Physics Comprehensive Exam
Quantum Mechanics 2009

(Do 3 of the following 4 problems. They have equal value)

1. A free particle of mass "m" and energy $E_0$ is incident from negative infinity onto a barrier of constant height $2V_0$, beginning at $x = 0$ and extending to $x = a$, and a connected barrier of constant height $V_0$, beginning at $x = a$ and extending to $x = 2a$, after which the potential returns instantly to zero.

   a. Sketch this potential
   b. State why bound states can or cannot exist for:
      1. $E_0 > 2V_0$
      2. $V_0 < E_0 < 2V_0$
      3. $0 < E_0 < V_0$
      4. $E_0 < 0$
   c. Write down the wavefunctions for the above cases.
   d. Apply the appropriate boundary conditions and write down the resulting equations for the $V_0 < E_0 < 2V_0$ case. Do not attempt to solve them.
   e. In terms of the variables you used in c above, write down the energy of the particle in each of the following regions for the $V_0 < E_0 < 2V_0$ case:
      1. $x < 0$
      2. $0 < x < a$
      3. $a < x < 2a$
      4. $x > 2a$
   f. In the real world, what might this problem apply to?

2. To a good physics student, the famous Kolmogorov solution for the propagation of laser light in a turbulent atmosphere is nothing more than the Born approximation for a carefully chosen scattering potential. Kolmogorov chose this potential to be of the form:

$V(r) = A/r^p$ where A is a constant equal to about $2 \times 10^{-28}$ (in mks units) and $1 < p < 2$

   a. What is the essential requirement for making the Born approximation and why might that make sense for laser light propagating in the atmosphere? You do not have to be an optics student to figure this one out.
   b. Write down the scattering amplitude integral equation for the potential given above and solve it.
   c. Find the ***differential*** cross-section for $p = 3/2$ .
   d. Show that the ***total*** cross-section for the $p = 3/2$ case is infinite. What does an infinite cross-section mean?
   e. What is the physical interpretation when p approaches 1 and 2?
   f. Experimentalists are much more interested in the differential cross-section because it can be written in terms of measurable quantities. One expression is:

$$D(\theta) = 1/L \; dN/d\Omega$$

where L is the "Luminosity" (particles per area per time, or irradiance to a good optics student- if you multiply by the energy of the particle). N is the number of detected particles per time and $\Omega$ is the solid (acceptance) angle of the detector.

Assume the Luminosity of the particle beam is about $6 \times 10^{17}$ particles per second per square meter and that your detector has an acceptance solid angle of 0.1Sr and it is placed at a 45 degree angle from the horizontal. Using your calculation in c above, about how many particles (they are electrons of energy 2 Mev) per second should you expect to capture?