Survival of Moving Polymers Among Obstacles

with Siva Athreya, Mathew Joseph

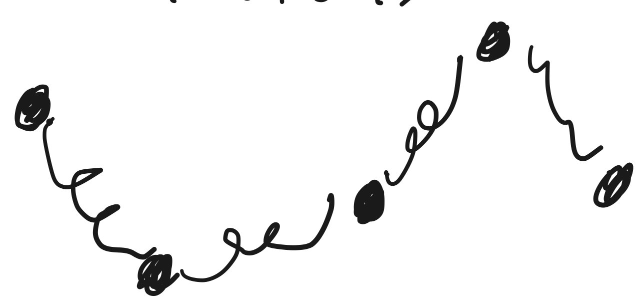
$$\partial_t u = \partial_x^2 u + \dot{W}(t,x)$$

$x \in [0, J]$ (circle)

$u \in \mathbb{R}^d$, $\dot{W} = (\dot{W}_1, \ldots, \dot{W}_d)$

Model for a moving polymer

Rouse model: (Funaki)



Newton's law $\partial_t^2 u = F$

Aristotle's law $\partial_t u = kF$

Properties of u

Tools for BM

Generator G

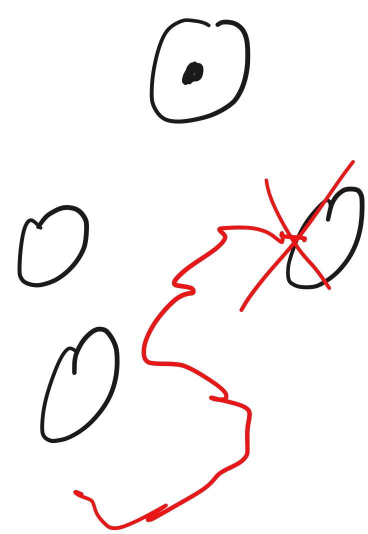
Harmonic functions

Potential theory

Much harder for u

BM among obstacles

Poisson point ensemble, intensity ν in $\mathbb{R}^d$, surround each point y by a ball $B_a(y)$



- Soft vs hard obstacles
- Condition on obstacles (Quenched)

  Or just study rate of killing overall (Annealed)

Survival prob. up to time t $\approx e^{-\beta t}$

β is different in annealed and quenched cases

History: Varadhan
Sznitman (enlargement of obstacles)

Based on eigenvalues of Δ on $\mathbb{R}^d \setminus$ obstacles

- Send a polymer through the field of obstacles
- Studying eigenvalues of generator seems too hard
- We study the annealed prob. of survival, with both hard and soft obstacles

- Focus on hard obstacles for this talk.

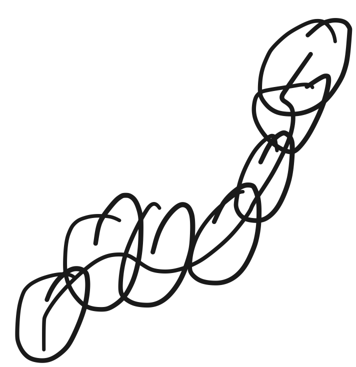
<u>Theorems</u> 1 For $a, \nu > 0$ $\exists\, C_1, \ldots, C_4$ s.t. for T large enough and $J \geq 1$, $S_t = P\left(\text{survival up to time } t\right)$

$$C_1 \exp\left(-C_2 \left(\frac{T}{J^2}\right)^{\frac{d}{d+2}}\right) \leq S_t$$

$$\leq C_3 \exp\left(-C_4 \left(\frac{T}{J^2}\right)^{\frac{d}{d+2}}\right)$$

Wiener sausage:

$$\bigcup_{0 \leq s \leq t} B_a(W_t)$$



$$\mathcal{S}_t^{(a)} = \bigcup_{0 \leq s \leq t} \; \bigcup_{x \in [0,J]} B_a(u(t,x))$$

---

u survives up to time t, iff no Poisson points in $\mathcal{S}_t$

Thm 2 Let $J = 1$. $\exists\, C > 0$ s.t.

Thm 2 Let

$\forall \varepsilon, T > 0$

$$E\left[\exp\left(-\varepsilon^d |\mathcal{S}_T(\varepsilon)|\right)\right] \quad T \to \infty$$

$$= \exp\left(-c T^{\frac{d}{d+2}} + O(1)\right)$$

Some ideas of proof of Thm 1

Previous work with Athreya, Joseph (for $\dot{W}$ replaced by $\sigma(u)\dot{W}$)

Xiao if $\sigma \equiv 1$

Small ball: let $u_0 = 0$

$$C_0 \exp\left(-C_1 \frac{TJ}{\varepsilon^6}\right) \le P\left(\sup_{\substack{0 \le t \le T \\ x \in [0,J]}} |u(t,x)| < \varepsilon\right)$$

$$\le C_2 \exp\left(-C_3 \frac{TJ}{\varepsilon^6}\right)$$

Annealed survival prob

Lower bound

$$P(\text{survival up to time } T)$$

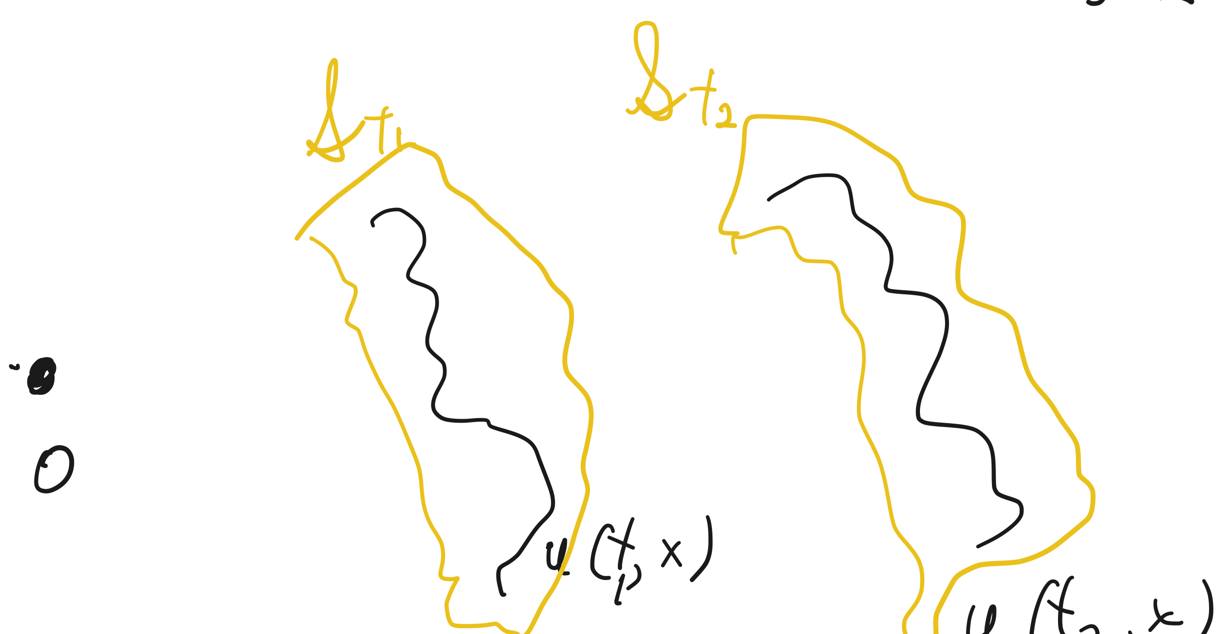
$$\geq P\left(\begin{array}{c}\text{no obstacles intersect} \\ B_R(0)\end{array}\right) \cdot P\left(\begin{array}{c}u \text{ stays in } B_R(0) \\ \text{up to time } T\end{array}\right)$$

Easy Poisson calculation

Thm 1

Upper bound on survival prob.

Range of $(u(t,x))_{\substack{0 \le t \le T \\ x \in [0,J]}}$



If we can find $t_1, \dots, t_N$ (random) such that $\mathcal{S}_{t_1}, \dots, \mathcal{S}_{t_N}$ are disjoint, and we have lower bounds on volumes of $(\mathcal{S}_{t_i})_{i=1}^{N}$

Then we can get upper bound.