TITLE: The Fundamental Bifurcation Theorem and Darwinian Matrix Models
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Matrix models of the form $\hat{x}(t+1) = P\hat{x}(t)$, $\hat{x}(0) \geq \hat{0}$ are commonly used to described the dynamics of structured populations. If the $m \times m$ (nonnegative and irreducible) projection matrix P is constant and r is its dominant eigenvalue, then the population goes extinct if $r < 1$ and persists (and is unbounded) if $r > 1$. The only non-extinction and unbounded states occur when $r = 1$ when there is an infinite continuum of equilibria (which are neutrally stable). This Fundamental Bifurcation Theorem has been extended to nonlinear matrix models when $P = P(\hat{x}(t))$. Under general conditions there bifurcates from $\hat{x} = \hat{0}$ at $r = 1$ an unbounded continuum of non-extinction equilibria as a function of the dominant eigenvalue $r = r(\hat{0})$ of $P(\hat{0})$ (the inherent growth rate). Near the bifurcation point, these bifurcating equilibria are asymptotically stable (or unstable) depending on the direction of bifurcation. This Fundamental Bifurcation Theorem has been proved for numerous other types of nonlinear population models, including periodically forced matrix models, autonomous and periodically forced continuously structured PDE models, and continuous spatial models for discrete time structured populations (integro-difference equations). In this talk I will show two things: (1) how the Fundamental Bifurcation Theorem can be extended to Darwinian matrix models, i.e., to evolutionary game theoretic (EGT) extensions of matrix population dynamic models that account for Darwinian evolution of populations and their phenotypic traits under natural selection and (2) how the basic properties of the bifurcation can be described equivalently by means of the inherent net reproductive number $R_0 = R_0(\hat{0})$. The latter result can be an aid to the study of Darwinian matrix models since R_0 is typically more mathematical tractable than r.