QUANTUM MECHANICS

Do any 3 of the 4 problems.

1. Suppose that a delta-function potential $aV_0\delta(x)$ is placed inside an infinite 1-D square well, which extends from $x = -a$ to $x = +a$. The quantity V_0 is a measure of the delta-function potential strength, and has units of energy. Assume here that $V_0 > 0$.

   (a) Consider the even and odd states separately, and derive the conditions that give the bound state energies. You may wish to express your result in terms of the dimensionless parameters $g = \hbar^2/ma^2V_0$ and $y = ka$.

   (b) Find the solutions to the above conditions when $V_0 \to 0$ and $V_0 \to \infty$ and comment on the results.

potential

2. Consider a particle of mass m confined to an infinite cylindrical well: Inside the cylinder of radius a and length L, the potential is zero; outside the cylinder, the potential is infinity. The axis of the cylinder lies along the z axis and its base is on the x-y plane.

   (a) Assuming the particle is in a zero angular momentum state, find the time-independent Schrödinger equation for the particle.

   (b) Now find the allowed energies of the particle. At some point, you may want to employ the dimensionless variable $x = \pi r/L$.

   (c) Find the allowed energies in the limit of $a \to \infty$. Comment.

Possibly Useful Information for Problem 2

In cylindrical coordinates,

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\phi^2} + \frac{\partial^2}{\partial z^2} \quad .$$

Bessel's Differential Equation is given by

$$x^2R'' + xR' + k^2x^2R = 0 \quad ,$$

with solutions $R = J_0(kx)$ and $N_0(kx)$.

3. Consider particle scattering from the "soft sphere" potential $V(r) = V_0$ for $r \leq a$ and $V(r) = 0$ for $r > a$.

(a) Find the scattering amplitude in the Born approximation for arbitrary energies. Define all quantities that are used.

(b) Derive the equation that predicts when the scattering amplitude is zero.

(c) Suppose that a detector is placed at 60° away from the direction of the incident particles. Show that the scattering amplitude is zero when the product mEa^2 is approximately $10\hbar^2$.

(d) Show that in the low-energy limit, the scattering amplitude is proportional to the volume of the sphere and independent of scattering angle and incident particle energy.

<u>Possibly Useful Information for Problem 3</u>

The first three zeroes of $\tan x = x$ are 0, 4.493, and 7.725. The first three zeroes of $\tan x = -x$ are 2.029, 4.913, and 7.979. The first three zeroes of $\cot x = x$ are 0.860, 3.426, and 6.437.

4. You have prepared an experiment containing a single electron and you measure the x, y, and z components of its spin angular momentum. You then repeat this many times for an ensemble of identical experiments, and find the average values to be $0\,\mathrm{kg\,m^2\,s^{-1}}$ for the x component, $-5.0619 \times 10^{-35}\,\mathrm{kg\,m^2\,s^{-1}}$ for the y component, and $-1.4764 \times 10^{-35}\,\mathrm{kg\,m^2\,s^{-1}}$ for the z component. You also measure the variance in the data for each component and find that it is $2.7803 \times 10^{-69}\,\mathrm{kg^2\,m^4\,s^{-2}}$ for the x component, $2.1798 \times 10^{-70}\,\mathrm{kg^2\,m^4\,s^{-2}}$ for the y component, and $2.5623 \times 10^{-69}\,\mathrm{kg^2\,m^4\,s^{-2}}$ for the z component. From the data, you want to determine the spin state χ of the electron.

(a) Derive the four equations which determine the components a and b of the spinor χ. You may want to put the raw data in terms of $\hbar$ and $\hbar^2$. Also remember that χ must be properly normalized and that a and b are complex, in general.

(b) Solve these equations and thus determine the properly normalized χ.

(c) Verify that your data do not violate the Heisenberg Uncertainty Principle.