The following may be useful

$$\mathfrak{F}\{f(x)\} = F(\xi) = \int_{-\infty}^{+\infty} f(x) e^{-i2\pi\xi x} dx$$

$$\mathrm{Gaus}(x) = e^{-\pi x^2}, \qquad \mathfrak{F}\left\{\mathrm{Gaus}\left[\frac{x}{b}\right]\right\} = |b|\,\mathrm{Gaus}(b\xi)$$

$$\mathrm{rect}(x) = \begin{cases} 1 & |x| < \frac{1}{2} \\ \frac{1}{2} & |x| = \frac{1}{2} \\ 0 & |x| > \frac{1}{2} \end{cases}, \qquad \mathfrak{F}\{\mathrm{rect}(x)\} = \mathrm{sinc}(\xi) = \frac{\sin(\pi\xi)}{\pi\xi}$$

$$\mathfrak{F}\left\{\exp(\pm i\pi x^2)\right\} = \exp\left(\pm i\tfrac{\pi}{4}\right)\exp(\mp i\pi\xi^2)$$

$$f(x) * g(x) = \int_{-\infty}^{+\infty} f(\alpha) g(x-\alpha) d\alpha$$

(1) Given that $F(\xi) = \mathfrak{F}\{f(x)\}$ and $G(\xi) = \mathfrak{F}\{g(x)\}$ Work out the following problems. **<u>PLEASE USE WORDS</u>** to explain the physical meaning of each.

(A) $\mathfrak{F}\{f(x) * g(x)\} =$

(B) $\mathfrak{F}\{f(x) \cdot g(x)\} =$

(C) $\mathfrak{F}\left\{\mathfrak{F}\{f(x)\}\big|_{\xi=x}\right\} =$

(D) $\mathfrak{F}\left\{f(x) \cdot e^{i2\pi\xi x}\right\} =$

(E) $\displaystyle\int_{-\infty}^{+\infty} \frac{\sin(\pi a\alpha)}{\pi a\alpha} \cdot \frac{\sin[\pi b(x-\alpha)]}{\pi b\alpha} d\alpha =$

(2) For describing monochromatic light propagating in free-space (vacuum) in the z-direction, we would specify its frequency ν, angular frequency $\omega=2\pi\nu$, vacuum wavelength λ_0, or vacuum wavevector magnitude $k=2\pi/\lambda_0$. When this same monochromatic light (or electro-magnetic radiation) propagates in a (nearly) transparent dielectric media (air, glass, H_20 etc.), the propagation depends on a single material parameter, the index of refraction n, which varies with the light's wavelength or frequency (i.e. we would write either $n(\omega)$, $n(\lambda)$, or etc.).

(A) We can mathematically represent this field in terms of its phase and complex amplitude

$$\underset{\to}{E}(z,t) = \underset{\to}{E}_0\, e^{i(Kz-\omega t)}$$

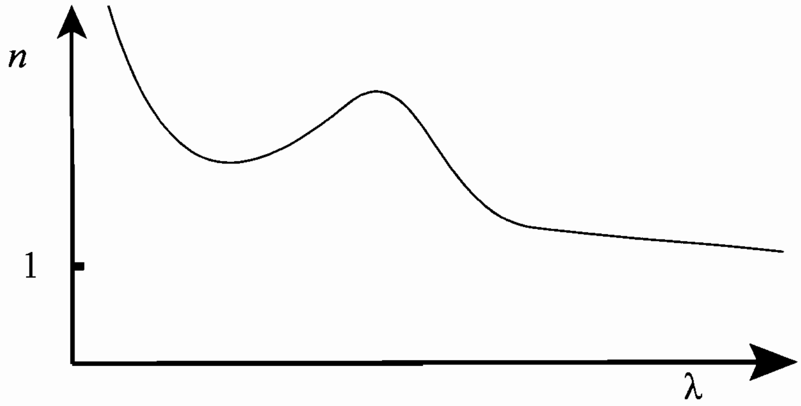
where K is the magnitude of the wavevector within the nearly transparent media. In terms of n, λ_0, ω, etc., give the relationship or definitions for the phase velocity v_φ, wavevector K and wavelength λ of the monochromatic field within the media.

(B) For a narrow-band, poly-chromatic field, the superposition of plane waves leads to packets or groups of waves in space where the field amplitude is high. By interfering two equal amplitude plane waves of nearly equal frequency ($\omega_1=\omega_0+ \frac{1}{2}\Delta\omega$ & $\omega_2=\omega_0-\frac{1}{2}\Delta\omega$), show that the resultant field can be written in the form

$$E_{Total} = E \cos\left[\Delta\omega\left(\frac{1}{v_g}z-t\right)\right] e^{i(\overline{K}z-\omega_0 t)}$$

or a *Group Term* times a *Phase Term*. Give a formal definition for the group velocity, v_g and the value of $\overline{K}$

(C) A plot of the index of refraction vs. wavelength for a certain transparent material is given below



Derive an expression for the group velocity in terms of the phase velocity, index and wavelength. Point out regions in the above plot that correspond to the phase velocity being greater than the group velocity.

(3) On the following page there is a diagram of a simple optical system composed of two thin lenses and an aperture stop. The lenses are identical having a focal length of 50mm and are 50mm apart. We place a 20mm high object 50mm in from of the first lens.

(A) Use the ray chart and trace conditional a-ray and b-ray through the system (as given). Find the image location.

(B) Find the Chief and marginal rays and trace them through the system. Draw them on the figure (which is to scale).

(C) On the figure locate and label: the image, the chief ray, the marginal ray, the principal planes, the focal points, the exit pupil and the entrance pupil. Supply the data requested on the next page.

Surface #	Object	1	2-stop	3	4-Image
$-\varphi_j$	0		0		0
t_j/n_j	50mm	25mm	25mm		
y_A	0mm				
nu_A					
y_B	20mm				
nu_B	0				
Marginal Ray					
y	0		2.5		
nu					
Chief Ray					
$\bar{y}$	5		0		
$\overline{nu}$					

If Additional space is needed to explain what you are doing please show this work on a separate 8½"x11" sheet of paper. Make a note here that such a sheet is attached.

Front Principal Plane: S#1→P_1=

Rear Principal Plane; P_1→S#3=

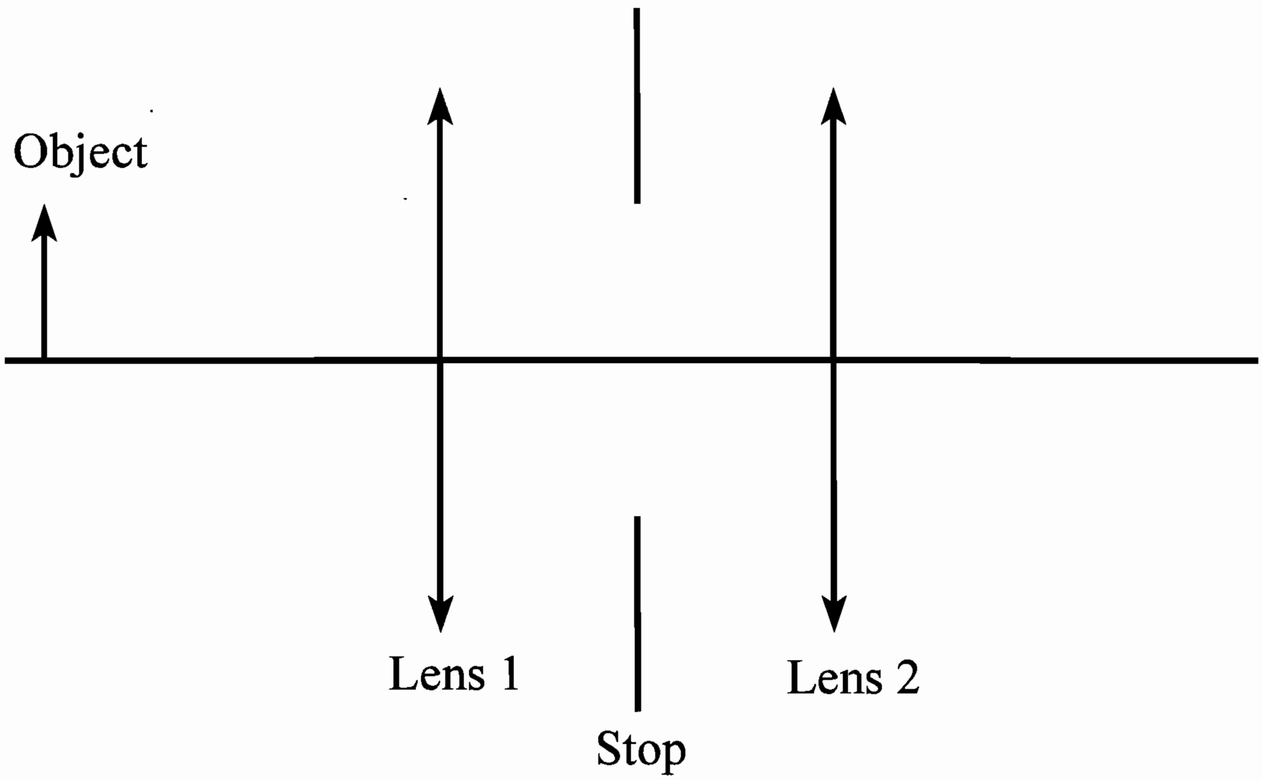
Entrance Pupil: S#1→E_1=

Diam of E_1=

Exit Pupil: E_2→S#3=

Diam of E_2=

Focal Length f=



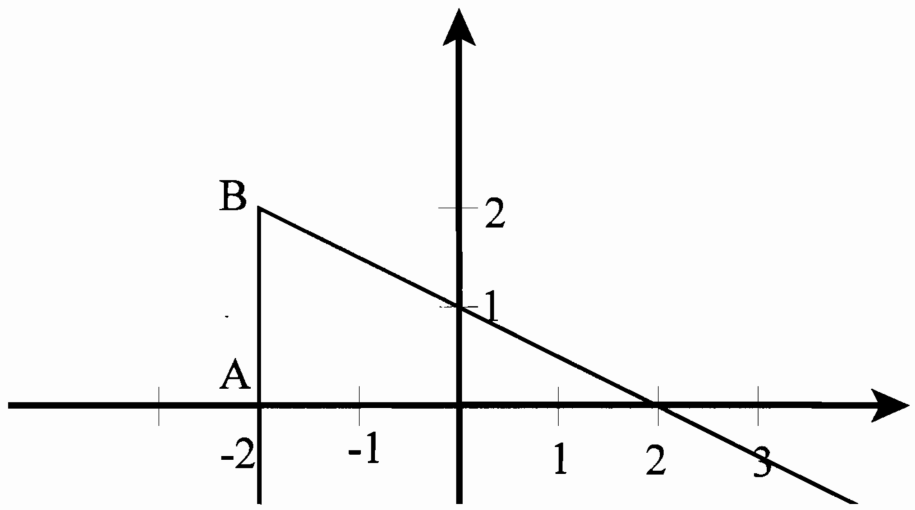
(4) A free-pion laser (a close relative to the free-electron laser) emits a Gaussian beam with a wavelength of 3.24250265μm. The y-$\bar{y}$ diagram for a beam from this laser is given below. The units are in millimeters. The output mirror plane is labeled A on the diagram. A lens is locate at the junction labeled B on the diagram.

(A) What is the size of the beam at the output mirror (plane-A) and at the lens (plane-B)?

(B) Indicate the locations of the beam waist planes on the y-$\bar{y}$ diagrams (both before and after the lens). What are the sizes of the beam waists and their locations relative to the lens?

(C) What is the radius of curvature of the beam just before and just after it passes through the lens?

(D) What is the focal length of the lens?



(5) Application of Moire to aberration analysis.

(A) Define the meaning of the wavefront aberration function. For a cylindrically symmetric optical system discuss the invariants that this function is expanded about.

(B) The first four terms of the wavefront aberration function expansion are:

$$W(h,\rho,\phi) = W_{000} + W_{200}\, h^2 + W_{020}\, \rho^2 + W_{111}\, h\, \rho \cos\phi + \cdots$$

Complete the expansion to the next order (i.e. include third-order aberrations) and identify all the terms.

(C) For a fixed field height, h, we reduce the wavefront aberration function to

$$W(\rho,\phi) = W_{00} + W_{20}\, \rho^2 + W_{11}\, \rho \cos\phi + W_{40}\, \rho^4 + W_{31}\, \rho^3 \cos\phi + W_{22}\, \rho^2 \cos^2 \phi + \cdots$$

Where the reduced terms are piston, defocus, y-tilt, spherical, coma, and astigmatism. We can use an interferometer to measure the wavefront aberration function. Tell how this is done and show that the resulting interference pattern is of the form

$$T(\rho,\phi) = T(\rho_x,\rho_y) = 1 + V \cdot \cos\left(\frac{2\pi}{\lambda} W(\rho,\phi)\right).$$

You are given such an interferogram pattern on the next page (which is hard limited). Recall that a Moire interference pattern is obtain by multiplying two transmission function together,

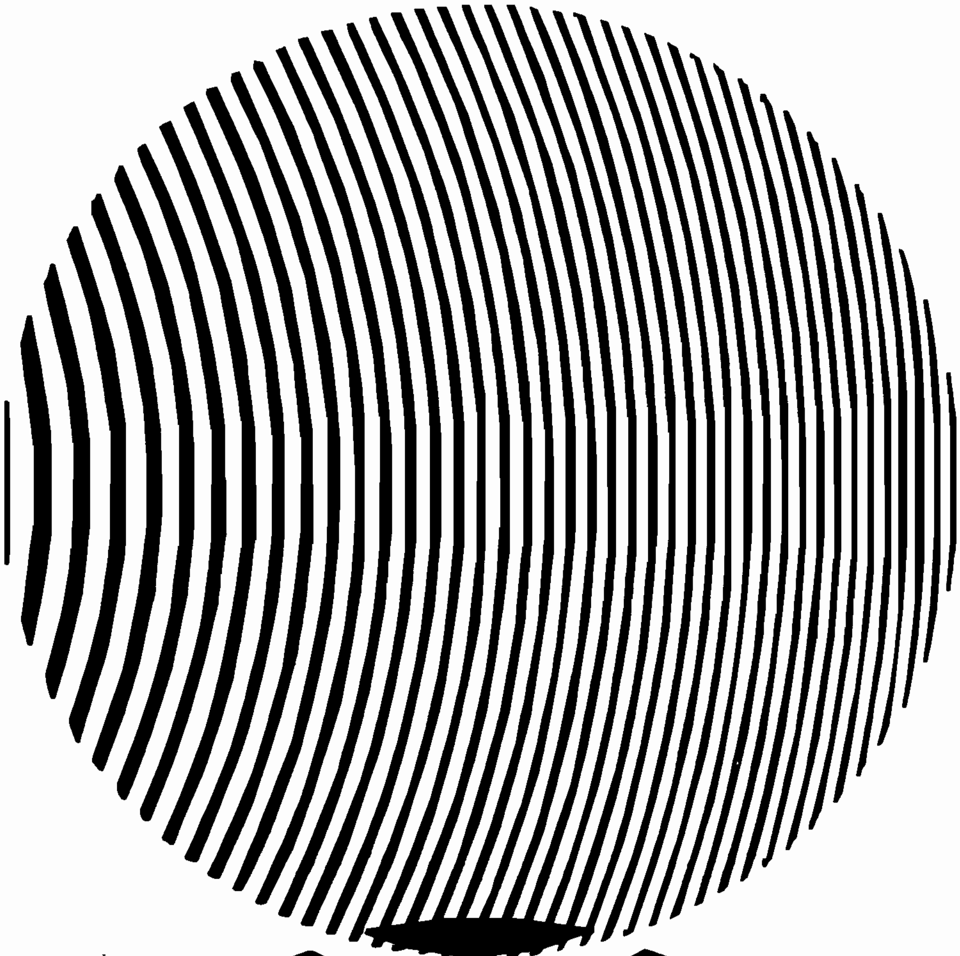
$$T_{Moire}(\rho,\phi) = T_1(\rho,\phi) \cdot T_2(\rho,\phi).$$

(D) Show that by forming the Moire pattern between the interferogram and its y-flipped self,

$$T(\rho_x,\rho_y) \cdot T(\rho_x,-\rho_y) = \left[1 + V \cdot \cos\left(\frac{2\pi}{\lambda} W(\rho_x,\rho_y)\right)\right] \cdot \left[1 + V \cdot \cos\left(\frac{2\pi}{\lambda} W(\rho_x,-\rho_y)\right)\right]$$

a Moire interference fringe pattern results that cancels the y-tilt. Use this method to find the aberration present in the interferograms on the next page. (Interferograms with y-tilt and x-tilt are given for the system.)

Interferogram
with x-tilt



with y-tilt