QUANTUM MECHANICS - 2008

Do any 3 of the 4 problems. Circle and clearly indicate final answers.

1. Consider a particle of mass m in a two-dimensional infinite square well (extending from $x = 0$ to $x = L$ and from $y = 0$ to $y = L$).
   (a) Write down the wavefunction(s) and energy(ies) for the particle in the first excited state.
   (b) Now the well gets a perturbation of the form $H' = Cxy$, where C is a constant. Find the first-order energy correction(s) to the first excited state.

2. For a one-dimensional quantum mechanical simple harmonic oscillator, find the expectation values of x, x^2, p, and p^2 in the n-th stationary state. Now find the product of the standard deviations of x and p in the ground state. Is this as expected?

Possibly Useful Information for Problem 2

$$a = \sqrt{\frac{m\omega}{2\hbar}}\left(x + i\frac{p}{m\omega}\right)$$

$$a^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(x - i\frac{p}{m\omega}\right)$$

3. Consider a system of two particles, one with spin quantum number $s_1 = 1$ and the other with spin quantum number $s_2 = 1/2$. The Hamiltonian for the system is $H = A\boldsymbol{S}_1 \cdot \boldsymbol{S}_2$, where A is some constant.
   (a) What are the units of A?
   (b) Find all the eigenvalues and eigenstates of H.
   (c) The system is now placed in a magnetic field in the z direction, and the new Hamiltonian is

$$H_1 = A\boldsymbol{S}_1 \cdot \boldsymbol{S}_2 + B\left(S_{1z} + S_{2z}\right) \quad .$$

   Find the exact eigenvalues of H_1.

4. A (spin-1/2) electron is placed in a magnetic field $\boldsymbol{B} = B\hat{\boldsymbol{z}}$, so that the Hamiltonian $H = -\boldsymbol{\mu}\cdot\boldsymbol{B}$, where

$$\boldsymbol{\mu} = -g_s \frac{e}{2mc}\boldsymbol{S}$$

is the magnetic dipole moment of the particle (in cgs units) and g_s is the usual Landé g factor.

(a) Which, if any, of the following quantities are conserved? S^2, S_x, S_y, and S_z.

(b) The spin state of the electron at time $t = 0$ is $|\psi\rangle = a|\alpha\rangle + b|\beta\rangle$, where $|\alpha\rangle$ and $|\beta\rangle$ are the usual spin-up and spin-down states, and a and b are (real) constants (constrained by normalization, of course). Calculate the expectation values of S_x, S_y, and S_z at later time t.