Final Exam: Section 1

Full Name: ....

Student ID Number...

Note: You need to **SHOW** all your **WORK** in order to have full **CREDIT**.
The use of **CALCULATOR is prohibited** during the exam.

There are 10 problems (30 points each) on the Exam and one bonus problem worths 30 points on this Final Exam. Read each question carefully

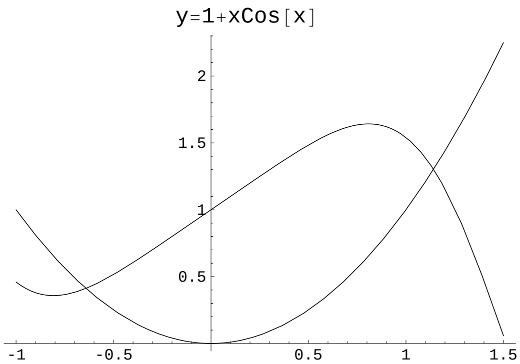
The Final is 2 and 1/2 hours long

Homework and Quizzes: 150 pts	Midterms: 550 pts	Final Exam : 300 pts

Final Grade	Letter Grade

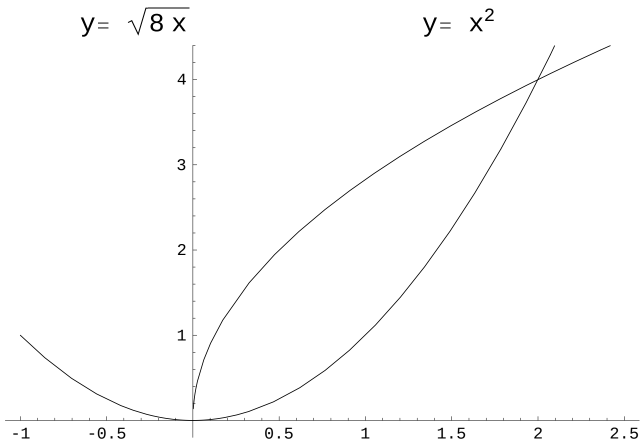
Exercise 1. *(30 points)*
Find the area of the region bounded by the curves

$$y = 1 + x\cos(x^2), \quad y = x^2, \quad \mathbf{x} = -\frac{\mathbf{1}}{\mathbf{2}} \quad \textit{and} \quad \mathbf{x} = \mathbf{1}$$



Exercise 2. *(30 points)*

Find the volume of the solid obtained by rotating the region bounded by the curves $y = x^2$ and $y = \sqrt{8x}$ about the y-axis



Exercise 3. *(30 points)*
Find the area of the surface obtained by rotating the curve $x = 3 + y^2$ *with* $1 \leq y \leq 2$ *about the* x*-axis.*

Exercise 4. *(30 points)*
Find the length of the curve

$$x = e^t \cos(t), \quad y = e^t \sin(t), \quad \textit{with} \quad 0 \leq t \leq \pi$$

.

Exercise 5. *(30 points) Evaluate the following integrals*

1.
$$\int_1^2 \frac{\ln(x+1)}{(x+1)^3}\,dx$$

2.
$$\int_0^1 \frac{12}{(x-2)(x+2)(x+1)}\,dx$$

Exercise 6. *(30 points)*
Identify the type of conic section whose equation is given and sketch it's graph.

1. $x = 2 + 2\sec(\theta),\ y = -3 + 5\tan(\theta),\ -\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$ *(Hint: Eliminate the parameter)*

2. $x^2 = y + 1$

3. $4x^2 + 9y^2 - 36 = 0$

Exercise 7. *(30 points)*

1. *Which of the following integral are improper?* ***why ?***
   (a) $\int_1^2 \frac{1}{2x-1}dx$
   (b) $\int_0^1 \frac{1}{2x-1}dx$
   (c) $\int_{-\infty}^{\infty} \frac{\cos(x)}{1+x^2}dx$
   (d) $\int_1^5 \ln(x-1)dx$
2. *Determine if each integral is convergent or divergent. Evaluate those that are convergent.*
   (a) $\int_1^{\infty} \frac{27}{2x+1}dx$
   (b) $\int_0^{\infty} x^5 e^{-x^3}dx$

Exercise 8. *(30 points)*

1. *Which of the following series is convergent?* ***Why ?***

   (a) $\sum_{n=1}^{\infty} \frac{(\pi)^n}{3^{n+1}}$

   (b) $\sum_{n=1}^{\infty} 12n^{-1.2}$

   (c) $\sum_{n=0}^{\infty}(-0.6666)^{n-1}$

   (d) $\sum_{n=1}^{\infty} \frac{(-1)^n}{6^{n+1}}$

2. *Find the sum of the following series*

   (a) $\sum_{n=1}^{\infty} \frac{(\pi)^n}{5^{n+1}}$

   (b) $\sum_{n=2}^{\infty} \frac{6}{n^2-1}$

Exercise 9. *(30 points)*
The power series $\sum_{n=0}^{\infty} c_n(x-2)^n$ *is convergent when* $x = 5$ *and diverges when* $x = -3$.

1. *Use the number line to graph the regions where there is certainty for convergence and certainty for divergence.*

2. *Use your graph in part 1 to determine whether the following series are convergent or divergent.*

   (a) $\sum_{n=0}^{\infty} c_n 6^n$

   (b) $\sum_{n=0}^{\infty} c_n 2^n$

   (c) $\sum_{n=0}^{\infty} c_n (-6)^n$

   (d) $\sum_{n=0}^{\infty} c_n (-3)^n$

Exercise 10. *(30 points) Find the* ***radius*** *and* ***interval*** *of convergence of the power series*

$$\sum_{n=0}^{\infty} \frac{(5x-1)^n}{3^n\, n^5}$$

Exercise 11. *(Bonus; 30 points) Let* $\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$ *be the Maclaurin series representation of* f.

1. *Find the Maclaurin series of* $f(x) = e^x$.

2. *Use part 1 to find an approximation of* $\int_0^{0.1} e^{-x^2}\, dx$ *to 4 decimal places.*

3. *Find the sum of the series* $1 - \ln(3) + \frac{(\ln(3))^2}{2!} - \frac{(\ln(3))^3}{3!} + \frac{(\ln(3))^4}{4!} - \cdots$