QUANTUM MECHANICS

Do any 3 of the 4 problems. Circle and clearly indicate final answers.

1. A mass m is attached by a massless rod of length ℓ to a pivot, which allows the mass to swing freely in a vertical plane under the influence of gravity. Let θ be the angle between the rod and the vertical.
   (a) In the small-angle approximation, find the energy levels of the system.
   (b) Find the lowest-order correction to the ground state energy resulting from the inaccuracy of the small-angle approximation.

2. Consider a particle of mass m confined to an infinite cylindrical well: Inside the cylinder of radius a and length L, the potential is zero; outside the cylinder, the potential is infinity. The axis of the cylinder lies along the z axis and its base is on the x-y plane.
   (a) Assuming the particle is in a zero angular momentum state, find the time-independent Schrödinger equation for the particle.
   (b) Now find the allowed energies of the particle. At some point, you may want to employ the dimensionless variable $x = \pi r/L$.
   (c) Find the allowed energies in the limit of $a \to \infty$. Comment.

Possibly Useful Information for Problem 1

In cylindrical coordinates,

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \quad .$$

Bessel's Differential Equation is given by

$$x^2 R'' + x R' + k^2 x^2 R = 0 \quad ,$$

with solutions $R = J_0(kx)$ and $N_0(kx)$.

3. An electron in an infinite square potential well of width $a = 100$ nm has the following initial wavefunction:

$$\psi(x, 0) = Ax(a - x) \quad .$$

(a) Sketch this wavefunction and verify that it might be valid. Find A.

(b) How does this wavefunction evolve in time? Find $\psi(x, t)$. Will the wavefunction ever return to the above initial state? If so, how long will it take?

(c) What is the probability that a measure of the energy of the electron will result in a value of 1 meV? The mass of an electron is 9×10^{-31} kg and $h = 6.6 \times 10^{-34}$ J-s.

4. Consider two negative ions, with different charge in general, implanted in a dielectric medium. They are separated a distance $2a$.

(a) Derive the transcendental equation that determines the bound-state energies a positively-charged particle may have in this configuration. Hint: Make a simplifying assumption about the potential of the ions; do not use the Coulomb potential.

(b) Sketch the bound state wavefunctions. As the separation between the ions becomes small, find the energy of the only bound state that survives. Verify that when the separation is exactly zero, you get the bound state energy you might have guessed.