UNIVERSITY OF ALABAMA IN HUNTSVILLE
Mathematical Sciences Colloquium

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Classification Theorems and Blow-up Analysis for Solutions to Critically Nonlinear Elliptic Equations

DATE: Friday, September 5, 2014

TIME: 3:00 p.m. – 4:00 p.m.

PLACE: Shelby Center Room 218

For $n \geq 3$, variants of the semilinear elliptic equation

$$\Delta u + u^{(n+2)/(n-2)} = 0 \qquad \text{in } \Omega \subset \mathbb{R}^n, \tag{1}$$

arise in conformal geometry in relation to so-called *prescribing scalar curvature* problems. Perhaps the most well-known example of this kind of problem is the Yamabe problem. The Yamabe problem seeks to prove the following conjecture 'For any compact Riemannian manifold (M^n, g) there exists a metric $\hat{g}$ conformal to g such that the scalar curvature of $\hat{g}$ is constant'. Proving existence of such a metric $\hat{g}$ is equivalent to proving the existence of a solution to a certain variant of equation (1). Through the works of Yamabe, Trudinger, Aubin and Schoen, and over the course of more than twenty years, the Yamabe conjecture was proven affirmative.

One of the interesting aspects of equation (1) is the exponent $(n+2)/(n-2)$. If this exponent were replaced with a new exponent $1 < p < (n+2)/(n-2)$, then one could use standard variational theory to prove the existence of a solution. However, for the particular nonlinearity shown in equation (1), the standard variational theory does not apply. In order to understand equation (1) thoroughly, one must understand the dynamics of blow-up solutions. In this talk, some aspects of blow-up analysis including classification theorems and Harnack-type inequalities will be discussed for variations of equation (1).

Refreshments will be served at 2:30 p.m. in SC 201 suite landing..