QUANTUM MECHANICS

Do any 3 of the 4 problems.

1. Consider a spinless particle of mass m and charge q confined to the surface of a sphere of radius R.

(a) Find the Hamiltonian H^0 for the particle along with the energy eigenvalues and eigenfunctions. Comment on degeneracy.

(b) Now suppose a constant magnetic field $\boldsymbol{B} = B_0\hat{\boldsymbol{z}}$ is added, in which case the vector potential $\boldsymbol{A} = -(1/2)\boldsymbol{r} \times \boldsymbol{B}$. By considering the Hamitonian H for a particle in a magnetic field, find the perturbation H' caused by the magnetic field to the unperturbed Hamiltonian H^0. (At this point, H' should be second order in B_0. Leave H' in terms of L_z.)

(c) Find the correction E_0' to the unperturbed ground state energy due to H'.

(d) Now assume that B_0 is small, so that the second order term in H' can be neglected. Find the correction(s) E_1' to the unperturbed first-excited-state energy.

Possibly useful information for Problem 1

The φ component of momentum p_φ in spherical coordinates (r, θ, φ) can be found by noting that $L_z = (r \sin\theta)p_\varphi$; hence,

$$p_\varphi = \frac{L_z}{r\sin\theta} = \frac{\hbar}{i}\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi} \quad .$$

2. Consider a particle of mass m in a one-dimensional double-delta-function potential $V(x) = -\alpha[\delta(x-a) + \delta(x+a)]$, where α and a are both positive, and $2a$ is thus the separation between the wells. If the separation distance is very small, show that there is only one bound state. Sketch the wavefunction and find the bound state energy, and then compare this energy to the energy that the particle would have $(-m\alpha^2/2\hbar^2)$ if there were only one delta-function well of strength $-\alpha$.

3. To a good approximation, the Hamiltonian for a positronium atom in the 1S configuration state in a magnetic field $\boldsymbol{B} = B_0\hat{\boldsymbol{z}}$ is given by

$$H = \frac{\alpha}{\hbar^2}\boldsymbol{S}_1 \cdot \boldsymbol{S}_2 + \frac{eB_0}{mc}\left(S_{1z} - S_{2z}\right) \quad ,$$

where $\boldsymbol{S}$ is a spin angular momentum, α is a characteristic energy, and the electron is labeled as particle 1 and the positron as particle 2.

(a) Experimentally, it is known that for $B_0 = 0$ the frequency of the triplet-to-singlet transition is 2.0338×10^5 MHz. Use this information to find α in eV. ($\hbar = 6.59 \times 10^{-16}$ eV-s.)

(b) In the basis consisting of the *coupled representation* spin states, find the matrix representation of H when the magnetic field is present. (From the form of H, you see that the magnetic field mixes the $m = 0$ singlet and triplet states, but that $\boldsymbol{B}$ does not affect the other two triplet states.)

(c) Find the eigenenergies in the presence of the magnetic field. Sketch the energy levels when $B_0 = 0$ and when $B_0 > 0$.

Possibly useful information for Problem 2

For two spin-1/2 particles, the states $|sm\rangle$ in the coupled representation can be written in terms of the states in the uncoupled representation as

$$\begin{aligned}
|00\rangle &= \frac{1}{\sqrt{2}}\left(\alpha_1\beta_2 - \beta_1\alpha_2\right) \\
|1+1\rangle &= \alpha_1\alpha_2 \\
|10\rangle &= \frac{1}{\sqrt{2}}\left(\alpha_1\beta_2 + \beta_1\alpha_2\right) \\
|1-1\rangle &= \beta_1\beta_2
\end{aligned}$$

where α is the spin-up state and β is the spin-down state.

4. Suppose that a one-dimensional potential $V(x)$ is given by

$$V(x) = \begin{cases} \infty, & \text{if } x < 0 \\ \alpha\delta(x-a), & \text{if } x \geq 0 \end{cases} \quad ,$$

where $a > 0$ is the width of the well and $\alpha > 0$ is the strength of the delta function. A particle of mass m is placed inside the well at time $t = 0$.

(a) Explain qualitatively what happens as time progresses.

(b) Derive the equation from which the particle energy can be determined.

(c) Notice that the energy E in part (b) is complex, and write it as $E = E_0 - i\gamma$ (but don't find E_0 and γ!). Use this to now quantitatively justify your answer for part (a).