonus problem worth 20 points on the Midterm.

Exercise 1. *(30 points)*

a)List the first 5 terms of the sequence $\{a_n\}$ *if* $a_1 = 2$ *and* $a_{n+1} = \frac{a_n}{2a_n+1}$

b) Determine if the following sequence converges or diverges. (Give reasons. Think of Sandwich Theorem or Squeeze Theorem.)
$a_n = \cos(n)\, e^{-n}$

Exercise 2. *(30 points)*
Determine whether the series is convergent or divergent. If it is convergent, find its sum

1. $$\sum_{n=0}^{\infty} \frac{1}{5} \frac{7^n}{4^{n+1}}$$

2. $$\sum_{n=0}^{\infty} \frac{1}{n^2 - n}$$

Exercise 3. *(30 points)*

1. *Suppose* $\sum_{n=1}^{\infty} a_n$ *and* $\sum_{n=1}^{\infty} b_n$ *are series with positive terms and* $\sum_{n=1}^{\infty} a_n$ *is known to be convergent.*
   a) If $a_n > b_n$ *for all* n*, what can you say about* $\sum_{n=1}^{\infty} b_n$*? Why?*

   b)If $a_n < b_n$ *for all* n*, what can you say about* $\sum_{n=1}^{\infty} b_n$*? Why?*

2. *Determine whether the series converges or diverges. (You can use part 1.)*

$$\sum_{n=0}^{\infty} \frac{\cos(n\frac{\pi}{2})}{n!} \qquad \textit{(note that} \quad \cos\left(n\frac{\pi}{2}\right) = \begin{cases} (-1)^k, & \textit{if } n{=}2k; \\ 0, & \textit{if } n{=}2k{+}1. \end{cases} \textit{)}$$

Exercise 4. *(30 points)*

- *Find The radius and interval of convergence of the series*

$$\sum_{n=1}^{\infty} \frac{(5x-1)^n}{n^3 5^n}$$

- *suppose that* $\sum_{n=1}^{\infty} c_n(x+1)^n$ *converges when* $x=-3$ *and diverges when* $x=3$. *what can you said about the convergence or divergence of the following? Why? (Clearly mention on a number line the regions where there is a certainty of convergence or divergence)*

1. $\sum_{n=1}^{\infty} c_n$

2. $\sum_{n=1}^{\infty} c_n(-1)^n$

3. $\sum_{n=1}^{\infty} c_n(-3)^n$

4. $\sum_{n=1}^{\infty} c_n(7)^n$

Exercise 5. *(30 points)*

1. *Use Differentiation Theorem to find a power series representation for* $f(x) = \log(1 - x^3)$.
   For your information we know that $\big(\log(1 - x^3)\big)' = \frac{-3x^2}{1-x^3}$ *and* $\log(1+0) = 0$

2. *Approximate the definite integral to 3 decimal places. (you don't need to simplify your answer.)*

$$\int_0^{1/3} \log(1 - x^3)\, dx$$

Exercise 6. *(Bonus;20points)*
We have the following Maclaurin series;
$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, $\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$, *and* $\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$.
Use these series to find the following limits

1.
$$\lim_{x \to 0} \frac{\sin(x) - x + \frac{1}{6}x^3}{x^5}$$

2.
$$\lim_{x \to 0} \frac{1 - \cos(x)}{1 + x - e^x}$$

3.
$$\lim_{x \to 0} \frac{\sin(x^2) - x^2}{x^6}$$