# NSF-CBMS Research Conference
# Small Deviation Probabilities: Theory and Applications
# June 4-8, University of Alabama in Huntsville
# Tentative Schedule and Abstracts

## TENTATIVE SCHEDULE

All the lectures and talks will be in Room 109, Shelby Center (SC), and all the coffee breaks will be in Room 301, SC.

**Sunday, June 3**

6:30p.m.-8:00p.m., Informal Reception at Bevill Center Hotel (Cash Bar)

**Monday, June 4** (Chair: Kyle Siegrist)

8:00-8:45 Registration and Coffee

8:45-9:00 Opening, **Dr. Fix** (Dean, College of Science)

9:00-10:00 **Wenbo V. Li**, Lecture 1

10:00-10:20 Coffee Break

10:20-11:20 **Wenbo V. Li**, Lecture 2

11:20-11:30 Break

11:30-12:30 **Xia Chen** (University of Tennessee Knoxville), *Laplace asymptotics and Brownian functionals*

12:30-2:00 Lunch (Room 301, SC)

2:00-3:00 **Wenbo V. Li**, Lecture 3

3:00-3:20 Coffee Break

3:20-3:40 **Paul Jung** (University of Alabama at Birmingham), *Symmetry breaking in Quasi-1D Coulomb systems*

3:45-4:05 **Fei Xing** (University of Tennessee Knoxville), *Almost sure asymptotics for Ornstein-Uhlenbeck processes of Poisson potenial*

4:10-4:30 **Guoqing Liu** (Harbin Institute of Technology), *Semiparametric bounds on completely monotone functions*

**Tuesday, June 5** (Chair: Zhijian Wu)

9:00-10:00 **Wenbo V. Li**, Lecture 4

10:00-10:20 Coffee Break

10:20-11:20 **Wenbo V. Li**, Lecture 5

11:20-11:30 Break

11:30-12:30 **James Kuelbs** (University of Wisconsin Madison), *A CLT for empirical processes and empirical quantile processes involving time dependent data*

12:30-2:00 Lunch at Garden View Cafe (Bevill Center Hotel)

2:00-3:00 **Tai Melcher** (University of Virginia), *Second-order chaos and processes on Heisenberg-like groups*

3:00-3:20 Coffee Break

3:20-3:40 **Weijuan Chu** (Nanjing University), *Small value probabilities for continuous state branching processes with immigration*

3:45-4:05 **Sergios Agapiou** (University of Warwick), *Posterior consistency of the Bayesian approach to linear ill-posed inverse problems*

4:10-4:30 **Jebessa B. Mijena** (Auburn University)*Strong analytic solutions of fractional Cauchy problems*

**Wednesday, June 6** (Chair: Shannon Starr)

9:00-10:00 **Wenbo V. Li**, Lecture 6

10:00-10:20 Coffee Break

10:20-11:20 **Wenbo V. Li**, Lecture 7

11:20-11:30 Break

11:30-12:30 **Thomas Kühn** (Universität Leipzig), *Metric entropy in learning theory and small deviations*

12:30-2:00 Lunch (Room 301, SC)

2:00-3:00 **Frank Gao** (University of Idaho) *Interplay between convex geometry, bracketing entropy and small ball probability*

3:00-3:20 Coffee Break

3:20-3:40 **Zhijian Wu** (University of Alabama Tuscaloosa), *Most likely paths of shortfalls in certain hedging problems*

3:45-4:05 **Dawei Lu** (Dalian University of Technology), *The first exit time of a Brownian motion from the minimum and maximum parabolic domains*

4:10-4:30 **Tanja Kramm** (TU Berlin, Germany), *First passage times of Lévy processes over a one-sided moving boundary*

4:35-4:55 **Peter Pivovarov** (Texas A&M University), *Small-ball probabilities for the volume of random convex sets*

**Thursday, June 7** (Chair: Paul Jung)

9:00-10:00 **Wenbo V. Li**, Lecture 8

10:00-10:20 Coffee Break

10:20-11:20 **Wenbo V. Li**, Lecture 9

11:20-11:30 Break

11:30-12:30 **Yaozhong Hu** (University of Kansas), *Malliavin calculus and convergence in density*

12:30-2:00 Lunch at Garden View Cafe (Bevill Center Hotel)

2:00-3:00 **Hoi Nguyen** (University of Pennsylvania), *Littlewood-Offord estimates and applications to random matrix theory*

3:00-3:20 Coffee Break

3:20-3:40 **Christoph Baumgarten** (TU Berlin), *Survival probabilities of weighted random walks*

3:45-4:05 **Fangjun Xu** (University of Kansas), *Central limit theorem for an additive functional of the fractional Brownian motion*

4:10-4:30 **Ruslan Pusev** (Saint-Petersburg State University), *Exact small deviation asymptotics for some Brownian functionals*

4:35-4:55 **Dan Cheng** (Michigan State University), *Some results on the excursion probabilities of Gaussian random fields*

5:30-7:00 Conference Reception at Bevill Center Hotel (Cash Bar)

**Friday, June 8** (Chair: Dongsheng Wu)

9:00-10:00 **Wenbo V. Li**, Lecture 10

10:00-10:20 Coffee Break

10:20-11:20 **Yimin Xiao** (Michigan State University), *Small ball properties and fractal propereties of Gaussian random fields*

11:20-11:30 Break

11:30-12:30 **Michael Lacey** (Georgia Institute of Technology), *Discrepancy, small balls, and harmonic analysis*

12:30-2:00 Lunch (Room 301, SC)

2:00-3:30 **Problems and Discussion**

# ABSTRACTS

**Ten Lectures on Small Deviation Probabilities: Theorey and Applications**

*Wenbo V. Li* (University of Delaware)

**Lecture 1: Introduction, overview and applications.** We first define the small deviation (value) probability in several settings, which basically study the asymptotic rate of approaching zero for rare events that *positive* random variables take smaller values. Many applications discussed in the scientific justification section are given. Benefits and differences of various formulations of small deviation probabilities are examined in details, together with connections to related fields.

**Lecture 2: Basic estimates and equivalent transformations.** We first formulate several equivalent results for small deviation probability, including negative moments, exponential moments, Laplace transform and Taubirean theorems. The basic techniques involved are various useful inequalities, motivated from large deviation estimates. Some refinement of known results are given, including the classical Paley-Zegmund inequality. Applications to regularity and smoothness of probability laws via small deviation estimates of the determinant of Malliavin matrix are discussed in the setting of stochastic (partial) differential equations.

**Lecture 3: Techniques associated with independent variables.** We start with probabilistic arguments for algebraic properties of small deviation probability, such as independent sums and products. These estimates are non-asymptotic and hence they can be applied in the setting of conditional probability. Separate treatments are analyzed for exponential and power decay rates. A newly discover symmetrization inequality is proved by Fourier analytic method. Littlewood and Offord type problems are discussed. We end with Komlos Conjecture on balancing vectors in discrepancy theory.

**Lecture 4: Blocking techniques for the sup-norm.** We first present the vary useful blocking techniques for the maximum of the absolute value of partial sums in both upper and lower bound setting. The lower bound is more involved since the end position of each block has to be controlled also. The resulting estimates play a critical role in the Chung's type strong limit theorems for sample paths. Similar techniques are applied to weighted and/or controlled sup-norms for Brownian motion and stable processes. Applications to the two-sided exit time and Wichura type functional limit theorems are indicated.

**Lecture 5: Links between small ball probabilities and metric entropy.** For a continuous centered Gaussian process, the generating linear operator is compact and so is the unit ball of the associated reproducing kernel Hilbert space. The fundamental links between small ball probability for Gaussian measure and the metric entropy are given and various far-reaching implications are explored. Several purely probabilistic results, obtained via the analytic connection without direct probabilistic proofs, are analyzed.

**Lecture 6: Small deviation (ball) estimates for sums of correlated Gaussian elements.** We treat the sum of two *not* necessarily independent Gaussian random vectors in a separable Banach space. The main ingredients are Anderson's inequality and the weaker correlation inequality developed by the lecturer. Various applicants are provided to show the power of the method. As a direct consequence, under the sup-norm or $L_p$-norm, Brownian motion and Brownian bridge have exact the same small ball behavior at the log level, and so do Brownian sheets and various tied down Brownian sheets including Kiefer process.

**Lecture 7: More lower bound techniques.** We first establish a commonly used general lower bound estimate for the supremum of non-differentiable Gaussian process via the chaining argument, as well as improvements for smooth Gaussian processes. Then we present a connection between small ball probabilities, discovered recently, that can be used to estimate small ball probabilities under any norm via a relatively easier $L_2$-norm estimate.

**Lecture 8: More upper bound techniques.** We present three techniques: locally non-determinant method, determinant method for smooth processes and Riesz representations for Gaussian random fields. The key ideas are illustrated by several important processes: fractional Brownian motion, L-process (infinitely differentiable), and Brownian and/or Slepian sheet.

**Lecture 9: Evaluation and existence of precise constants.** Most of techniques discussed so far are for the asymptotic decay rate (up to a constant factor). Here we present a few known methods in which the exact constants can be obtained or shown to exist. In the Hilbert space $l_2$, the full asymptotic formula is developed. And with the help of a comparison result, most small deviation probabilities under the $L_2$-norm can thus be treated, and in particular when the Karhunen-Loeve expansion for a given Gaussian process can be found in some reasonable form. This is the case for Brownian motion, fractional Brownian motion and Brownian sheets, etc. A scaling argument, similar to the well known subadditive method, is established for the sup-norm of the fractional Brownian motion

**Lecture 10: Lower tail probabilities and one-sided exit asymptotics.** There are only a handful of known examples (specific Gaussian processes) for the one-sided exit asymptotics and it is intellectual challenging to work out more examples in order to find a theory. We focus on extending classical results from Brownian motion to the fractional Brownian motions. The main motivations are not only the importance of these processes, but also the need to find proofs that rely upon general principles at a more fundamental level by moving away from crucial properties (such as the Markov property) of Brownian motion. Fractional Brownian motion might not be an object of central mathematical importance but abstract principles are.

**Posterior consistency of the Bayesian approach to linear ill-posed inverse problems**, *Sergios Agapiou* (University of Warwick, UK)

**Abstract:** We consider the Bayesian approach to a family of linear inverse problems in a separable Hilbert space setting, with Gaussian prior and noise distribution. An alternative method of identifying the posterior distribution using its precision operator is presented. Working with the unbounded precision operator, enables us to use partial differential equations (PDE) methodology to study posterior consistency in a frequentist

sense and in particular to obtain rates of contraction of the posterior distribution to a Dirac measure centered on the true solution in the small noise limit. Our methods assume a relatively weak relation between the prior covariance operator, the forward operator and the noise covariance operator, more precisely, we assume that appropriate powers of these operators induce equivalent norms. We compare our results to known sharp rates of convergence in the case where the forward operator and the prior and noise covariances are all simultaneously diagonalizable, and confirm that the PDE method provides the same rates in many situations.

**Survival probabilities of weighted random walks**, *Christoph Baumgarten* (TU Berlin, Germany)

**Abstract:** I will present some results on the asymptotic behaviour of the probability that a weighted sum of centered i.i.d. random variables $X_k$ does not exceed a constant barrier, i.e.

$$\mathbb{P}\left(\sup_{n=1,\ldots,N}\sum_{k=1}^{n}\sigma(k)X_k \leq x\right), \quad x \geq 0,$$

where $\sigma$ is a positive function with $\sigma(x) \to \infty$ as $x \to \infty$. For regular random walks it is well-known that this survival probability decays like $N^{-1/2}$ as $N \to \infty$ if $X_1$ is centered and has finite variance.

First I discuss the case of a polynomial weight function and determine the rate of decay of the above probability for Gaussian $X_k$. This rate is shown to be universal over a larger class of distributions that obey suitable moment conditions.

Finally we discuss the case of an exponential weight function. The mentioned universality does not hold in this setup anymore so that the rate of decay has to be determined separately for different distributions of the $X_k$. I present some results in the Gaussian framework where the survival probability corresponds to that of a discrete Ornstein-Uhlenbeck process.

The talk is based on a joint work with Frank Aurzada (TU Berlin).

**Laplace asymptotics and Brownian functionals**, *Xia Chen* (University of Tennessee)

**Abstract:** The method of Laplace transformation is also known as Tauberian theorem or time-exponentiation, and has been applied to a variety of the problems associated to Brownian motions (and other models) such as small ball probabilities, large deviations, moment computation of local and intersection local times, and Ray-Knight theorem. In this talk, examples will be given to demonstrate how this powerful tool is used in different settings and related questions will be asked.

**Some results on the excursion probabilities of Gaussian random fields**, *Dan Cheng* (Michigan State University)

**Abstract:** In this talk, we consider the excursion probabilities of two types of Gaussian random fields:those with stationary increments and smooth sample functions, and those with anisotropic and non-smooth (or fractal) sample functions. For the first type Gaussian random fields, it is shown that the "Expected Euler Characteristic Heuristic" still

holds; and for the second type of Gaussian random fields, we prove an asymptotic result which extends those of Pickands (1969), Chan and Lai (2006).

This talk is based on joint work with Yimin Xiao.

**Small value probabilities for continuous state branching processes with immigration**, *Weijuan Chu* (Nanjing University, China)

**Abstract:** In this paper, we consider the small value probability of supercritical continuous state branching processes with immigration, $(\mathcal{Z}_t, t \geq 0)$. It is well known that under some condition on the branching mechanism and immigration mechanism, $e^{-mt}\mathcal{Z}_t$ converges to a non-degenerate finite and positive limit $\mathcal{W}$ as $t$ tends to infinity, with proper positive constant $m$. Our goal is to estimate the asymptotic behavior of $\mathbb{P}(\mathcal{W} \leq \varepsilon)$ as $\varepsilon \to 0^+$ by studying the Laplace transform of $\mathcal{W}$. We also reprove the small value probability of $\mathcal{W}$ via the prolific backbone decomposition for continuous state branching processes in the non-subordinator case.

**Interplay between convex geometry, bracketing entropy and small ball probability**, *Frank Gao* (University of Idaho)

**Abstract:** In this talk, I will present a number of examples/problems in convex geometry and in bracketing entropy where small ball probability is a powerful tool, and examples where tools from these areas are useful to estimate small ball probability.

**Malliavin calculus and convergence in density**, *Yaozhong Hu* (University of Kansas)

**Abstract:** The classical central limit theorem in probability theory states that if $X_1, \cdots, X_n$, are iid with mean $\mu$, then (under some mild conditions)

$$F_n = \sqrt{n}\left(\frac{X_1 + \cdots + X_n}{n} - \mu\right)$$

converges *in distribution* to a normal distribution. This is true for many other random variables $F_n$ such as multiple Wiener-Itô integrals. In this talk we shall discuss under what condition, $F_n$ have densities and the densities of $F_n$ converge to the normal density. More precisely, we will consider the problem of finding conditions such that there are integrable positive functions $f_n(x)$ such that

$$P(a \leq F_n \leq b) = \int_a^b f_n(x)dx$$

and

$$\lim_{n\to\infty} \int_{-\infty}^{\infty} |f_n(x) - \phi(x)|^p dx = 0$$

for all $p \geq 1$, where $\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ is the density of standard normal. The tool that we use is the Malliavin calculus.

This is an ongoing joint work with Fei Lu and David Nualart.

**Symmetry breaking in quasi-1D Coulomb systems**, *Paul Jung* (University of Alabama at Birmingham)

**Abstract:** Quasi one-dimensional systems are systems of particles in domains which are of infinite extent in one direction and of uniformly bounded size in all other directions, e.g. on a cylinder of infinite length. The main result proven here is that for such particle systems with Coulomb interactions and neutralizing background, the so-called "jellium", at any temperature and at any finite-strip width there is translation symmetry breaking. This extends the previous result on Laughlin states in thin, two-dimensional strips by Jansen, Lieb and Seiler (2009). The structural argument which is used here bypasses the question of whether the translation symmetry breaking is manifest already at the level of the one particle density function. It is akin to that employed by Aizenman and Martin (1980) for a similar statement concerning symmetry breaking at all temperatures in strictly one-dimensional Coulomb systems. The extension is enabled through bounds which establish tightness of finite-volume charge fluctuations.

**First passage times of Lévy processes over a one-sided moving boundary**, *Tanja Kramm* (TU Berlin, Germany)

**Abstract:** In this talk we discuss the one-sided exit problem with a moving boundary for Lévy processes. The main focus of this talk concerns the question: For which functions $f$ does the one-sided exit problem with a constant boundary, i.e.

$$\mathbb{P}\left(X(t) \leq 1,\ t \leq T\right) = T^{-\delta+o(1)}, \quad \text{as } T \to \infty,$$

with some $\delta \geq 0$ imply

$$\mathbb{P}\left(X(t) \leq 1 \pm f(t),\ t \leq T\right) = T^{-\delta+o(1)}, \quad \text{as } T \to \infty\ ?$$

Our main result states that if the boundary $f$ behaves asymptotically as $t^\gamma$ for some $\gamma < 1/2$ then the probability that the process stays below the boundary behaves as in the case with a constant boundary. Both positive and negative boundaries are considered.

After presenting our main results we compare it to previously known ones and briefly sketch the main idea of the proof.

The talk is based on joint work with Frank Aurzada (Berlin) and Mladen Savov (Oxford).

**A CLT for empirical processes and empirical quantile processes involving time dependent data**, *James Kuelbs* (University of Wisconsin-Madison)

**Abstract:** We establish empirical quantile process CLTś based on $n$ independent copies of a stochastic process $\{X_t : t \in E\}$ that are uniform in $t \in E$ and quantile levels $\alpha \in I$, where $I$ is a closed sub-interval of $(0,1)$. Typically $E = [0,T]$, or a finite product of such intervals. Also included are CLTś for the empirical process based on$\{I_{X_t \leq y} - \mathbb{P}(X_t \leq y) : t \in E, y \in \mathbb{R}\}$ that are uniform in $t \in E, y \in \mathbb{R}$. The process $\{X_t : t \in E\}$ may be chosen from a broad collection of Gaussian processes, compound Poisson processes, stationary independent increment stable processes, and martingales.

**Metric entropy in learning theory and small deviations**, *Thomas Kühn* (Universität Leipzig, Germany)

**Abstract:** In the first part of the talk I will give a short introduction into learning theory, in order to show the importance of metric entropy in this very active field. A particular problem - which is related, e.g., to support vector machines - consists in finding good upper bounds for entropy numbers (or covering numbers) in reproducing kernel Hilbert spaces. In the second part I will determine the exact asymptotic behaviour of covering numbers in Gaussian RKHSs. On one hand, this improves earlier results by Ding-Xuan Zhou, and on the other hand it has an interpretation in terms of small deviations of certain smooth Gaussian processes. This part is based on my paper "Covering numbers in Gaussian reproducing kernel Hilbert spaces" *J. Complexity* **27** (2011), 489–499.

**Discrepancy, small balls, and harmonic analysis**, *Michael Lacey* (Georgia Institute of Technology)

**Abstract:** We will survey the remarkably close connection with small ball problems in probability theory, and the classical bounds associated with the Discrepancy function. Tools to analysize these questions arise from Harmonic Analysis. But don't seem strong enough to complete the proofs of outstanding conjectures in the subject.

**Semiparametric bounds on completely monotone functions**, *Guoqing Liu* (Harbin Institute of Technology, China)

**Abstract:** Given any random variable $S \geq 0$ and a completely monotone function $f(s)$, various bounds are derived on the mean and variance of $f(S)$. The techniques are based on domination by exponential functions, Cauchy-Schwarz inequality and symmetrization method for variance.

**The first exit time of a Brownian motion from the minimum and maximum parabolic domains**, *Dawei Lu* (Dalian University of Technology, China)

**Abstract:** Consider a Brownian motion starting at an interior point of the minimum or maximum parabolic domains, namely, $D_{\min} = \{(x, y_1, y_2) : \|x\| < \min\{(y_1+1)^{1/p_1}, (y_2+1)^{1/p_2}\}\}$ and $D_{\max} = \{(x, y_1, y_2) : \|x\| < \max\{(y_1+1)^{1/p_1}, (y_2+1)^{1/p_2}\}\}$ in $\mathbb{R}^{d+2}$ $(d \geq 1)$, respectively, where $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^d$, $y_1$, $y_2 \geq -1$, and $p_1$, $p_2 > 1$. Let $\tau(D_{\min})$ and $\tau(D_{\max})$ denote the first times the Brownian motion exits from $D_{\min}$ and $D_{\max}$. Estimates with exact constants for the asymptotics of $\log \mathbb{P}(\tau(D_{\min}) > t)$ and $\log \mathbb{P}(\tau(D_{\max}) > t)$ are given as $t \to \infty$, depending on the relationship between $p_1$ and $p_2$, respectively. The proof methods are based on Gordon's inequality and early works of Li, Lifshits, and Shi in the single general parabolic domain case.

**Second-order chaos and processes on Heisenberg-like groups**, *Tai Melcher* (University of Virginia)

**Abstract:** Smoothness properties of measures in infinite-dimensional spaces, in particular the laws of Brownian motions in these spaces, have been the subject of much research in various settings, including certain curved examples. We will consider the setting of infinite-dimensional "Heisenberg-like" groups. The Brownian motions in this case may be realized as a flat infinite-dimensional Brownian motion along with its second-order chaos. We will discuss recent smoothness results for the law of these processes; reverse log Sobolev inequalities play a critical role in the proof of these results. I will provide all basic definitions as well as some background to put these results in context. This is joint work with F. Baudoin and M. Gordina.

**Strong analytic solutions of fractional Cauchy problems**, *Jebessa B. Mijena* (Auburn University)

**Abstract:** Fractional derivatives can be used to model time delays in a diffusion process. When the order of the fractional derivative is distributed over the unit interval, it is useful for modeling a mixture of delay sources. In some special cases distributed order derivative can be used to model ultra-slow diffusion. We extend the results of Baeumer and Meerschaert in the single order fractional derivative case to distributed order fractional derivative case. In particular, we develop the strong analytic solutions of distributed order fractional Cauchy problems.

**Littlewood-Offord estimates and applications to random matrix theory**, *Hoi Nguyen* (University of Pennsylvania)

**Abstract:** In the first half of the talk I will introduce several versions of the Erdös and Littlewood-Offord inequality. In the second half I will then present an application to bound the least singular value and to establish the circular law in random matrix theory.

**Small-ball probabilities for the volume of random convex sets**, *Peter Pivovarov* (Texas A&M University)

**Abstract:** The focus of the talk will be distributional inequalities for the volume of random convex sets. Typical models include convex hulls and Minkowski sums of line segments generated by independent random points. I will outline an approach to small-deviation estimates that makes use of rearrangement inequalities and tools from classical convexity such as intrinsic volumes and natural generalizations.

This is joint work with Grigoris Paouris.

**Exact small deviation asymptotics for some Brownian functionals**, *Ruslan Pusev* (Saint-Petersburg State University, Russia)

**Abstract:** We find exact small deviation asymptotics with respect to weighted $L_2$-norm for a rather wide class of Gaussian processes. Our approach does not require the knowledge of eigenfunctions of the covariance operator of a weighted process. Such a peculiarity of the method makes it possible to generalize many previous results in this area. We also obtain new relations connected to exact small deviation asymptotics for a Brownian excursion, a Brownian meander, and Bessel processes and bridges.

**Most likely paths of shortfalls in certain hedging problems**, *Zhijian Wu* (University of Alabama Tuscaloosa)

**Abstract:** With or without the constraint of the terminal risk, an optimal strategy to minimize the running risk in hedging a long-term commitment with short-term futures can be solved explicitly if the underline stock follows the simple stochastic differential equation

$$dS_t = \mu dt + \sigma dB_t$$

where $B_t$ is the standard Brownian motion. In this talk, the most likely paths of shortfalls associated with the hedging are discussed. We typically focus on the shortfalls corresponding to the optimal strategies established to minimize the running risk with or without the terminal constraint. These paths give information about how risky events occur instead of just their probability of occurrence.

**Almost sure asymptotics for Ornstein-Uhlenbeck processes of Poisson potenial**, *Fei Xing* (University of Tennessee Knoxville)

**Abstract:** The objective of this paper is to study the long time behavior of the following exponential moment:

$$\mathbb{E}_x \exp\left\{\pm \int_0^t V(X(s))\, ds\right\}$$

where $\{X(s)\}$ is a $d-$dimensional Ornstein-Uhlenbeck process and $\{V(x)\}_{x\in\mathbb{R}^d}$ is a homogeneous ergodic random field which will be defined in the paper. It turns out that the positive /negative exponential moment has $e^{ct}$ growth/decay rate, which is different from the Brownian motion model studied by Carmona and Molchanov (1995) for positive exponential moment and Sznitman (1993) for negative exponential moment.

**Small ball properties and fractal propereties of Gaussian random fields**, *Yimin Xiao* (Michigan State University)

**Abstract:** Small ball probabilities are very important for investigating fine structires of the sample functions of Gaussian random fields. In this talk we present applications of small ball probabilities in establishing results on exact Hausdorff measure functions, exact packing measure functions and multifractal analysis for Gaussian random fields.

**Central limit theorem for an additive functional of the fractional Brownian motion**, *Fangjun Xu* (University of Kansas)

**Abstract:** We prove a central limit theorem for an additive functional of the $d$-dimensional fractional Brownian motion with Hurst index $H \in (\frac{1}{1+d}, \frac{1}{d})$, using the method of moments, extending the result by Papanicolaou, Stroock and Varadhan in the case of the standard Brownian motion.

**Comparison results for large deviations of random series**, *Xiangfeng Yang* (Hubei University of Arts and Science, China)

**Abstract:** Let $\{\xi_n\}$ be a sequence of independent and identically distributed random variables whose common distribution satisfies $\lim_{u\to\infty} u^{-p} \log \mathbb{P}\{|\xi_1| \geq u\} = -c$ for some constants $p \geq 1$ and $0 < c < \infty$. The following comparison

$$\frac{\log \mathbb{P}\{\sum_{n=1}^{\infty} a_n|\xi_n| \geq r||a||_q\}}{\log \mathbb{P}\{\sum_{n=1}^{\infty} b_n|\xi_n| \geq r||b||_q\}} \sim 1 \qquad \text{as } r \to \infty$$

is proved with $||a||_q = (\sum_{n=1}^{\infty} a_n^q)^{1/q}$ and $\frac{1}{p} + \frac{1}{q} = 1$ under suitable conditions on $\{a_n\}$ and $\{b_n\}$. Exact comparison results without logarithm are also obtained in the form

$$\frac{\mathbb{P}\{\sum_{n=1}^{\infty} a_n|\xi_n| \geq r||a||_2\beta + |\alpha| \sum_{n=1}^{\infty} a_n\}}{\mathbb{P}\{\sum_{n=1}^{\infty} b_n|\xi_n| \geq r||b||_2\beta + |\alpha| \sum_{n=1}^{\infty} b_n\}} \sim 1 \qquad \text{as } r \to \infty$$

for $\{\xi_n\}$ being Gaussian random variables $N(\alpha, \beta^2)$ under more restrictive conditions on $\{a_n\}$ and $\{b_n\}$. For several non-Gaussian random variables, exact comparisons are derived as well. We believe that, after suitable scaling the comparisons for large deviations are always equivalent, which forms a conjecture by us.