QUANTUM MECHANICS

Do any 3 of the 4 problems. Circle and clearly indicate final answers.

1. Consider a particle of mass m bound by the delta-function potential $V(x) = -g\delta(x)$, where $g > 0$.

   (a) Derive the bound state energy and the corresponding properly-normalized energy eigenfunction.

   (b) Suppose that the momentum of the particle is now measured. What is the probability that the momentum will be larger than $mg/\hbar$?

2. Recall that a spin-1/2 particle with gyromagnetic ratio γ at rest in a static magnetic field $B_0\hat{z}$ precesses at the the Larmor frequency $\omega_0 = \gamma B_0$. Now suppose we apply an oscillating radiofrequency (RF) magnetic field $\boldsymbol{B_r} = B_r\hat{\boldsymbol{x}}\cos\omega t - B_r\hat{\boldsymbol{y}}\sin\omega t$.

   (a) Find the Hamiltonian matrix for the particle.

   (b) If χ is the matrix representation of the spin state, derive time-differential equations for $a(t)$ and $b(t)$, the amplitudes for being found in the spin up and spin down configurations, respectively. Let $\Omega = \gamma B_r$.

   (c) The solution of these equations is not trivial; however, if the particle starts out in the spin-up state, then $b(t) = (i/\omega')\Omega\sin(\omega' t/2)\exp(-i\omega t/2)$, where $\omega' = \sqrt{(\omega - \omega_0)^2 + \Omega^2}$. Calculate the probability of finding the particle in a spin-down state as a function of time.

   (d) Find the frequency where P is a maximum, as well as the full width at half maximum $\Delta\omega$. Using this technique of magnetic resonance, the gyromagnetic ratio (or the dipole moment) of a particle can be measured.

3. In a low-energy neutron-proton system (no orbital angular momentum), the potential energy is given by

$$V(r) = V_1(r) + V_2(r)\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + V_3(r)S_{12},$$

where $\boldsymbol{r}$ is the vector connecting the two particles and $\boldsymbol{\sigma}_i$ is the Pauli operator for particle i. The Pauli operator is composed of the usual Pauli matrices, and is related to the spin by $\boldsymbol{S} = (\hbar/2)\boldsymbol{\sigma}$. The term $S_{12} = 3(\boldsymbol{\sigma}_1 \cdot \widehat{\boldsymbol{r}})(\boldsymbol{\sigma}_2 \cdot \widehat{\boldsymbol{r}}) - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$.

(a) In early studies of the neutron-proton interaction, it became clear that the interaction had a spin dependence: for example, the deuteron has a total spin of 1, while the singlet state did not bind. This can be taken into account with a spin-dependent central force, like the first two terms in $V(r)$. Neglecting the third term, find $V(r)$ for the triplet and singlet states of the neutron-proton system. What must generally be true of the sign of V_2? (Hint: $\boldsymbol{S} = (\hbar/2)\boldsymbol{\sigma}$ defines $\boldsymbol{\sigma}$ for the total spin angular momentum, too.)

(b) The operator S_{12} depends on the relative orientation of the particles and their spins, and produces the tensor-force component (which has the longest range of the nuclear force components, by the way). One could construct the tensor force without the $\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$ term, but it is conventional to employ one whose average over all directions is zero, which is what this term ensures.

The form of this expression is very familiar from classical physics. What is it similar to? Describe how it produces a force that can be either attractive or repulsive, depending on the orientation of the spins and the particles (assume triplet state and $V_3 < 0$). What is a consequence of this force?

4. Consider scattering off of the delta-function potential $V(r) = aV_0\delta(r - a)$, where the distance $a > 0$.

(a) Find the radial wavefunction $u(r)$ inside and outside the shell, in terms of the S-wave partial phase shift δ_0. Do not apply the boundary conditions on the shell yet. (Recall: $j_0(x) = \sin x/x$ and $n_0(x) = -\cos x/x$.)

(b) Now find an expression for δ_0, in terms of the dimensionless parameter $\beta = 2ma^2V_0/\hbar^2$. (Hint: remember that ka is small for S-wave scattering.)

(c) Find the S-wave total cross section. Does it reduce to the correct result when $V_0 \to \infty$?