Midterm #2: Section 1

Full Name: .... **Signature**..

Note: You need to **SHOW** all your **WORK** in order to have full **CREDIT**.
The use of **CALCULATOR is prohibited** during the exam.

There are 8 problems on the Midterm.

Exercise 1. *(30 points)*

Use a) Trapezoid rule, b) Simpson's rule and c) Midpoint rule to approximate the integral. (Do not simplify your final answer.)

$$\int_0^8 \frac{d\,x}{1+x^4}, \quad n = 8$$

Exercise 2. *(40 points)*
Explain why the following integral is improper, then evaluate the integral

1.
$$I \;=\; \int_0^{\infty} x^3\, e^{-x^2}\, dx$$

2.
$$\int_{-1}^{1} \frac{8}{(x+1)(x+3)(x-1)}\, dx$$

Exercise 3. *(30 points)*
Find the length of the curve

1. $y = \ln(\sin(x)), \qquad \frac{\pi}{4} \leq x \leq \frac{\pi}{2}$

2. $$x = e^{2t} + e^{-2t}, \qquad y = 3 + 4t, \qquad 0 \leq t \leq 1$$

Exercise 4. *(20 points)*
Find the area of the surface obtained by rotating the curve

$$y \;=\; 5 \;+\; \frac{1}{2}x^2, \qquad 0 \le x \le 1 \quad \text{about} \quad \text{the} \quad y-\text{axis}$$

Exercise 5. *(40 points)*

1. *Find an equation to the tangent to the curve at the given point.*

$$r \;=\; 3 + \;\cos(\theta), \qquad \theta \;=\; \frac{\pi}{2}$$

2. *The following parametric equation is an equation of a conic section*

$$x \;=\; 1 \;+\; 3\cos(\theta), \quad y \;=\; -2 \;+\; 5\sin(\theta), \qquad 0 \leq \theta \leq 2\pi$$

Eliminate the parameter to identify the conic section. (give the Vertices , Foci, sketch the curve and indicate with an arrow the direction in witch the curve is traced as the parameter increases.)

Exercise 6. *(20 points)*
Identify the curve by finding the cartesian equation of the curve

1. $r\cos(\theta) \;=\; 3$

2. $r \;=\; -3\sin(\theta)$

Find the exact length of the polar curve $\quad r \;=\; 2\theta^2\,, \qquad 0 \le \theta \le \pi$

Exercise 7. *(20 points)*
Identify the type of conic section whose equation is given and find the vertices and the foci.

1. $9x^2 = 4y^2 + 36$

2. $x^2 = 4y - 2y^2$

3. $x^2 = y + 1$

Exercise 8. *(Bonus)*
A curve is defined by the parametric equation

$$x(t) = \int_1^t \frac{\cos u}{u}\, du, \qquad y(t) = \int_1^t \frac{\sin u}{u}\, du$$

find the length of the arc of the curve from the origin to the nearest point where there is a vertical tangent line.
Note that if a function is defined by

$$f(t) = \int_a^t g(u)\, du, \quad \text{then} \quad f^{'}(t) = g(t)$$