Problem 1

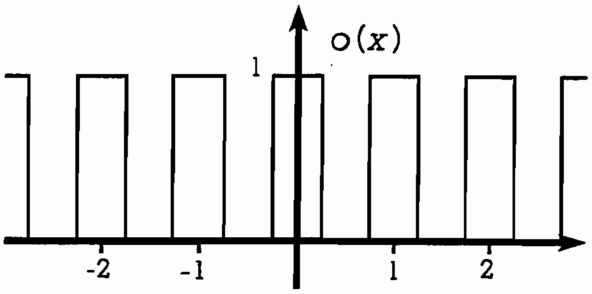
Consider a one-dimensional incoherent imaging experiment where the object is a square-wave

$$o(x) = rect(2x) * comb(x)$$

where

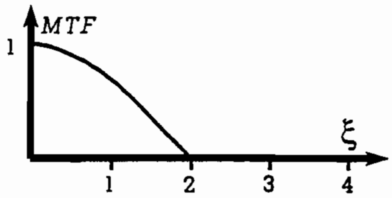
$$rect(x) = \begin{cases} 1 , & |x| < \frac{1}{2} \\ \frac{1}{2} , & |x| = \frac{1}{2} \\ 0 , & |x| > \frac{1}{2} \end{cases}$$

$$comb(x) = \sum_{n=-\infty}^{+\infty} \delta(x-n)$$



and $*$ signifies convolution. Let this object be imaged with unit magnification by an optical system with unit magnification with a real *OTF* and modulation transfer function (*MTF*) given by

$$MTF(\xi) = \cos\left(\frac{2\pi\xi}{8}\right) \bullet rect\left(\frac{\xi}{4}\right) .$$



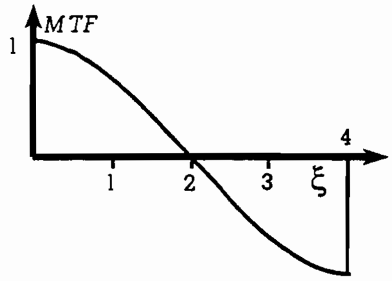
(a) Determine the spatial frequency content of the object $O(\xi) = \Im\{o(x)\}$, where $\Im$ indicates the Fourier transform,

$$\Im\{f(x)\} = \int_{-\infty}^{+\infty} f(x) e^{-j2\pi\xi x} dx .$$

(b) Determine the spatial frequency content of the image $I(\xi)$. Calculate and sketch the image $i(x)$.

(c) By changing the optical system the *MTF* becomes:

$$MTF(\xi) = \cos\left(\frac{2\pi\xi}{8}\right) \bullet rect\left(\frac{\xi}{8}\right)$$



Determine the resulting image $i_2(x)$ for the changed system.

Problem 2

(a) Plot the general from of the s and p intensity reflectance coefficients R_s and R_p for reflection in air from an air/water interface, $n_{H20}=1.33$. For reference, the s and p amplitude reflection coefficients (Fresnel coefficients) are given below.

$$r_s = \frac{n_1\cos(\theta_1) - n_2\cos(\theta_2)}{n_1\cos(\theta_1) + n_2\cos(\theta_2)}$$

$$r_p = \frac{n_2\cos(\theta_1) - n_1\cos(\theta_2)}{n_2\cos(\theta_1) + n_1\cos(\theta_2)}$$

On your plot indicate the values at normal incidence, Brewster's angle, and grazing incidence. Your plot does not need to be numerically accurate but should show the general features of the reflectance.

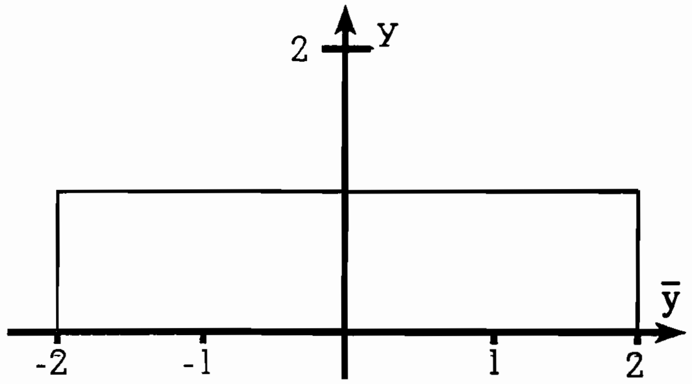
(b) Calculate Brewster's angle for this interface.

(c) Consider a still pond illuminated in all direction with unpolarized light from a cloudy sky. The reflected light has its polarization altered in reflection. Describe the polarization of this reflected light over the entire 2π steradian on incidence angles. Show with diagrams and explain all significant characteristics of the polarization of this light. Where is the reflected light fully polarized and how is the electric field oriented? Where is the reflected light unpolarized?

(d) Now consider a sky where the illumination is uniform and the polarization is oriented along a north-south (N-S) line everywhere in the sky. Describe with words and figures the intensity and reflected polarization state along three meridians: (i) N-S, (ii) E-W, and (iii) NE-SW.

Problem 3

Pictured below is the y-$\bar{y}$ diagram of a doubly telecentric radiometer based on two thin lenses. The object is a 4 cm diameter disk, Lambertian source with a brightness (sterance or radiance) of $L_p = \frac{1}{\pi} \times 10^5$ nt. (Note 1 nt = 1 nit = $1 \frac{\text{lumin}}{\text{meter}^2\ \text{steradian}} = 1 \frac{\text{lm}}{\text{m}^2\ \text{sr}}$).



(a) It is required that the illuminance (or incidence) on the detector be equal to $E_p = 10$ lx. (Note that 1 lx = 1 lux = 1 lm/m^2.) Find the numerical aperture, NA, of the system at the image. What is the optical invariant, ϰ, of the system? What is the optical throughput or étendue of the system?

(b) Find the minimum clear apertures of the two lenses. What is the transverse magnification of the system? What is the size of the aperture stop?

(c) Given the locations and sizes of the entrance and exit pupils.

(d) Find the focal lengths of the two lenses and their locations. Sketch the optical system, label important features.

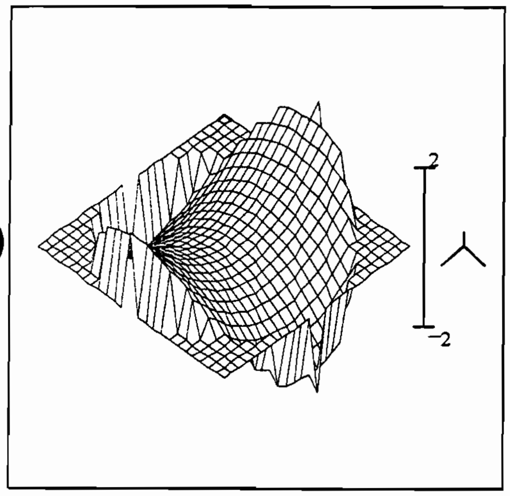
Problem 4

To third-order, the wavefront aberration function has the following form:

$$W=W_{00} + W_{20}\cdot\rho^{2} + W_{11}\cdot\rho\cdot\cos(\phi+\theta) + W_{40}\cdot\rho^{4} + W_{31}\cdot\rho^{3}\cdot\cos(\phi) + W_{22}\cdot\rho^{2}\cdot\cos^{2}(\phi).$$

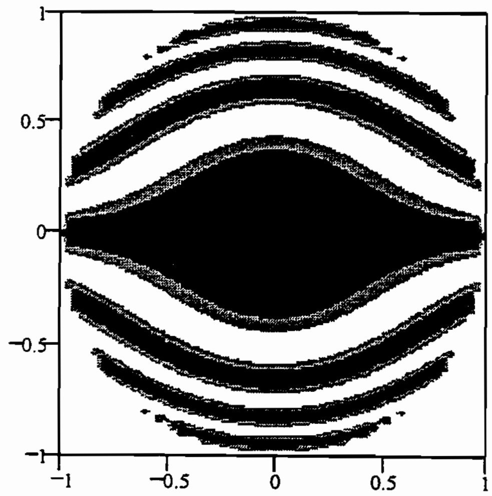
In this shorthand, the aberrations are piston W_{00}, defocus W_{20}, tilt W_{11}, spherical W_{40}, sagittal coma W_{31}, and sagittal astigmatism W_{22}. The angle of tilt, θ, is relative to the y-axis ($\theta=0°$). ρ is the normalized radial component in the pupil and ϕ is the pupil's angle coordinate relative to the y-axis. Below and on the next page are a series of profiles of third-order wavefront aberrations and interferograms of these wavefronts taken with a Mach-Zehnder interferometer. You are to match the wavefront profile with its corresponding interferogram. Tell which aberrations are present in each group and provide an estimate value for the aberration coefficients. Thoroughly explain your choices.

(A)



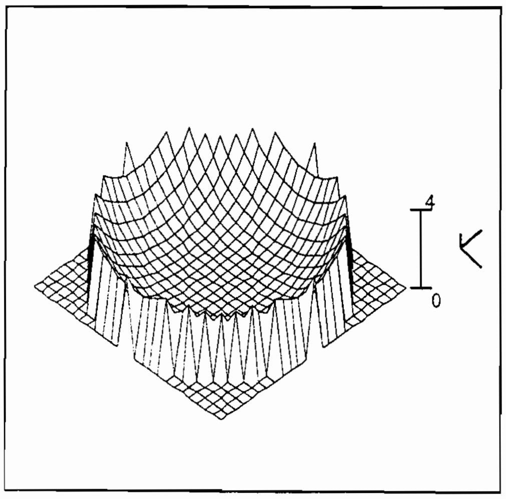
WAF

(a)



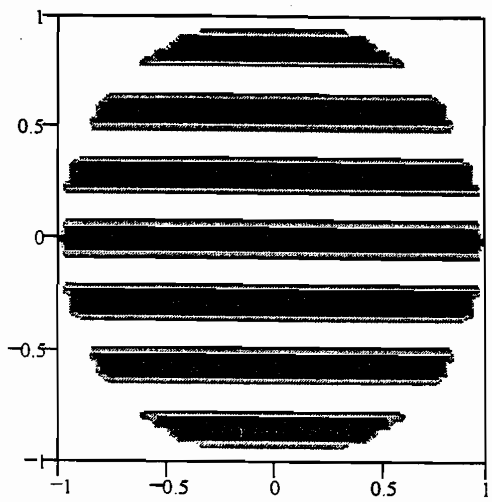
I

(B)



WAF

(b)



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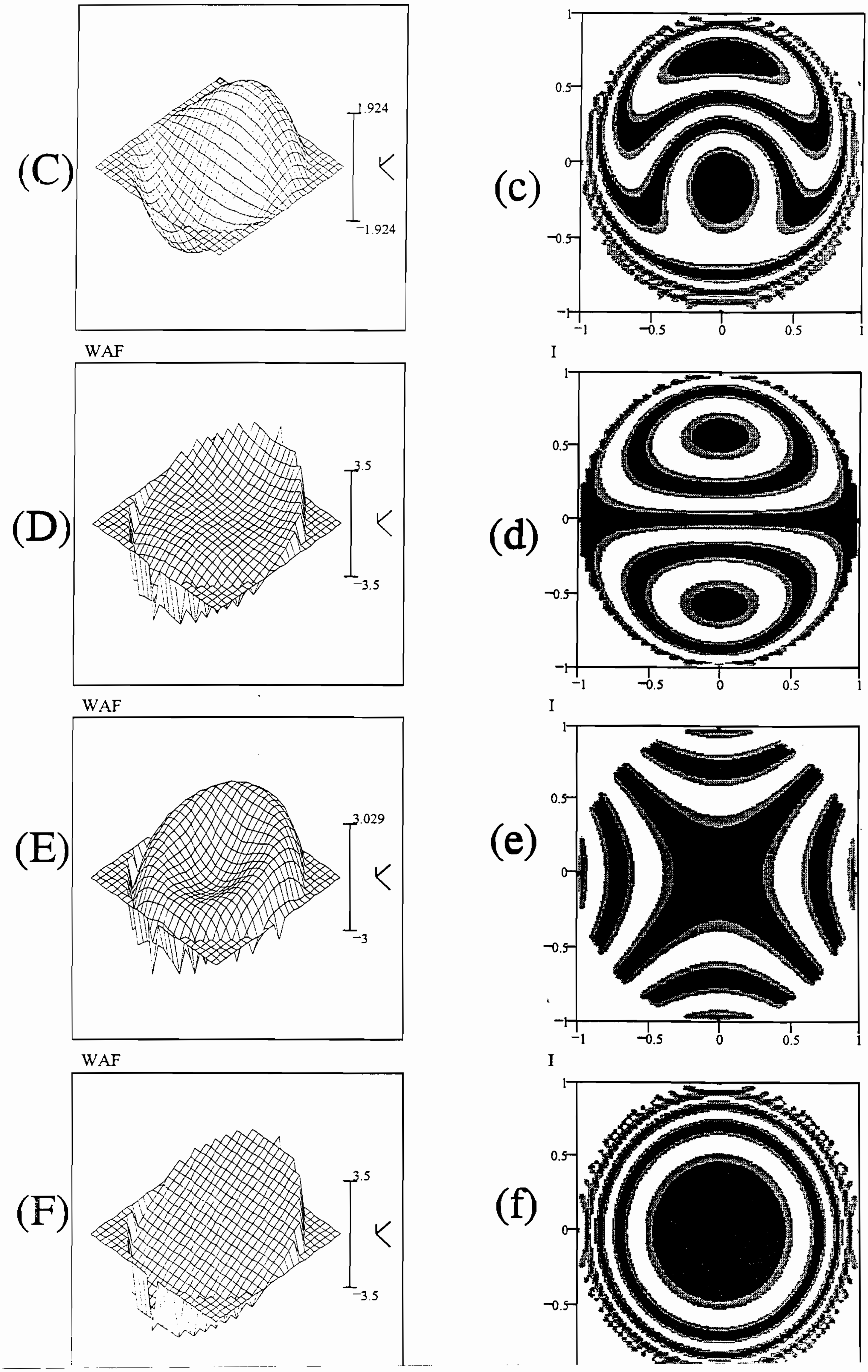

(C)
1.924
−1.924
(c)
WAF
I
(D)
3.5
−3.5
(d)
WAF
I
(E)
3.029
−3
(e)
WAF
I
(F)
3.5
−3.5
(f)
1
0.5
0
−0.5
−1