QUANTUM MECHANICS

Do any 3 of the 4 problems.

1. A quantum mechanical "rigid rotator" consists of two identical masses (each of mass m) separated by a constant distance d, and is free to rotate about the center of mass. (Such a configuration could be used to model the rotation of a linear diatomic molecule, for example.) Define the origin of the coordinate system such that it coincides with the center of mass. (Note: Half of the credit is with part (d).)

(a) What values can the angular momentum magnitude L assume?

(b) Derive an expression for the quantized energy levels in terms of d and m. Clearly define any other quantities used. What is the degeneracy of each energy level?

(c) Suppose that both ends are equally charged, so that the rotator now forms a magnetic dipole with dipole moment $\boldsymbol{\mu} = (q/2m)\boldsymbol{L}$, where q is the charge of either end. Suppose a perturbation in the form of a uniform magnetic field $\boldsymbol{B} = B\hat{\mathbf{z}}$ is now introduced. Using the fact that the interaction energy between the dipole and $\boldsymbol{B}$ is $H' = -\boldsymbol{\mu} \cdot \boldsymbol{B}$, find the first-order correction to the ground-state energy.

(d) Show that the degeneracy of the first excited state is broken, and find the first-order corrections to the unperturbed energy.

(e) Argue as to whether the degeneracy of all other excited states will be broken as well.

2. A particle moves in one dimension with energy E in the field of a potential given by the sum of a step function and a delta function; that is, $V(x) = V_0\eta(x) + g\delta(x)$, with V_0 and g both positive. (The step function $\eta(x)$ is zero for $x < 0$ and unity for $x \geq 0$.) Assume $E > V_0$.

(a) Find the reflection coefficient R for a particle incident from the left. You can leave the answer in terms of wavenumbers, as long as they are clearly defined.

(b) Find R (in terms of E) in the limit $E \gg V_0$. Is this result as expected, given that $R = [1 + (2\hbar^2 E/mg^2)]^{-1}$ when no step function is present?

(c) Find R when $g = 0$. Check your result by considering the case when $E = V_0$ and $E \gg V_0$. Are the results what you expected?

3. Consider the potential $V(x) = \infty$ for $x < 0$ and ax for $x \geq 0$, where the constant $a > 0$.

(a) Sketch the potential and the approximate shape of the ground-state wavefunction. Write down a functional form, in terms of a normalization constant and one free parameter, for this wavefunction and justify your choice. Now normalize this trial wavefunction.

(b) Estimate the ground state energy of a particle of mass m in this potential well. (Simplify your expression as far as possible; express any fractions as decimal numbers.) Is the actual ground-state energy less than or greater than this estimate?

4. The magnetic dipole moment of an electron is $\boldsymbol{\mu} = -(e/m)\boldsymbol{S}$, where e is the charge magnitude, m is the mass, and $\boldsymbol{S}$ is the spin angular momentum.

(a) What are the eigenenergies of an electron in the presence of a magnetic field $\boldsymbol{B} = B_0\hat{\boldsymbol{z}}$ ($B_0 > 0$)? Which energy corresponds to which spin state?

(b) If $B_z = 0.5$ T, what is the wavelength of the radiation that is absorbed/emitted when an electron flips its spin orientation?

(c) Now suppose that all N_0 electrons in the magnetic field are in the spin-down state. They could be placed in the spin-up state by bathing the system in radiation of wavelength determined in part (b). Instead, flip their spin by turning on a small x component to the magnetic field $B_1\hat{\boldsymbol{x}}$ ($0 < B_1 \ll B_0$) for a time t. Find the perturbation Hamiltonian.

(d) Derive an expression for the number of electrons that have flipped their spin as a function of t.

(e) Derive an expression for the time at which the number of spin-up electrons is a maximum. If $B_1 = 0.1B_0$, what fraction of the electrons will be spin up at this optimal time?

(f) Suppose that instead of introducing a B_x component, we just changed the magnitude of B_0. Can we get a higher fraction with spin up this way?

Possibly Useful Result

$$P_{nm} = \frac{4|H'_{mn}|^2}{(E_m - E_n)^2} \sin^2\left[\frac{E_m - E_n}{2\hbar}t\right]$$