Global Bifurcation in Systems of Ordinary Differential Equation

James Ward
The University of Alabama at Birmingham

Let $F \in C(\mathbb{R} \times \mathbb{R}^N \times [0,\pi], \mathbb{R}^N)$ and suppose $F(\lambda, 0, t) = 0$ for all $\lambda \in \mathbb{R}$ and $t \in [0,\pi]$. We will consider boundary value problems of the form

$$\begin{aligned} &\frac{dw}{dt} = F(\lambda, w, t),\ t \in [0,\pi] \\ &Bw = 0 \end{aligned}$$

where B is a linear boundary operator. For example, if $N = 2$, with $w = (u, v)$, then examples include $B(u,v) = (u(0), u(\pi))$, $B(u,v) = (u(0), v(\pi))$, and $B(w) = w(\pi) - w(0)$. The set $\{(\lambda, 0) : \lambda \in \mathbb{R}\}$ forms the line of trivial solutions. We will discuss the global bifurcation of nontrivial solutions from the line of trivial solutions. In doing this we will make use of the rotation number associated with a nontrivial solution. For example, if $N = 2, w(t) = (u(t), v(t))$ and $B(u,v) = (u(\pi), v(\pi)) - (u(0), v(0))$ then the rotation number of a nontrivial solution w is given by

$$Rot(w) = \frac{1}{2\pi} \int_0^\pi \frac{v'u - u'v}{u^2 + v^2}\, dt.$$

In this case $R(w)$ will be an integer. It is simply the number of times $w(t)$ wraps around the origin as t varies from 0 to π. We will emphasize two dimensional problems of the form

$$\begin{aligned} \frac{du}{dt} &= -\lambda v + f(\lambda, u, v, t) \\ \frac{dv}{dt} &= \lambda u + g(\lambda, u, v, t) \end{aligned}$$

where $f(\lambda, 0, 0, t) = g(\lambda, 0, 0, t) = 0$ and both $f(\lambda, u, v, t)$ and $g(\lambda, u, v, t)$ approach 0 faster than $|u| + |v|$ approaches 0.