QUANTUM MECHANICS

Do any 3 of the 4 problems. Circle and clearly indicate final answers.

1. Consider a particle of mass m in a one-dimensional infinite square well located between $x = 0$ and $x = a$. You are, of course, very familiar with this situation. Here we will explore the consequences of a "hidden" extra dimension, say y, upon the energy eigenvalues.

(a) Suppose that the dimension y closes in upon itself, like a circle of radius R. Find the energy eigenvalues of such a system.

(b) What is the ground state energy with the extra dimension?

(c) The question is now whether this extra "hidden" dimension can be revealed or observed. Let's say that, in order to be easily observable, one of the energies must lie between the ground state and the first excited state of the system *if the extra dimension is not present.* Calculate the value of R above which this condition can be satisfied. Hence, you see that, for sufficiently-small R, this extra dimension will remain "hidden". Extra dimensions are a characteristic of, for example, string theory.

2. Consider low-energy scattering off of an attractive "soft sphere": $V(r) = -V_0$ for $r \leq a$, and $V(r) = 0$ for $r > a$.

(a) Find the S-wave phase shift δ_0.

(b) Find the S-wave scattering cross section.

(c) Discuss the resonances in this cross section.

Possibly useful information for Problem 2

$$j_0(x) = \frac{\sin(x)}{x}, \qquad n_0(x) = -\frac{\cos(x)}{x}$$

3. For a potential hill centered at $x = 0$, which has $V = 0$ beyond a certain distance a from the origin, the following relations hold: If

$$\psi(x) = \begin{cases} Ae^{ikx} + Be^{-ikx}, & \text{if } x < -a, \\ Ce^{ikx} + De^{-ikx}, & \text{if } x > a, \end{cases}$$

then define a (transfer) matrix M such that

$$\begin{pmatrix} C \\ D \end{pmatrix} = \begin{pmatrix} s & u \\ v & w \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = M \begin{pmatrix} A \\ B \end{pmatrix} .$$

(a) Find the elements of the S matrix in terms of the elements of M.

(b) Consider an identical hill centered at $x = -d$, where $d > 2a$. Find the M matrix for this hill in terms of existing quantities.

(c) Find the overall transmission coefficient from left to right for both hills.

4. Consider an electron of charge $-e$ and mass m in a constant magnetic field $\boldsymbol{B} = \hat{z}B_0$. The electron's magnetic moment $\boldsymbol{\mu} = -(ge/2mc)\boldsymbol{S}$, where g is Landé g-factor.

(a) Find a general expression (i.e., one before initial conditions are applied) for the time evolution of $\langle S_x \rangle$

(b) Suppose a measurement at time $t = 0$ shows the electron spin to be aligned along the positive x direction. Find the time evolution of $\langle S_x \rangle$ in this case.

(c) What is the frequency of $\langle S_x \rangle$? Comment on this result.