Direction: Work 3 out of the following 4 problems.

(1) **Basic elements of Geometrical Optics:** (Please use the military standard notation for specifying components and values in this problem.)

(A) In specifying the requirements for an optical system you are given, the size, distance and radiance of the object, the detector elements size and total image area, the NEP (Noise Equivalent Power) of the detector elements, and the required signal to noise ratio. This information allows you to calculate the single most important parameter of the system. Identify this parameter, give its definition, tell why it is important and any other embellishments you wish to add.

Given the specification of an optical systems, we often begin our analysis of the system by tracing two provisional rays, ray-A and ray-B. Starting at the object we perform a paraxial ray trace, where the two rays originate at the object with values

$$\begin{pmatrix} y_A \\ nu_A \end{pmatrix}_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} y_B \\ nu_B \end{pmatrix}_0 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}.$$

(B) What is the optical invariant $\mathcal{K}_{AB}$ between the A & B rays at object plane? What is it at the image plane?

(C) Tell how the A & B rays are used in image space to find

(i) the image location
(ii) the rear focal point
(iii) the rear principal plane
(iv) system magnification

Note that at the image plane $\begin{pmatrix} y_A \\ nu_A \end{pmatrix}_k = \begin{pmatrix} 0 \\ nu_{A,k-1} \end{pmatrix}$ and $\begin{pmatrix} y_B \\ nu_B \end{pmatrix}_k = \begin{pmatrix} y_{B,k} \\ nu_{B,k-1} \end{pmatrix}$.

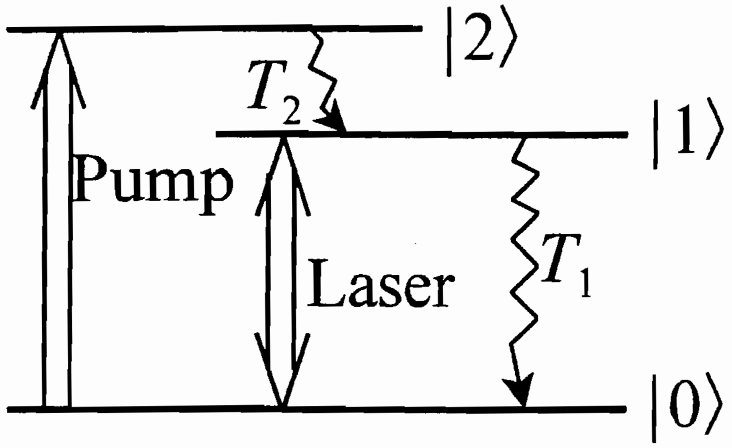
(D) Show that a new ray can be constructed in image space with unit height and parallel to the optical axis in terms of a superposition of the A & B ray:

$$\begin{pmatrix} y_C \\ nu_C \end{pmatrix} = \alpha_1 \begin{pmatrix} y_A \\ nu_A \end{pmatrix} + \alpha_2 \begin{pmatrix} y_B \\ nu_B \end{pmatrix}$$

(E) Tell how this C ray is used in object space to find

(i) the front focal point
(ii) the front principal plane

(2) **Laser Physics**: Consider the three-level laser system below, which determine all the optical properties of the gain medium.



In this system, the gain medium is excited by the pump radiation from the ground state $|0\rangle$ to the excited state $|2\rangle$ and the system rapidly decays to the intermediate level $|1\rangle$ with a decay lifetime $T_2 \approx 1$ picosecond. Level $|1\rangle$ is relatively meta-stable for decay to the ground state with a lifetime $T_1 = 10\ \mu$s. The number density of gain centers in the medium is $\mathcal{N}$ = 4 x 10^12 per mm^3. The gain medium length is $\ell = 1$ cm, and the gain modal volume is $\mathcal{V}$. The standing-wave laser cavity consists of a 100% reflector and output coupler with reflectivity $\mathcal{R} = 0.95$ and transmission $\mathcal{T} = 0.05$ (where $\mathcal{R} + \mathcal{T} = 1$.) The cavity length is L = 20 cm. The output coupler has a radius of curvature of 40 cm and the 100% reflector is a planar mirror. The gain medium is located near the plane mirror. The pump transition angular frequency is ω_P and laser angular frequency ω_L, with corresponding wavelength of $\lambda_P = 1.23\ \mu$m and $\lambda_L = \pi\ \mu$m = 3.1415 μm (i.e. this is the infamous free pi-laser.) (Recall $\hbar$ = 1.05 x 10^-34 J s = 6.58 x 10^-16 eV s)

(A) Calculate the transition resonance frequencies and energies for the pump and laser fields.

(B) Find the fundamental mode Gaussian beam waist, w_0. (Hint: The cavity is one-half a confocal cavity, the waist is located at the planar mirror, and the cavity is a Rayleigh distance in length, i.e. $L = z_0$. Think how simple y-ybar makes this problem.)

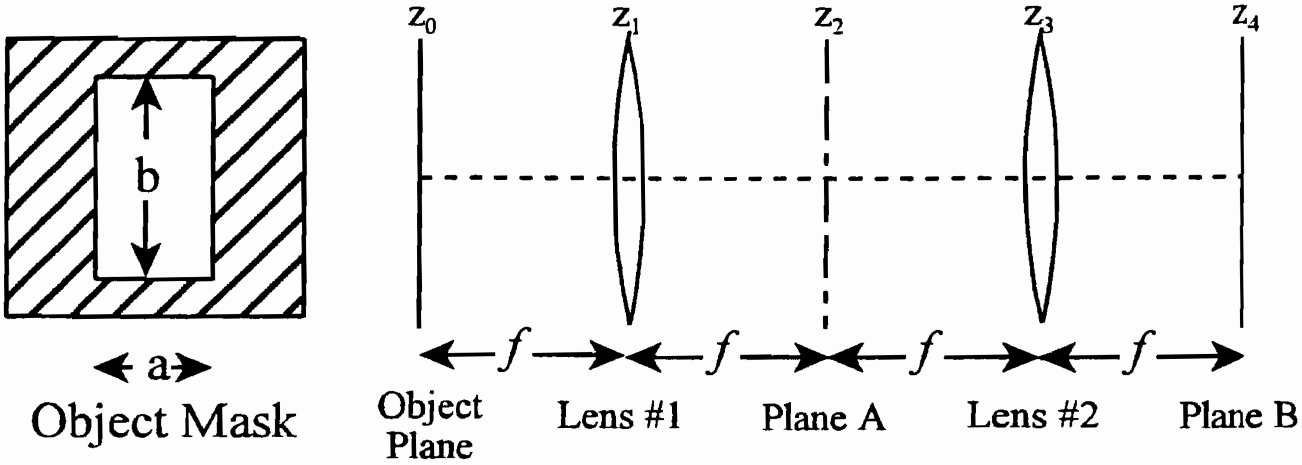
(C) The population density in the three levels are ρ0, ρ1, & ρ2 where ρ0 + ρ1 + ρ2 = $\mathcal{N}$. Assume that the total pump flux Φ_P (measured in watts) is absorbed by the media. With only the pump field present (i.e. the system is not lasing) develop the three rate equations for the population densities.

(D) Discuss why and show that these equations can be reduced to a single equation for the population difference (or inversion) between the level $|1\rangle$ and the ground state $|0\rangle$.

(E) What pump power makes the medium transparent at the laser transition wavelength? For this problem assume that the gain mode volume is $\mathcal{V} = 8\, w_0^2\, \ell$.

(F) When the material is not pumped (i.e. all the gain centers are in the ground state $|0\rangle$), the 1 cm thick gain material absorbs 25% of incident light from a beam resonant to the lasing transition, $|0\rangle$ to $|1\rangle$. Find the empty cavity round-trip loss and the cavity decay time. Determine the threshold pumping power for the laser.

(3) **Propagation and Diffraction:** Consider the optical system pictured below. The two lenses are identical having a focal length of f. The distance between the object plane, Lens #1, Plane A, Lens#2, & Plane B are all equal to f. The diameters of the lenses are large, so that they do not contribute to diffraction in this problem. The input wavelength is λ.



(A) First consider a Gaussian beam as the input. The beam waist size is w_0 and is located at the object plane. Describe the Gaussian beam before and after lens #1, at Plane A, before and after lens #2 and at Plane B, by giving the beam size and the wavefront's radius of curvature.

(B) Now consider that a mask with a rectangular opening is placed at the object plane. This mask is illuminated with a spatially coherent, monochromatic planewave. In the clear aperture of the object mask, the exitance is M, hence the total flux that passes through the rectangular mask is $\Phi = M \cdot a \cdot b$. Find the optical field and the flux irradiance before and after lens #1, at Plane A, before and after lens #2 and at Plane B. These expressions can be left in the from of convolution integrals, except at Plane A & B where explicit expressions must be given along with a clearly labeled diagram or plot of the irradiance in these planes.

(C) If the mask is illuminated with spatially incoherent light, how do the irradiances at Plane A and Plane B differ from the coherent case?

Fresnel Propagation Kernel: $u(x,y;z) = \dfrac{1}{i\lambda z}\exp\left(i\dfrac{\pi}{\lambda z}\left(x^2+y^2\right)\right)$

2-Dimentional Fourier Transform

$$\mathfrak{F}^{(2)}\{u(x,y;z)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u(x,y;z)\exp\left(-i\,2\pi(\xi x + \eta y)\right)dx\,dy$$

$$= U(\xi,\eta;z) = -\exp\left(i\pi\lambda z\left(x^2+y^2\right)\right)$$

Transform Property of Lens: $T_{lens}(x,y;f) = \exp\left(-i\dfrac{\pi}{\lambda f}(x^2+y^2)\right)$

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This page is to be turned in with your exam!!!!!

(4) An optical system consists of three lenses and a stop. The focal lengths and order of the lenses and stop are f_1 = 10 mm, stop, f_3 = 20 mm, and f_4 = 24 mm. The object is t_0 = 10 mm in front of the first. The spacing between element in order are t_1 = 10 mm, t_2 = 30 mm, and t_3 = 40 mm. The aperture stop radius is 2 mm.

(A) Enter the system data into the attached ynu Ray Tracing Worksheet.

(B) Trace the provisional A & B rays. Find the image location, system magnification, and focal length.

(C) Find and trace the marginal and chief rays. Determine the systems Optical Invariant.

(D) Make a $y - \bar{y}$ diagram of the system. Identify the location of the Front and Rear Principle Planes and the Front and Rear Focal Point Planes.

(E) Identify the size and location of Entrance and Exit Pupils. Specifically, find the location (i.e. distance) of the Exit Pupil from the last element.

Surface #	Object	Lens #1	Stop–2	Lens #3	Lens #4	Image
$-1/f_j$						
t_j						
y_A	0					
nu_A	1					
y_B	-1					
nu_B	0					
y	0					
nu						
$\bar{y}$	-1		2			
$\bar{n}\bar{u}$						